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Cartel dynamics and Leniency policy: Self-reporting to start over with a clean slate

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Abstract

This paper develops a dynamic discrete-time model of collusive behaviour in which firms can apply for leniency to reduce fines. We propose a sequential-move game inspired by the centipede game, capturing firms' incentives to be the first to self-report a cartel. The model examines cartel formation, stability, and recidivism, assuming that fines apply to the undiscovered record of collusion, not just current conduct. We find that when collusion is attractive but the leniency programme is not sufficiently generous, firms form a single cartel without self-reporting. However, when collusion is highly attractive and the leniency programme sufficiently generous, it can destabilize cartels but also foster recidivism: firms use leniency to "clean the slate" and restart collusion at a lower expected cost. This equilibrium behaviour may help explain the empirically observed prevalence of short-lived cartels and repeat offenders under existing leniency regimes.

Keywords: Antitrust; Cartels; Recidivism; Leniency; Dynamic Games

JEL CODES: D43; K21; K42; L40

1 Introduction

Cartels are considered the most serious and harmful form of anticompetitive behaviour among competitors. Competition authorities (CAs) use different tools to detect and combat them, with Leniency Programs (LPs) demonstrating notable success in recent years. These programs reward cartel members who voluntarily disclose their involvement and contribute to dismantling them, usually by offering fine reductions or even complete immunity.

Existing related literature[1] suggest that LPs make it more difficult for firms to sustain cartel behaviour by altering their incentives to collude, increasing the payoff from deviating and reducing collusion benefits by fostering competition to report first (see, e.g., Harrington, 2008). Nevertheless, some studies also suggest that LPs may make collusion more attractive by increasing the expected benefits from colluding, as firms can strategically use leniency applications to reduce future sanctions. This line of research

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[1]See Marvão and Spagnolo (2018) for a comprehensive review of the literature on corporate leniency programmes.

points to the risk that overly generous LPs may be exploited by cartels, raising concerns about possible perverse effects.[2] In such cases, firms may adopt repeated collude-and-report strategies (Chen and Rey, 2013; Spagnolo, 2004), as collusion becomes less costly for those that choose to self-report.

In this work, we develop a finite-horizon dynamic model in discrete-time to examine cartel formation, stability, and recidivism within a duopoly under different antitrust policies. Our model analyses firms' strategic decisions, focusing on the influence of two detection tools available to the CA: random investigations and a pre-investigation LP. Firms sequentially decide whether to collude for supra-competitive profits and whether to apply for leniency, assuming that the reduction for the first applicant (which may include immunity) is greater than that for the second. This setup allows us to study equilibrium strategies, the timing and duration of cartel formation, and how varying degrees of leniency influence firms' behaviour.

We analyse how firms' strategic behaviour adjusts to different anti-cartel policy parameters, delving into the underlying reasons for the potential perverse effects of leniency policies. Our finite-horizon approach provides a natural framework to explore strategic decisions over bounded time spans, including recidivism and the timing of cartel dissolution. Shifting from the standard framework of infinitely repeated games of tacit collusion—commonly used to analyse the effectiveness and optimal design of LPs—we are able to capture key dynamics that are overlooked in traditional models. This, in turn, allows us to derive policy recommendations depending on the enforcement objectives.

Building on previous theoretical literature on this topic, our approach is innovative in several ways. Firstly, most studies model cartel fines as exogenously fixed (see, e.g., Chen and Rey, 2013; Gärtner, 2022; Harrington, 2008, 2013; Motta and Polo, 2003; Spagnolo, 2004) or dependent solely on the degree of collusion in the period under consideration, represented by the chosen price (level of profits) (Houba et al., 2010) or the number of markets in which collusion occurs (Emons, 2020), without accounting for past collusion. Instead, our model ties penalties to the duration of collusion, introducing a state variable that records the total number of periods with collusive profits, as Harrington and Chang (2009) suggests.[3]

Additionally, we make cumulative collusive profits punishable, even if firms suspend cartel activity and return to competition. Undetected cartels (i.e. undiscovered non-active cartels) remain liable for fines based on the total accumulated profits from collusion up to the time of detection. We believe this approach better captures how collusive activities are penalized under competition law in real-world contexts.

Thirdly, in our model, firms decide each period whether to collude, accounting for the possibility that, once a cartel is broken down -either internally by its members or externally by a CA investigation- the dynamics of the equilibrium strategies reveal whether the firms may form a new cartel. In contrast, other studies assume exogenously that the cartel either immediately re-establishes itself or never reforms. Thus, our approach provides a closer representation of how cartels operate and helps to explain behaviours observed in actual cartelised industries.

Our findings show that, depending on the design of the anticartel policy and the tools available to the CA (such as the leniency scheme and the probability of cartel detection), firms may fall into three different equilibrium branches: (i) never colluding, (ii) colluding but not disclosing the collusive activities, and (iii) colluding and self-reporting (systematically).

[2] Isogai and Shen (2023) also highlights a perverse effect of LPs, where a multiproduct firm uses leniency applications to build a reputation for toughness, stabilising collusive agreements by leveraging the threat of reporting the cartel.

[3] Harrington and Chang (2009) do not include a state variable in their model due to the significant complication it introduces. Instead, they assume that the fine is proportional to the average increase in cartel profits, allowing the penalty to be sensitive to the average duration of collusion.

Specifically, when the benefits of collusion do not outweigh the expected fine from detection by the CA, i.e., when collusion is not sufficiently attractive, firms will opt for the first equilibrium branch, which is to always remain in the competitive environment. This result highlights that in certain industries, a policy that increases the likelihood of detecting cartels through "*ex-officio*" investigations, coupled with stricter sanctions, would be sufficient to deter cartel formation. However, when the benefits of collusion exceed the expected fine, i.e., when collusion is sufficiently attractive, two equilibrium strategies emerge, depending on how attractive firms find applying for leniency.

When the penalty for the first leniency applicant, after the fine reduction, remains higher than the expected penalty for detection without self-reporting, firms will opt for the second equilibrium branch and collude without applying for leniency. Firms maximise their expected profits by forming a unique cartel, the duration of which depends on the expected fine for detection by the CA. For a sufficiently large but finite interaction horizon, firms aiming to minimise exposure to CA investigations will initially remain competitive, forming a cartel from a certain point until the end of their lifespan. Tf the authority were to discover the cartel before it concluded, the firms would reform it (resulting in a subsequent cartel episode), leading to a situation of recidivism.

If collusion is attractive and the LP is generous, firms adopt a third equilibrium: they collude for a predetermined number of periods, apply for leniency to facilitate their conviction, and subsequently restart collusion. Firms leverage the LP to secure a clean slate and initiate a new episode of collusion, protected by the *non bis in idem* principle.[4] This result of consecutive unstable cartels (recidivism) in equilibrium is consistent with prior findings on the perverse effects of the LP, as Marvão (2023), which finds a significant proportion of repeat offenders in convicted cartels.

The incentives to apply for leniency in this equilibrium depend on the difference in fine reductions between the first and the second applicant. When this difference is small, firms are less inclined to report early. Instead, they tend to continue colluding until the expected cost of detection outweighs the gains from collusion. In this case, the small disparity in benefits discourages early defection and avoids the destabilising effect of mutual distrust (unravelling process), leaving only the "prosecution effect"-the risk of external detection-to drive firms' behaviour. As a result, the higher the detection probability and the greater the penalty gap between applying for leniency and being detected without applying, the shorter the cartel's expected duration (if not detected early).

When the disparity in fine reductions between the first and second applicants is large, sequential leniency applications create a dynamic akin to the centipede game (Aumann, 1995), where firms are incentivised to self-report first (the pre-emption effect). This can lead to full unravelling, as in Gärtner (2022), or partial unravelling if the disparity is smaller. The pre-emption effect shortens collusion but does not eliminate it. Cartel duration depends on the first applicant's advantage: a larger advantage results in shorter successive cartels, while a smaller one allows longer successive ones.

While in the long run the discovery of short-lived cartels aligns with the effectiveness of LPs in destabilising cartels, as suggested by Harrington and Chang (2009) and empirically supported by Borrell et al. (2024), our findings indicate that the short duration of these cartels would primarily be a consequence of the LP design. We further argue that this form of effectiveness (reducing cartel duration) does not necessarily imply that LPs deter cartel formation. In fact, the presence of cartel recidivism suggests that their deterrent effect on the initial decision to collude may be limited. Moreover, a lot of short cartels can be more harmful for the economic welfare than a single long one.

The rest of this paper is organised as follows: Section 2 introduces the model, which is solved in

[4] The use of leniency applications to "clean the slate" by removing past cartel activities has been recognized in practice by real-world actors, as highlighted by Jaspers (2020). This practice is seen in contexts such as mergers, acquisitions, or changes in management, where firms use leniency applications to mitigate future liability risks.

Section 3. In Section 4 we derive the policy implications of different configurations (parameter values) of the model. Section 5 wraps up the main conclusions.

2 The Model

In this section, we are going to present a model with two firms deciding whether to reach a collusion agreement or not, allowing them to make higher profits if they do so. These collusive profits, however, will be accumulated over time and can be subject to a penalization if a CA detects these practices. Besides this monitoring mechanism, firms may also apply for the benefits of a LP by self-reporting their collusive activities, provided that the cartel has not been previously detected by the CA's own initiative (pre-leniency investigation). Leniency application would enable them to pay potentially lower fines than in the case without leniency, including the possibility of not paying any fine at all.

Let us consider $n = 2$ risk-neutral agents (firms), $i = \{A, B\}$, interacting for a finite number of periods, T. Despite an infinite horizon is relatively standard in the literature, firms usually only consider short time spans in their decision-making processes. For instance, it is common to observe how CEOs try to maximise their short-term profits without caring about future consequences. These myopic incentives are even stronger if the CEO does not expect to run the firm for a long period of time and hence, may only care about the profits that the firm makes during his/her leadership. In terms of the game, this implies a short T horizon.

Firm A (she) and Firm B (he) will sequentially decide whether to collude or to behave competitively.[5] Choosing to behave competitively provides firms profits of $c_i > 0\ \forall i$, whereas colluding allows firms to obtain higher profits of $m_i \geq c_i > 0\ \forall i$. We assume that firms are equal in size and structure so that both can reap the same profits from competition ($c_A = c_B = c$) and from collusion ($m_A = m_B = m$). This allows us to simplify the notation.[6]

Definition 1 (λ)**.** *We define λ as the ratio of competitive profits over the collusive ones: $\lambda = \frac{c}{m}$, where $0 < \lambda \leq 1$.*

Therefore, λ measures how profitable collusive agreements are. More specifically, $1 - \lambda$ is the (relative) increase in profits that each firm obtains within the cartel. As all results will depend only on this ratio, from now on, we will normalize m to 1.

If firms engage in collusion and successfully form a cartel, they have the chance of reporting the cartel and cleaning up their collusive records by using the leniency mechanism.[7] We assume that firms sequentially decide whether or not to disclose the cartel, therefore, in our model as in Motchenkova (2004) there is a first mover or leader (Firm A) and a second mover or follower (Firm B). Additionally, if neither of the firms reports, we assume that the CA detects and convicts the cartel with probability $0 < p < 1$. This reflects the probability of success of the traditional *ex-officio* investigation mechanisms generally used by CAs as a complement to the LP.

An essential characteristic of our model is that collusive profits are accumulated over time and in case a cartel is convicted, its members will pay proportionally to the collusive profits obtained until that

[5] In this paper, we focus on the dynamic aspects of cartel formation and stability. Consequently, instead of considering a price-setting mechanism as many other studies on cartels, firms will decide whether to collude or not.

[6] Without loss of generality, asymmetry in payoffs can be easily considered, given that differences in size are not going to influence the strategic environment of the game. This kind of asymmetry will only be relevant for the model if the relative profits of collusion (λ as defined below) differ across firms.

[7] As in real-life cartels, CAs usually require firms to have stopped colluding when applying for leniency.

period. To capture this, we create a state variable τ that registers the number of sanctionable periods, as suggested by Harrington and Chang (2009).

Definition 2 (τ). *Let us denote by τ_t the number of previous (undiscovered) periods of collusion at the beginning of period t.*

It is important to point out that these periods must not necessarily be consecutive. For instance, firms can collude for a certain number of periods, stop colluding and form a second cartel afterwards. As long as there is no self-reporting and the CA has not detected the collusive behaviour, the firms' τ_t will be equal to the total number of collusion periods. However, if the cartel is exogenously dismantled by any CA's means in period t, the value of τ_{t+1} will reset to $\tau_{t+1} = 0$.

The amount of sanctionable profits, $P_{tA} = P_{tB} = P_t$, depends on this state variable τ_t and the size of collusive profits, m. Given that collusive profits m have been normalized to 1, $P_t = 1 \cdot \tau_t$, if there has been no collusion during period t and $P_t = 1 \cdot (\tau_t + 1)$ if there has.

Assumption 1. *We assume that firms start the game without having colluded before, that is $\tau_1 = 0$.*

Definition 3 (γ_k). *We denote as γ_k, where $k = \{s, ss, n, f\}$, the fine rate paid by cartel members. Which fine rate is actually paid will depend on the circumstances under which the cartel has been uncovered, such that:*

- *γ_s is the fine rate paid by the first firm who reports collusion, in case at least one of the firms applies for leniency.*
- *γ_{ss} is the fine rate paid by the second firm who reports collusion, in case both firms apply for leniency.*
- *γ_n is the fine rate paid by a firm when it does not report collusion but the other firm does, in case just one of the firms applies for leniency.*
- *γ_f is the fine rate paid by any firm that colluded and is discovered by the CA, in case nobody applies for leniency.*

Assumption 2. *We assume the following order of the different fine rates:* $0 \leq \gamma_s \leq \gamma_{ss} \leq \gamma_f \leq \gamma_n \leq 1$

Notice that we are assuming that the worst possible case for a firm is to be betrayed by the other one, $\gamma_f \leq \gamma_n$, that is, not applying for leniency while the other one does (as the CA will have a strong evidence against firms). If only one of the two firms reports, that firm pays a lower fine than in the case neither firm reports and they were caught and convicted by the CA, $\gamma_s \leq \gamma_f$. If both firms report, there is an advantage for the first reporter, $\gamma_s \leq \gamma_{ss}$, but the fine paid in case of being the second reporter is still lower than the fine paid in case of being discovered by the CA in absence of reporting, $\gamma_{ss} \leq \gamma_f$.

Remark. Some CAs, like the European Commission (EC) and those operating in most EU countries, offer a particular LP where first self-reporting firms receive full immunity for disclosing their existing cartels. These are called Full Leniency Programmes. In our model this would be a particular case characterized by $\gamma_s = 0$. Any other case ($\gamma_s > 0$), will be equivalent to a Partial Leniency Programme.

The specific timing of the game in each period is as follows (see also Figure 1):

Stage 1 - *Collusive Agreement*

- Firm A decides whether to enter a collusive agreement with Firm B, E, or not enter, NE.

- After observing Firm A's decision, Firm B decides whether to enter a collusive agreement with Firm A, E, or not enter, NE.

Both firms observe the decisions made in this stage. Only if *both* firms choose E, there will be collusion and firms will accumulate $m = 1$ to the amount of sanctionable profits and will move onto Stage 3. Otherwise, firms are unable to reach a collusive agreement and each one earns $c = \lambda$ in this period and move onto Stage 2.[8]

Stage 2 - *Reporting*

This stage is reached if at least one of the two firms has decided not to enter the collusive agreement. The firm/firms that have chosen NE in the first stage, can now report the anticompetitive behaviour.

- If Firm A has chosen NE in Stage 1, she will decide whether to report anticompetitive behaviour R, or not report, NR. If, instead, she has chosen E in Stage 1, she has no action to take in Stage 2.
- Firm B observes Firm A' action (or lack of action). If he had chosen NE in Stage 1, he will decide whether to report, R, or not report, NR. If he had chosen E, he has no action to take in Stage 2.

Both firms observe the decisions made in this stage. Notice that for a firm to have a chance to report a given cartel, he or she has to defect from the collusive agreement, i.e. only firms choosing NE in Stage 1 can self-report. If any firm self-reports, the cartel is dismantled and payoffs are realised according to the following scheme:

- If both Firms A and B report: $\begin{cases} \pi_t^A = \lambda + (1-\gamma_s)\tau_t \\ \pi_t^B = \lambda + (1-\gamma_{ss})\tau_t \end{cases}$
- If only Firm A reports: $\begin{cases} \pi_t^A = \lambda + (1-\gamma_s)\tau_t \\ \pi_t^B = \lambda + (1-\gamma_n)\tau_t \end{cases}$
- If only Firm B reports: $\begin{cases} \pi_t^A = \lambda + (1-\gamma_n)\tau_t \\ \pi_t^B = \lambda + (1-\gamma_s)\tau_t \end{cases}$

In all these cases, where at least one firm reports the cartel, firms' collusive records are cleaned up ($\tau_{t+1} = 0$), and the period ends. Instead, if neither firm self-reports, firms will move onto Stage 3.

Remark. Notice that if none of the firms decide to enter the collusive agreement there is an intrinsic first-mover advantage of Firm A over Firm B in terms of the opportunity to report previous cartel conduct. This could, for instance, be explained by Firm A's better understanding of the leniency procedure. This is the sole asymmetry between firms that we assume in our model and, as we will see later, it will generate unravelling in the leniency–seeking process.

Stage 3 - *Competition Authority*

After the opportunity of self-reporting, the CA can randomly carry out an investigation process.[9] With probability p, the CA discovers and convicts the cartel and both firms have to pay a fine rate, γ_f,

[8] We make this stage sequential as to give Firm A a first-mover advantage that we will also give in Stage 2.

[9] We assume that the CA does not investigate cartels that have already been self-reported, as self-reporting leads directly to conviction without the need for a post-leniency investigation.

over the amount of sanctionable profits, P_t.[10] Additionally, they clear their collusive records for the next period, such that $\tau_{t+1} = 0$. If the cartel is not discovered, which happens with a probability $1-p$, firms earn the corresponding profits for that period, free of any penalty. They also retain their cartel status, which is $\tau_{t+1} = \tau_t + 1$ if there has been collusion in the current period and $\tau_{t+1} = \tau_t$ otherwise.

Therefore, expected profits of Firm $i = \{A, B\}$ at the beginning of this stage are:

$$\pi_t^i = \begin{cases} p\big[(1-\gamma_f)(\tau_t+1)\big] + (1-p) & \text{if} \quad \{E,E\} \\ p[\lambda + (1-\gamma_f)\tau_t] + (1-p)\lambda & \text{otherwise} \end{cases}$$

For tractability, we are assuming that collusive profits add to P_t every time there is collusion but are not collected by the firms until the cartel is dismantled by their own or by the CA.[11] For this reason, if firms collude in a specific period and are not caught, they do not directly receive any profit in that period, but enlarge P_t. Nevertheless, if collusion is broken down by any of the two firms, i.e. at least one of them decides not to enter the collusion agreement but do not report it, they will receive the competitive profits of λ in that period.

In any case, the period ends at this point. Figure 1 shows the different stages explained above.

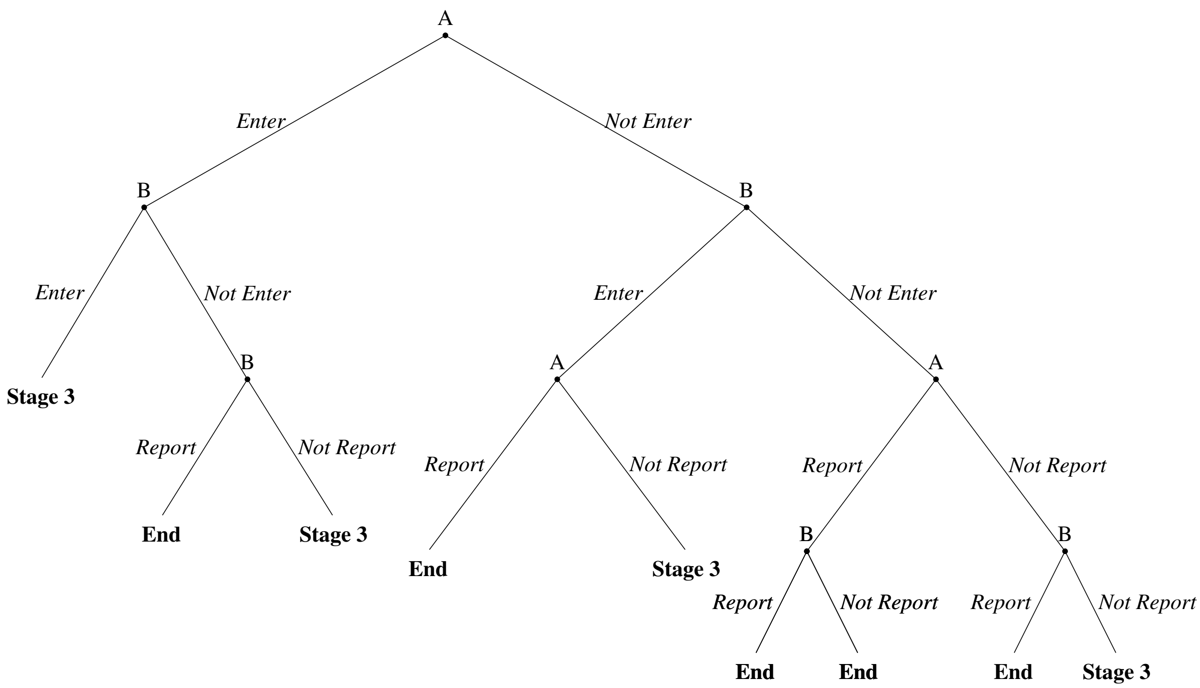

Figure 1: Stages 1-2

Remark. Notice that, in this setup, periods are not strategically independent since payoffs depend on past behaviour through the state variable τ_t.[12] When $\tau = 0$, all outcomes but $\{E, E\}$ are payoff equivalent to

[10] Another way of understanding this probability p is as the probability of inspection, where the inspection tool is perfect. That is, when inspection is carried out, present and past misbehaviour is completely discovered.

[11] In mathematical terms, as we assume a discount rate of one, this assumption is equivalent to a model where profits (and fines) are realised each period. However, it simplifies the characterisation of equilibrium strategies.

[12] This is the reason why, although the game has a repetitive structure, it does not have the structure and properties of a supergame.

not entering the collusive agreement, with $\pi_i = \lambda \; \forall i$. For this reason and for the sake of simplicity, we have not introduced an alternative action for staying competitive in Stage 1. Moreover, notice that even when $\tau_t = 0$ and there are no collusion profits to fine, firms can still self-report or be investigated by the CA. This, however, would have no consequences in terms of payoffs and $\pi_i = \lambda \; \forall i$.

3 Results

3.1 Classification of the equilibrium strategies

There are multiple sets of equilibria in this T-period game. However, as there are equivalent actions in terms of outcomes (i.e. different sets of actions that take to the same endpoint in the one-shot game), we can make the analysis tractable by grouping them in an appropriate manner.

Broadly speaking we will show that, in equilibrium, firms may prefer (i) to never collude, (ii) to collude for a certain number of periods and report collusion systematically, or (iii) to collude for a certain number of periods but never report collusion.

Following this reasoning, we classify the equilibrium strategies into three branches, which we define below. From now on, with slight abuse of notation, we will use τ_t and τ indifferently.

Definition 4 (Competitive branch)**.** *The Competitive branch is composed by the set of actions in which firms stay competitive, that is, those where their payoffs are equal to λ. In any case, to be in the Competitive branch, it must hold that $\tau = 0$ for the whole game.*

Notice that in the Competitive branch, in equilibrium firms do not have incentives to form cartels and, therefore, to increase τ. That is, they never reach a collusive agreement in Stage 1 of the game. Any set of actions where $\{E, E\}$ is never chosen along the game is part of this branch.

Definition 5 (Leniency branch)**.** *The Leniency branch is composed by the set of actions in which (i) for a specific number of periods both firms enter the collusive agreement (therefore they cannot report), followed by (ii) a period in which both firms choose not to enter the collusive agreement and report collusion, after which (iii) they start colluding again. This pattern will repeat for the whole game.*

In this case, firms choose $\{E, E\}$ until τ reaches a certain threshold. Once this occurs, firms will choose: $\{NE, R; NE, R\}$. Notice that the collusive profits accumulation sequence can be stopped by the CA with a probability p in each period. In this case, both firms will restart collusion immediately after and will do so until they reach the threshold without interruption. Once reached, they will stop it themselves. For this reason, the threshold can be expressed in an explicit form (see below), but the number of collusion periods can only be expressed in expected terms.

As we are going to show in Proposition 1, despite there being many strategies that are similar to the ones presented in Definition 5, they are not going to be equilibrium strategies. For instance, stopping collusion but not reporting the cartel (silent exit) would not be part of any equilibrium strategy.

Definition 6 (Authority branch)**.** *The Authority branch is composed by the set of actions in which (i) for n periods both firms act as in the Competitive branch, followed by (ii) $T - n$ periods in which both firms choose to collude (and will not be able to report).*

Now, in equilibrium, firms only collude in the last periods of the game and there is no self-reporting. After the competitive sequence (the first n periods), the cartel will only be stopped in two scenarios: (i) when the cartel is discovered by the CA and (ii) when the game ends ($t = T$). Notice that in the latter case, firms would earn the total collusive profits, while in the former one the collusive profits accrued until that moment would be discounted by the fine rate γ_f. Furthermore, if the cartel is convicted before T, firms will restart collusion immediately.

We use these definitions in the following proposition.[13]

Proposition 1. *Given a specific combination of parameter values, the set of actions chosen by rational players are always going to belong to one and only one of the three branches.*

Proof. In order to demonstrate that players are always going to belong to one and only one of the three branches, let us start by analysing the incentives of being in the Competitive branch rather than in any of the other two branches.

In any period t, given τ previous periods of collusion, firms will prefer to compete and obtain λ instead of colluding and increasing P_t by 1 if $\lambda + (1 - \gamma_f p)\tau > (1 - \gamma_f p)(\tau + 1)$ holds, i.e. if $\lambda > 1 - \gamma_f p$. Therefore, the decision just depends on these parameters, but does not depend on the period of the game in which players are at, nor the size of P_t. By backward induction, if this inequality holds, firms will not collude in any period, that is, they will remain in the Competitive branch for the whole game.

Let us now show that, in case of there being incentives to collude, firms will belong to one and only one of the two remaining branches.

If, on the contrary, $\lambda < 1 - \gamma_f p$ holds, firms will collude and, therefore, they will either be in the Leniency branch or in the Authority branch. At any given period t, the probability of a cartel of *not ever being discovered* before the end of the game is $(1 - p)^{T+1-t}$, so if $\lambda + (1 - \gamma_s)\tau < \lambda + ((1 - \gamma_f)(1 - (1 - p)^{T+1-t}) + (1 - p)^{T+1-t})\tau$ holds for all t, players have no incentive to report the cartel. That is, if ϕ is the cartel length in the leniency branch (as defined below), provided that $\gamma_s > \gamma_f(1 - (1 - p)^{T+1-\phi})$ holds, leniency will never be used and, consequently, firms will be in the Authority branch. Otherwise, they will belong to the Leniency branch.

□

3.2 The Competitive branch

This section characterises the Competitive branch. In the following lemma, we give the condition for firms to be in the Competitive equilibrium branch, derived from the proof of Proposition 1.

Lemma 1. *Firms will choose an equilibrium strategy from the Competitive branch if $\lambda > 1 - \gamma_f p$.*

The condition comes from the direct comparison of the payoffs obtained when there is no cartel and $\tau = 0$ ($c = \lambda$), and the expected payoffs when both firms decide to Enter, i.e. $p(1 - \gamma_f) + (1 - p)$. Notice that reporting an inexistent cartel in Stage 2, that is, applying for leniency when $\tau = 0$, is also

[13] In these results we disregard indifferences.

part of the Competitive branch given that there are no collusive agreements. As with any other strategy from the Competitive branch, this situation yields profits of c for both firms.

Using a competition policy interpretation, when the relative incremental value of profits from collusion are sufficiently low (λ is sufficiently high), firms will not form cartels. The condition from Lemma 1 can also be expressed as $\gamma_f p > 1 - \lambda$, where γ_f and p are the policy variables of the CA and $1 - \lambda$ is the relative increase in profits from collusion. Therefore, if the expected fines for cartelization ($\gamma_f p$) are sufficiently high, only when cartels are very profitable ($\lambda \to 0$), firms would have incentives to collude.

3.3 The Leniency branch

We now expose the characteristics and dynamics of the Leniency branch. With this end, we start by defining the optimal cartel length in this branch.

Definition 7 (ϕ). *We define ϕ as the maximum number of collusion periods per cartel in equilibrium within the Leniency branch.*

The condition for firms to be in the Leniency equilibrium branch, also derived from the proof of Proposition 1, is presented in the next lemma.

Lemma 2. *Firms will choose an equilibrium strategy from the Leniency branch if $\lambda < 1 - \gamma_f p$ and $\gamma_s < \gamma_f(1 - (1-p)^{T+1-\phi})$.*

This lemma results from two comparisons. On the one hand, to be in the Leniency branch, firms must have incentives to collude, i.e., $\lambda < 1 - \gamma_f p$. On the other hand, they must prefer to defect from the cartel agreement and report it to the CA instead of defecting without reporting. That is, after ϕ periods of collusion, the first firm compares the cost of self-reporting γ_s with the expected cost of being discovered and convicted by the CA without self-reporting: $\gamma_f(1 - (1-p)^{T+1-\phi})$.

Now, we present a series of propositions and corollaries showing the dynamics in the Leniency equilibrium branch. In what follows, we assume that $\lambda < 1 - \gamma_f p$ and $\gamma_s < \gamma_f(1 - (1-p)^{T+1-\phi})$, that is, that Lemma 2 holds.

In order to characterize the optimal cartel length in the leniency branch, we obtain the maximum number of consecutive periods that firms would be willing to collude according to the profit maximization rule.

Proposition 2. *In the Leniency branch, firms will be willing to collude until τ reaches a maximum value of τ^*, where $\tau^* = \text{floor}\left(\frac{p(1-\gamma_f)+(1-p)(1-\gamma_s)}{p(\gamma_f-\gamma_s)}\right)$.*

Proof. Given that there have been τ previous periods of collusion, Firm i has two strategic options: (i) reporting now or (ii) increasing the cartel duration for another period and then reporting (recall that defecting without reporting is a dominated action in the Leniency branch, given that $\gamma_s < \gamma_f(1 - (1-p)^{T+1-\phi})$ holds). The payoff in the first case is $\lambda + (1-\gamma_s)\tau$, while in the second one this payoff is $\lambda + (p(1-\gamma_f) + (1-p)(1-\gamma_s))(\tau + 1)$. By comparing these two payoffs we obtain that Firm i will maintain the cartel for another period if $\tau < \dfrac{p(1-\gamma_f)+(1-p)(1-\gamma_s)}{p(\gamma_f-\gamma_s)}$. Notice that as Firm A is the first mover, she will be the one who has incentives to self-report the cartel in first place.

$\square$

Intuitively, this value measures for how long are firms willing to maintain the cartel before they have incentives to disclose it and, consequently, clear their collusive records, even if this implies losing part of their profits: a proportion γ_s for the first reporter and γ_{ss} for the second reporter. The reason behind there being a threshold is that, as periods go by, the cartel value for each firm increases at a constant rate 1, but the losses that firms face if discovered are $\gamma_f p\tau$, that depends on the value of τ. At some point, the expected losses will surpass the gains and firms will disclose the cartel to the CA and in return benefit from fine reductions rather than waiting for the CA not to discover them. Notice that this is the maximum number of consecutive periods firms are *willing* to collude, but the ex-post number of periods depends on nature (the random CA's investigation success).

Corollary 1. *In the Leniency branch, if in a given period t, $\tau_t = 0$ and $t + \tau^* \geq T$, both firms will be willing to continue colluding for the remaining periods of the game.*

This result follows from Proposition 2. If firms are willing to collude for τ^* periods, but there are less (or equal) than τ^* periods left for the game to end, they are involved in a cartel agreement until the end while they are not uncovered by the CA. If discovered, firms will reach another cartel agreement in the following period.

Despite the optimal cartel length is τ^* there could be an unravelling process which would shorten the equilibrium cartel length.

Proposition 3. *In the Leniency branch, if in a given period t, $\tau_t = 0$ and $t+\tau^* < T$, firms will be willing to continue colluding for ϕ^* consecutive periods, where $\phi^* = \min(\tau^*, \rho^*)$, where $\rho^* = \text{floor}\left(\frac{1-\gamma_s}{\gamma_{ss}-\gamma_s}\right)$.*

Proof. Consider a period t in which $\tau_t = 0$ and $t + \tau^* < T$. Firm B expects Firm A to deviate and report in period $t + \tau^*$. Anticipating this, Firm B could have incentives to deviate and report as well in the previous period $t + \tau^* - 1$. Being more precise, Firm B will deviate in period $t + \tau^* - 1$ rather than wait for Firm A to do so in period $t + \tau^*$ if $\lambda + (1-\gamma_s)(t + \tau^* - 1) > \lambda + (1-\gamma_{ss})(t + \tau^*)$. Notice that Firm A will always anticipate to Firm B (if $\gamma_s < \gamma_n$). This process is similar to the backward induction reasoning of the centipede game (Aumann, 1995) and will continue until players have no more incentives to anticipate to the other firm. In other words, this unravelling process will continue until Firm B does not have incentives to anticipate any further. We define ϕ^* as the highest integer that fulfils the following inequality (considering that this number is equal or smaller than τ^*):

$$\lambda + (\phi^* - 1)(1-\gamma_s) < \lambda + \phi^*(1-\gamma_{ss}) \rightarrow \phi^* < \frac{1-\gamma_s}{\gamma_{ss}-\gamma_s}$$

$\square$

In contrast with Corollary 1, now we are analysing the game when there are more than τ^* periods ahead. In this case, firms may have incentives to defect and report the cartel before the end of the game, but not necessarily in period $t+\tau^*$. The reason behind this is that unravelling may occur. Firm B may find it profitable to stop the collusive profits accumulation sequence by anticipating to Firm A's actions if the fine for the first firm to report collusion is strictly lower than for the second one ($\gamma_s < \gamma_{ss}$). However, it will not necessarily be a complete unravelling, as it may reduce the number of collusion periods without driving them down to zero –unlike in the centipede game (Aumann, 1995), where payoffs eventually collapse to zero. In particular, firms will be willing to collude for $\phi^* \leq \tau^*$ consecutive periods.[14]

In terms of competition policy, we show that with a LP, cartels will still happen. Essentially, whether these cartels are shorter or longer will depend on the advantage of the first reporter with respect to the second reporter ($\gamma_{ss} - \gamma_s$). If the difference is large, then cartels will be shorter. This, for example, happens when there is Full Leniency for the first reporter and no leniency for subsequent reporters ($\gamma_s = 0, \gamma_{ss} = 1$). Alternatively, if the difference is small, cartels will be willing to last for longer.

The next corollary follows from the previous results (see also the example in Figure 2).

Corollary 2. *In the Leniency branch, equilibrium actions of firms will follow this scheme:*

1. *Firms will collude until the amount of sanctionable profits reaches a size of $P_t = \phi^*$. In case of being discovered by the CA before reaching this threshold, they will resume collusion until ϕ^* profits are accumulated.*
2. *Firm A and Firm B will report, obtaining $\phi^*(1-\gamma_s)$ and $\phi^*(1-\gamma_{ss})$ respectively.*
3. *Firms will restart collusion in the period immediately after.*
4. *When firms reach a period t with $\tau_t = 0$, such that $t \geq T - \tau^*$, firms collude until the end of the game.*

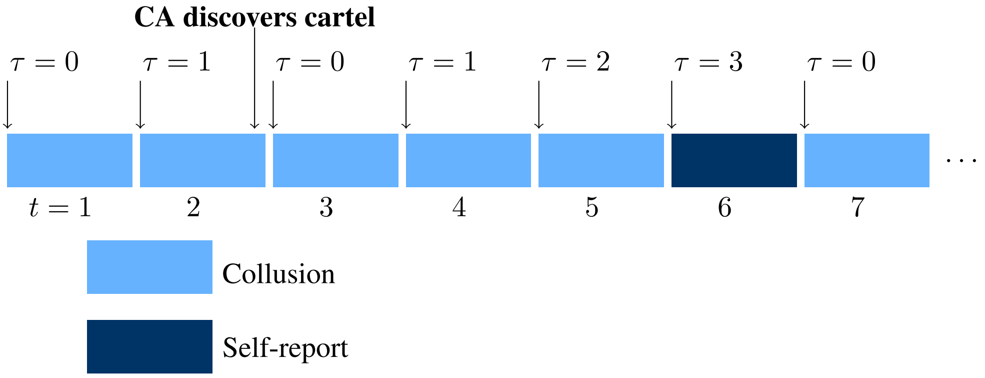

Figure 2: Example of dynamics for a game in the Leniency branch with $\phi^* = 3$

Therefore, introducing a LP results in a series of successive cartels that emerge throughout the T periods. The underlying reason is that, given an expected fine, $\gamma_f p$, when the CA discovers a cartel, as the LP allows firms to clear their records at a lower cost (pay γ_s and return to a clean slate at $t+1$, i.e. $\tau_{t+1} = 0$), firms will use the Leniency Programme to their own advantage. Once they have become clean, they form immediately another cartel.

Figure 2 shows an example of dynamics in the leniency branch.

[14]Notice that $\frac{1-\gamma_s}{\gamma_{ss}-\gamma_s}$ can be larger than τ^* if the difference $\gamma_{ss} - \gamma_s$ is sufficiently small. In this case, there will be no unravelling at all. The cartel will stop after τ^* periods of collusion, following Corollary 1.

3.4 The Authority branch

In this section we present the characteristics and dynamics of the Authority branch. To do so, we first give the condition for firms to be in this branch.

Lemma 3. *Firms will choose an equilibrium strategy from the Authority branch if* $\lambda < 1 - \gamma_f p$ *and* $\gamma_s > \gamma_f(1-(1-p)^{T+1-\phi})$.

This lemma is obtained using the same reasoning as in Lemma 2. Assuming that $\lambda < 1 - \gamma_f p$ such that firms prefer to collude rather than to compete, when γ_s is sufficiently high (the fine reduction derived from being the first firm to report is sufficiently low), firms will prefer not to report and take the risk of being discovered by the CA. That is, $\lambda + (1-\gamma_s)\tau < \lambda + \Big((1-\gamma_f)(1-(1-p)^{T+1-\phi}) + (1-p)^{T+1-\phi}\Big)\tau$.

We below show the dynamics of the Authority branch, assuming that both conditions from Lemma 3 hold.

Proposition 4. *In the Authority branch, firms will be willing to collude in the last* τ^{**} *periods of the game, where* $\tau^{**} = \text{Ceil}\left(\frac{ln(\gamma_f+\lambda-1)-\ln(\gamma_f)}{\ln(1-p)}\right)$

Proof. See Appendix A for the full proof.

Intuitively, there are now no incentives to break the collusive agreement by reporting it to the CA, as it is too costly, in expected terms, compared to the option of colluding and potentially being uncovered by a CA investigation. Once a cartel is formed, firms are exposed to the possibility of being discovered and sanctioned until the end of the game, irrespectively of the length of the cartel, so there is no reason to get back to competition if $\gamma_f p < 1 - \lambda$. Think as a counter example two firms which collude just in the first period of the game. These firms will carry that stain on their records and spend all their lifetime facing the risk about that (short) cartel being discovered. For this reason, in this branch firms will only do one collusive profits accumulation sequence, i.e. they will only form *one* cartel, that will resume immediately if discovered.

The next question is when will this cartel be formed. Suppose that incentives are only to collude for one period, i.e. $\tau^{**} = 1$. Then, firms will prefer to collude in the last period than in any other because by doing so they minimise the probability of being uncovered and the collusive profits P_t fined by the CA, which is $1-(1-p)^t$, being t the period in which they currently are. For the same reason, if $\tau^{**} = 2$, firms will prefer to collude in the last two periods, and so on. Hence, for any value of τ^{**}, the collusive behaviour sequence will begin in period $T - \tau^{**} + 1$.

The next corollary summarises the dynamics of the Authority branch (see also the example in Figure 3).

Corollary 3. *In the Authority branch, equilibrium actions of players will follow the following scheme:*

1. *If* $T < \tau^{**}$*, firms will form a cartel for the whole game.*
2. *If* $T > \tau^{**}$*:*
    i) *Firms stay in the Competitive branch for* $T - \tau^{**}$ *periods.*

ii) Firms will form a collusive agreement and collude until the end of the game. In case the CA discovers the cartel during this collusive phase, firms will restart collusion in the next period.

This result follows from Proposition 4. If the probability of inspection (p), the fine rate (γ_f) and/or the ratio of competitive profits over collusive ones (λ) are sufficiently low, such that $T < \tau^{**}$, it could be the case that firms collude for the entire game. Otherwise, collusion will only happen from a specific moment in time until the end of the game.

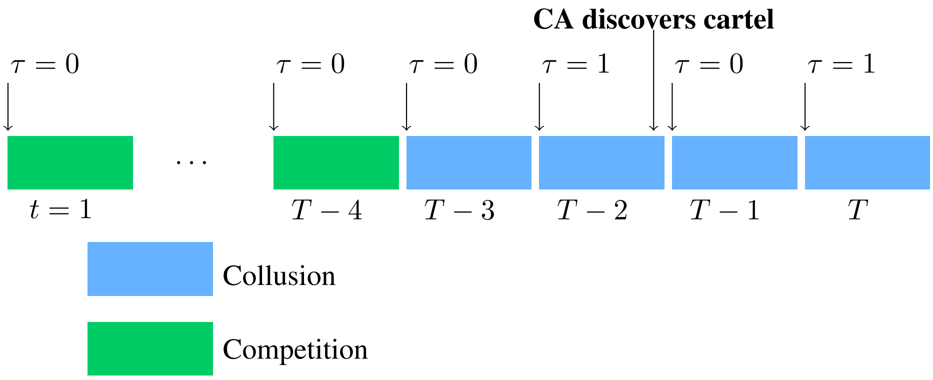

Figure 3: Example of dynamics for a game in the Authority branch with $\tau^{**} = 4$

Notice that when $\gamma_s \geq \gamma_f$ firms will always be in the Authority branch of the game (or in the competitive one if $\lambda > 1 - \gamma_f p$), that is, when the firms do not have incentives to apply for leniency or it is inconsequential, our model predicts cartels with the simple dynamics predicted in Corollary 3: some periods of competition followed by some periods of cartel.

If we take this result to cartels in the real world, notice that the interpretation of this branch depends on the time frame T considered, i.e. for how long do firms interact. If the interaction is short, we would observe two types of outcomes: collusion during the whole interaction or no collusion at all if there are no incentives to form cartels. Instead, when firms interact for long periods of time, $T \to \infty$, firms will basically be competing without a chance to form cartels because of the very low odds of going undetected until the end of their interaction.

3.5 Comparative statics

Assuming that firms have incentives to collude (i.e., $\lambda < 1 - \gamma_f p$ holds), for a given leniency policy, the inspection probability determines the equilibrium branch in which firms are. In particular, for low values of p, firms would not feel tempted to apply for leniency, and instead they would follow the dynamics of the Authority branch. In fact, as p approaches zero, the leniency policy becomes irrelevant, as firms do not need to use it. The risk is so low that they do not feel that they should clean their collusive records.

In the Authority branch, given λ, cartel length is going to depend negatively on both the inspection probability $\left(\frac{\partial \tau^{**}}{\partial p} < 0\right)$ and the fine paid by firms in case of being caught with collusive records by the CA $\left(\frac{\partial \tau^{**}}{\partial \gamma_f} < 0\right)$. Therefore, increasing the inspection probability and/or the fine paid in case of an inspection decreases cartel length and firms' profits.

However, as the inspection probability increases, the leniency policy gains importance. When conditions are such that firms have incentives to apply for leniency (Leniency branch), the optimal cartel length depends positively on the fine rate paid by the first reporter $\left(\frac{\partial\tau^*}{\partial\gamma_s} > 0 \text{ and } \frac{\partial\rho^*}{\partial\gamma_s} > 0\right)$, and depends negatively on the fine rate paid in case of being caught by the CA $\left(\frac{\partial\tau^*}{\partial\gamma_f} < 0\right)$, on the inspection probability $\left(\frac{\partial\tau^*}{\partial p} < 0\right)$ and on the fine paid by the second reporter $\left(\frac{\partial\rho^*}{\partial\gamma_{ss}} < 0\right)$.

If the CA focuses on the effectiveness of its pro-active detection tool (random audits) and its reactive one (leniency programme) in decreasing cartel length, it should aim to increase the inspection probability and/or decrease the fine rate paid by the first reporter in order to break cartels down. This, in turn, will decrease firms' profits.

However, this effect is unclear if the CA aims to (or must keep) the difference between the fine rates paid by the second and first self-reporters $(\gamma_{ss} - \gamma_s)$ constant. In this case, $\frac{\partial\rho^*}{\partial\gamma_s} < 0$, which implies that, for a sufficiently strong unravelling process ($\rho^* < \tau^*$), such that $\phi^* = \rho^*$, cartel length will increase if the CA reduces γ_s, which means a more generous leniency programme.

4 Simulations of the model and policy recommendations

In this section, we extend the comparative statics results of Section 3.5 deriving specific outcomes for certain parameter configurations. Since the policy implications of the model depend on the interrelationship between parameters, as demonstrated in the main model, we now show the outcomes of the model varying three policy variables: the probability of detection, p; the fine rate paid by the first self-reporter, γ_s; and the difference between the fine rates paid by the second and first self-reporters, $\gamma_{ss} - \gamma_s$.

To this end, we conduct four sets of simulations using the parameter choices presented in Table 1. The upper panel displays the parameter values that are common across all simulations, while the lower panel shows the range of values for the simulation variables. The parameters are selected based on the following considerations:

1. The length of the game, T, is set at 25 periods.
2. Monopoly profits m are normalized to $100/T$ (rather than to 1, as in the model), so that a "perfect" cartel (one that colludes throughout the entire game without being caught) earns a total profit of 100. Given that $\lambda = 0.2$, perfectly competitive firms earn total profits of 20 over the game. This parametrization facilitates a straightforward comparison of the welfare implications across different policies.
3. The fine rates γ_n and γ_f are set to 1 and 0.9, respectively. Note that for rational players, the value of γ_n does not affect equilibrium behaviour as long as $\gamma_n \geq \gamma_f$, consistently with the assumptions of the model.
4. The maximum detection probability in our simulations, 0.21 (21%), exceeds previous estimates, which range from 13% to 17% (Bryant and Eckard, 1991) and 12.9% to 13.3% (Combe et al., 2008). This higher value allows us to explore the outcomes of more successful interventions, while still remaining within a plausible range.
5. For the ex-post profit simulations, we run 30000 cartels for each combination of parameters to determine the actual average values of cartel rates and profits.
6. Each parameter dimension is discretized into 50 grid points.

Fixed parameters

T	25
m	100/T
λ	0.2
γ_n	1
γ_f	0.9
# Sampling Points	50 $\times$ 50
Resample	30000

Variable parameters

	p	γ_s	$\gamma_{ss} - \gamma_s$
Sim 1	0.01-0.21	0	0-0.6
Sim 2	0.01-0.21	0.3	0-0.6
Sim 3	0.01-0.21	0-0.6	0.3
Sim 4	0.01-0.21	0-0.6	0.1

Table 1: Parameter choice for the simulation models

Two types of outcomes are derived: (i) computed values obtained directly from the model (the first two variables listed below), and (ii) average ex-post values of the remaining two variables, assuming firms behave rationally, i.e. they follow best-response strategies based on the current state of each specific simulation (including the random realizations of Competition Authority inspections). These outcomes are particularly useful for interpreting the welfare implications of different policy choices. The variables under study are:

- *Branch of the game*. The selected equilibrium branch: Competitive (0), Authority (1) or Leniency(2). Although the competitive branch is a possible equilibrium solution, we have chosen our parameters so that we focus on the last two branches.
- *Average cartel length*. This is the average number of periods during which firms earn collusive profits, conditional on firms following the equilibrium strategy and accounting for the probability of detection by the Competition Authority. Values range from 0 (no cartel formation) to 25 (a single, undetected cartel lasting the entire game).
- *Average collusion rate*. The average rate of collusion observed across simulations, given equilibrium behaviour and the probability of detection. Values range from 0 (no collusion) to 1 (continuous undetected collusion).
- *Average profits*. Total average profits earned by firms, conditional on equilibrium strategies and detection probabilities.[15] As stated above, we have normalised these profits so that they go from 20 (no collusion) to 100 (full, undetected collusion).

The simulations presented in Figures 4 and 5 vary two parameters that are closely linked to the policy decisions of the Competition Authority: the probability of cartel detection (p), and the difference in fine reductions between the first and second self-reporters ($\gamma_{ss} - \gamma_s$). We consider two levels of leniency for the first self-reporter: full leniency ($\gamma_s = 0$) and partial leniency ($\gamma_s = 0.3$). This allows us to address the following question:

[15] When firms apply for leniency, the first applicant pays a lower fine and earns a higher profit. In all other cases, both firms receive equal profits.

Research Question 1: *Should the CA increase the investigation probability (p) or increase the difference in the benefits of being the first and the second self-reporter ($\gamma_{ss} - \gamma_s$)?*

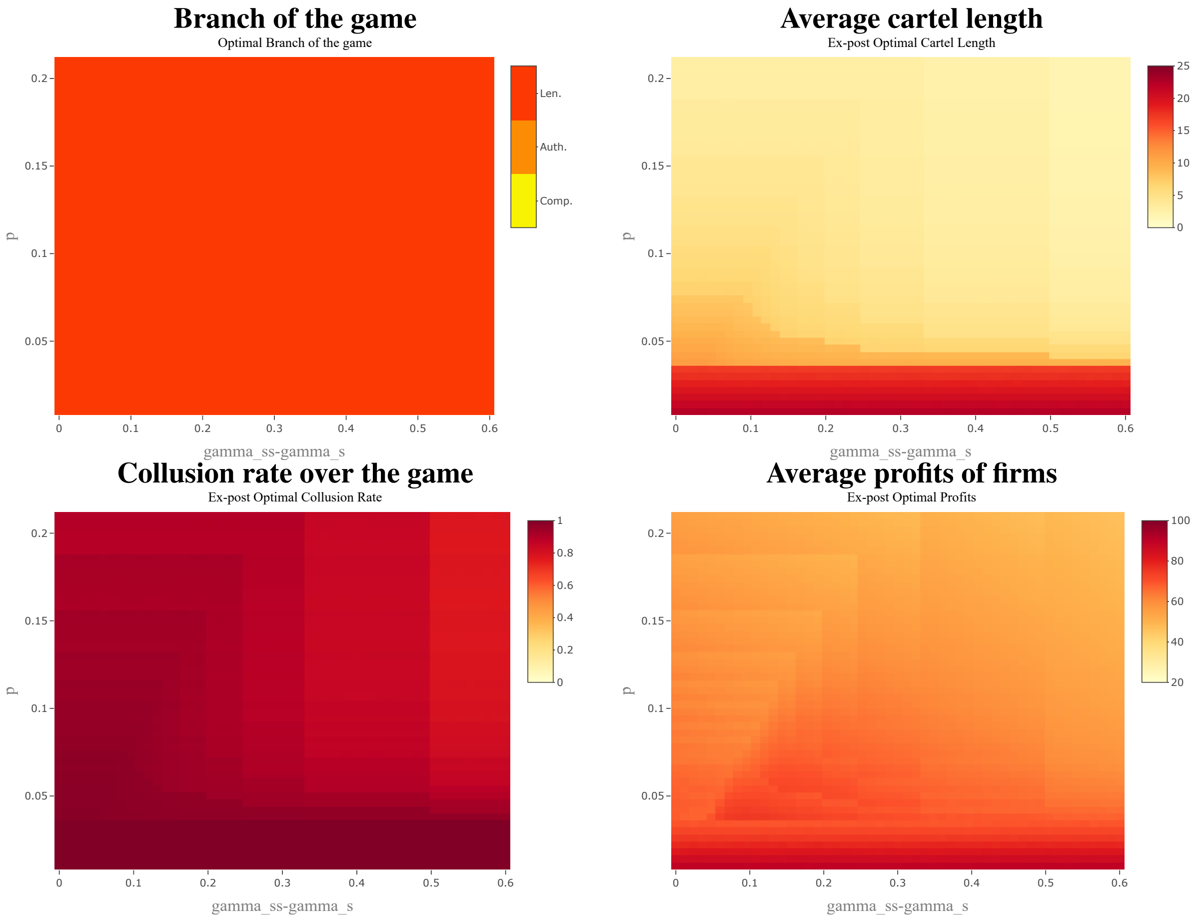

Figure 4: Simulation 1: $p \times (\gamma_{ss} - \gamma_s)$ with full leniency ($\gamma_s = 0$)

With full leniency, firms apply for it across the entire parameter region under consideration (top-left panel of Figure 4). Under partial leniency, the authority branch strategy begins to be adopted, but only when the probability of detection is low; i.e., when the expected cost of being caught is sufficiently small that it is not worthwhile to incur in leniency fines merely to clear one's record (top-left panel of Figure 5).

However, the graphs depicting average cartel length reveal more nuanced interactions. When the probability of detection is sufficiently low (depending on the degree of leniency granted to the first reporter), cartels tend to persist throughout the entire duration of the game, rendering the unravelling process inapplicable (see Corollary 1). In such cases, the difference in leniency between the first and second reporters becomes irrelevant. Once the probability of detection reaches a high enough threshold, the unravelling effect begins to operate, leading to a sharp reduction in cartel duration. Notably, the graphs display wide vertical bands, indicating that increasing enforcement resources significantly is required to reduce cartel length—unless such efforts are complemented by a greater disparity in leniency between reporters. These results suggest that differences in leniency degrees may or may not have a substantial impact, depending on the broader configuration of competition policy measures.

These results are reflected clearly in the collusion rates, which decline with both p and the difference $\gamma_{ss} - \gamma_s$. In all cases, and consistent with the general predictions of our model, collusion rates remain high—either because firms collude throughout the entire game or because they form successive cartels. This outcome is largely driven by the high leniency rates considered in the simulations ($0/0.3$).

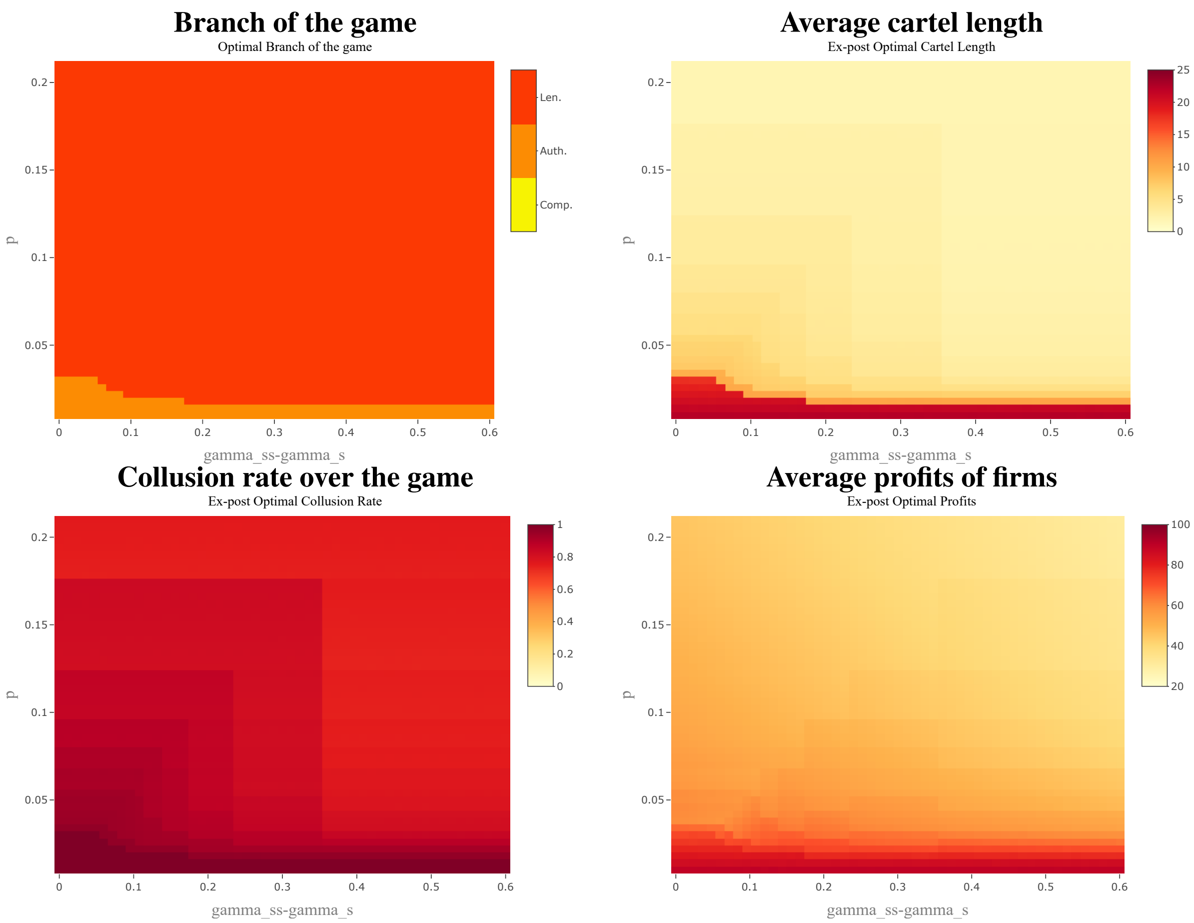

Figure 5: Simulation 2: $p \times (\gamma_{ss} - \gamma_s)$ with partial leniency ($\gamma_s = 0.3$)

The final graph in each set illustrates the welfare implications of the parameter configurations, represented by the average profits of the firms in the cartel. As expected, low probabilities of detection result in higher profits. However, note the substantial disparity between the two graphs: under full leniency, average profits are significantly higher than under partial leniency. More generally, the greater the difference in leniency degrees, the lower the profits extracted from the market. Nonetheless, under certain specific configurations (see the corresponding figure for full leniency), reducing the leniency difference may actually be beneficial. For instance, at values of p around 0.07 and $\gamma_{ss} - \gamma_s \to 0$, cartels persist longer than is optimal from the firms' own perspective, thereby reducing their profits and, as a result, enhancing consumer welfare.

The overarching lesson from these simulations is that Competition Authorities should prioritise increasing the probability of investigation, regardless of the specific design of the leniency programme. Our findings demonstrate that limited reliance on proactive detection tools leads to higher collusion rates, longer-lasting cartels, and increased profits for cartel members–irrespective of how fine reductions are structured for the first and second applicants. While increasing the disparity in fine reductions can be effective, its impact is limited and highly contingent on the broader configuration of the leniency programme.

In summary, in response to our first research question, we find that—regardless of the specific design of the leniency programme—Competition Authorities should adopt proactive measures to increase the probability of cartel detection, thereby shifting firms' equilibrium strategies. Indeed, once the probability of detection surpasses a certain threshold, firms prefer to self-report the cartels they form rather than risk detection and full sanctioning without eligibility for fine reductions. However, this threshold is not entirely independent of the leniency programme's design; it tends to be slightly lower when the programme is less generous to the second applicant (in relation to the first one).

Given that the structure of fine reductions between cartel members has demonstrated limited effectiveness in enhancing market competitiveness within our model, we now turn to our second research question and focus on analysing the degree of leniency granted to the first applicant, while holding constant the difference in leniency relative to the second applicant.

Research Question 2: *Should the CA increase the investigation probability (p) or increase the benefits of being the first self-reporter ($1 - \gamma_s$)?*

In Simulations 6 and 7, we once again vary the probability of cartel detection (p), but this time in relation to γ_s, the fine rate applied to the first self-reporter. This parameter also determines the fine paid by the second self-reporter. In these simulations, we fix the difference in leniency degrees ($\gamma_s - \gamma_{ss}$) at two levels: Low (0.1) and High (0.3).[16]

The main finding emerges from the top-left panels: even when the authorities are not particularly lenient, the instrument is still employed—unless the probability of inspection is so low that firms prefer to collude throughout the entire game. In such cases, the structure of fines (whether the difference $\gamma_{ss} - \gamma_s$ is low or high) has only a marginal influence on firms' optimal decision.

However, the fact that the leniency branch is chosen in equilibrium almost universally does not imply that the parameter configuration is irrelevant. As illustrated in the cartel length plots, particularly in the second figure, the interaction between the probability of inspection and the degree of leniency is clearly evident. Indeed, a trade-off emerges: a decrease in the probability of inspection must be offset by a reduction in the leniency degree in order to maintain cartel length at a constant level. According

[16] We choose to fix the difference in leniency degrees because the unravelling process depends on this differential, rather than on the absolute value of γ_{ss}.

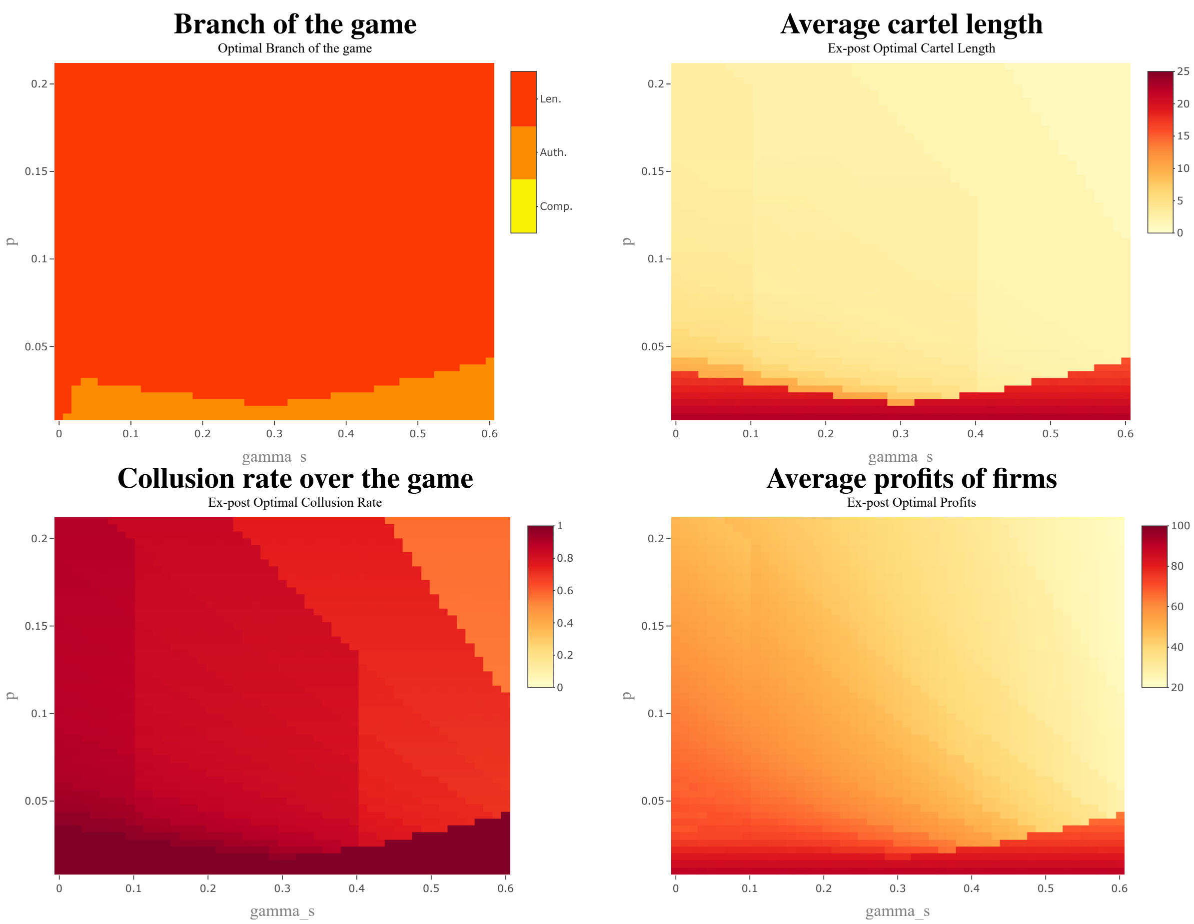

Figure 6: Simulation 3: $p \times \gamma_s$ with high difference in leniency ($\gamma_{ss} - \gamma_s = 0.4$)

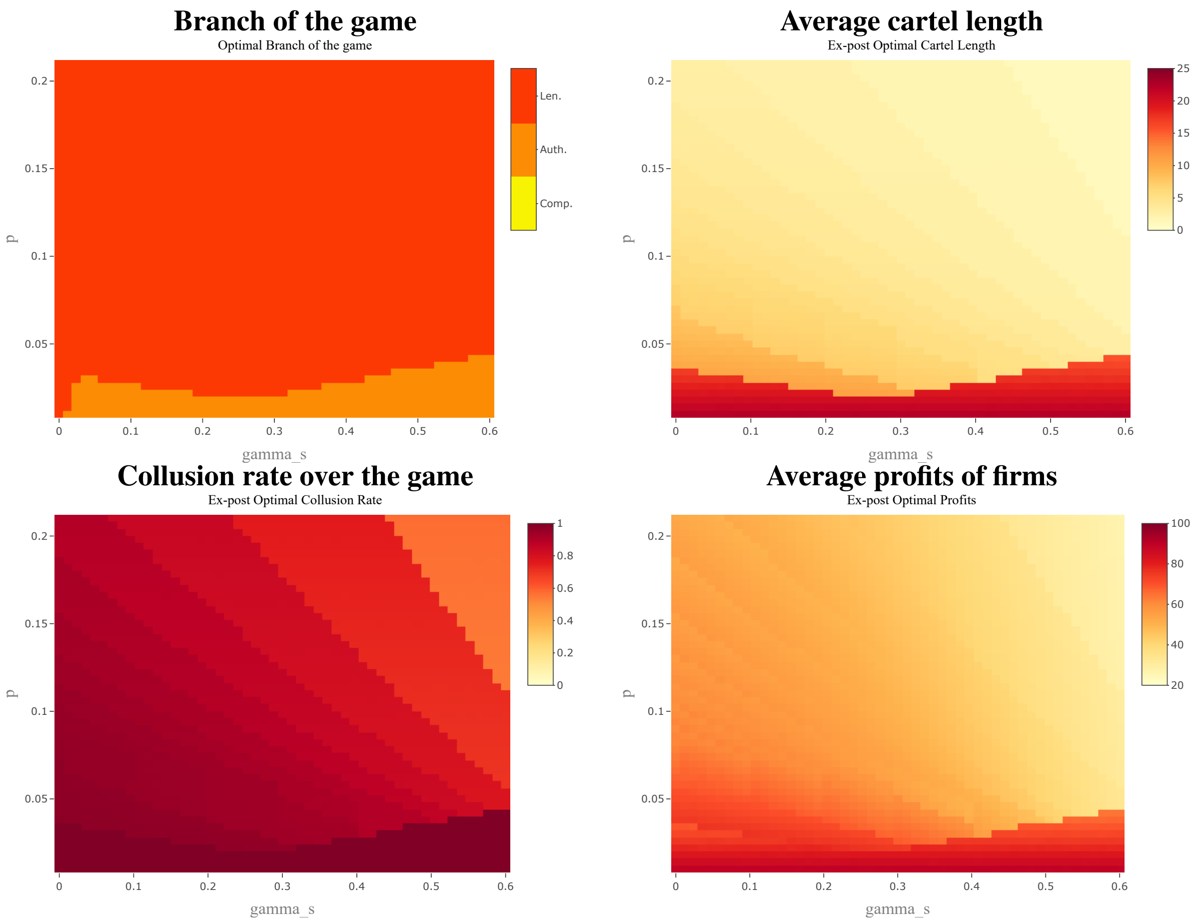

Figure 7: Simulation 4: $p \times \gamma_s$ with low difference in leniency ($\gamma_{ss} - \gamma_s = 0.1$)

to our predictions, the most detrimental configuration is one in which inspection probabilities are low and leniency is generous, as this leads to the longest-lasting cartels– especially when the difference in leniency degrees is small.

A similar pattern is observed in the predicted collusion rates. One of the key insights from our model, as highlighted in the simulations, is that the competition policy design can indeed shorten cartels. However, the incentives to reoffend introduce an important caveat: cartel length is not necessarily the best measure of the effectiveness of competition policy.

Finally, profits are higher under the Authority branch than under the Leniency branch. In the latter, profits increase as both p and γ_s decrease. The difference in the benefits granted to the first and second self-reporters does not affect this outcome. Therefore, higher leniency degrees result in greater profits. While leniency policies help firms uncover cartels, a sufficiently high p is also necessary to reduce profits. In other words, leniency can substitute for the inspection probability: as γ_s increases, p must also rise to prevent an increase in firms' profits. In this context, for the Competition Authority to optimise its resources, it should avoid implementing full leniency policies, as these require high inspection probabilities to prevent profits from rising.

Given that implementing competition policy instruments incurs both economic and political costs, one might hastily conclude that the best leniency policy is no leniency at all. In fact, the profit graphs from these simulations support this conclusion almost always. However, when the cost of monitoring firms is such that the probability of inspection is very low, it is better to have generous leniency programmes to ensure that cartels do not become permanent (as illustrated, for example, by the horizontal strip along $p = 0.05$ in both graphs).

5 Concluding remarks

This paper contributes to the analysis of the impact of leniency programmes on cartel formation and stability within a dynamic discrete-time framework. Our approach aims to shed light on the potentially perverse effects that may arise from leniency, especially when firms strategically use the programme to minimize fines while preserving incentives to collude.

Our findings reveal that the effectiveness of leniency programmes critically depends on their design and on their interaction with other enforcement instruments. When the probability of detection is high and fines are sufficiently strong, deterrence can be achieved without relying on leniency. However, when collusion remains attractive, firms may either sustain collusion without reporting or strategically use leniency to reduce penalties and reinitiate collusion with a clean slate.

In particular, when the fine reduction for the first leniency applicant is too generous, firms may engage in cyclical collusion, self-reporting periodically to benefit from leniency and avoid harsher fines. Rather than deterring anticompetitive behaviour, this dynamic leads to a series of short-lived cartels and contributes to recidivism, undermining the long-term effectiveness of leniency-based enforcement.

Regarding our simulation-based findings, we show that granting immunity or reductions close to full immunity makes participation in the leniency programme a dominant strategy, even when the perceived risk of detection is low. Nevertheless, detection probability still plays a critical role in shaping cartel duration, collusion rates, and firm profitability. When leniency is partial–i.e., when the fine reduction for the first applicant is moderate–firms require a higher perceived risk of detection to shift away from stable, long-lived collusion toward the strategy of self-reporting and restarting collusion. As the benefits from leniency decrease, so do the incentives to apply first.

These results emphasize the need for enforcement policies that strike a careful balance between offering leniency and maintaining credible detection threats. Our analysis shows that, when firms perceive detection risk as low, even a well-designed leniency programme may fail to destabilize cartels. An enforcement approach that downplays *ex officio* investigations would reduce firms' incentives to defect, ultimately weakening deterrence.

Unlike many prior studies focused on identifying optimal leniency policies (Harrington, 2008; Spagnolo, 2004), our contribution lies in characterizing the equilibrium dynamics and simulating firm behaviour under alternative enforcement regimes. Our model departs from the infinite-horizon assumption common in the literature, adopting a finite horizon that better captures the planning frame of CEOs or firm leadership. While this simplification limits some interpretations, it allows for cooperative outcomes without relying on infinite repetition and aligns more realistically with short- to medium-term strategic decisions. Nonetheless, our simulations confirm that the length of the planning horizon has significant implications, particularly in shaping firms' willingness to wait for exogenous cartel detection instead of applying for leniency.

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Appendices

A Proof of Proposition 4

Proof. Given the conditions, collusion along the game will never be stopped by firms and will only be stopped by the CA, which discovers the cartel with probability $Pr(p,t) = 1-(1-p)^{T-t}$, being t the current (collusive) period. As $\frac{\partial Pr(p,t)}{\partial t} \leq 0$, this probability is decreasing with t. Therefore, players will only take the risk of being discovered by the Authority as late in the game as possible. Also, as the records will not be cleaned unless discovered, firms have incentives to form a continuous cartel without silent exit periods as explained in the main text.

Now we obtain the number of collusion periods by backward induction. At $t = T$, if $\tau = 0$ (there is no previous collusion and collusion is starting in that period) both firms will accumulate in P_t if the expected payoffs are higher than the competitive ones, that is, if $p(1-\gamma_f)+(1-p) > \lambda \Rightarrow 1-\gamma_f p > \lambda$. This always holds when we are in the Authority branch, by Lemma 3. Therefore, collusion in the last period, $t = T$, will always occur.

Colluding in the second-to-last period ($t = T-1$) increases expected payoffs by:

$$\Big(p^2 2(1-\gamma_f) + p(1-p)((1-\gamma_f)+1) + (1-p)p2(1-\gamma_f) + (1-p)^2 2\Big)$$
$$-p(1-\gamma_f) - (1-p) = \Big(1 - p(2-p)\gamma_f\Big)$$

The alternative is to stay in the Competitive branch, which yields a payoff of λ. Firms will collude in the second-to last period if:

$$1 - p(2-p)\gamma_f > \lambda \Leftrightarrow 1-\lambda > p(2-p)\gamma_f$$

Notice that we can obtain the general condition for any period t recursively as a function of periods $t+1, t+2, \dots$ (see Example 1 for an algebraic step-by-step example of how to derive the proposition formula for $T = 4$). To do so, we have to define the "expected" payoff in period $t < T$, given that $\tau_t = 0$ as $W[t]$.[17] The value of $W[t]$ is the sum of two terms, i) with probability p firms are caught at t, in this case the expected value is $W[t+1]$ plus the profits kept after the fine $(1-\gamma_f)$ of the current collusion and ii) with probability $1-p$, a new tree where with probability p they are caught, so the expected value is $W[t+2]$ plus the profits kept after the two periods fine $2(1-\gamma_f)$, etc.

Next we will define a convenient function, that we will call $V[t]$ which is the expected value of starting colluding at period t and never being descovered by the authority until the last period T, that is:

$$V[t] = (1-p)^{T-t}(1+T-t)\big(p(1-\gamma_f)+(1-p)\big) = (1-p)^{T-t}(1+T-t)(1-\gamma_f p). \quad (1)$$

At period T, the value function is:

$$W[T] = \big(p(1-\gamma_f)+(1-p)\big) = V[T],$$

[17] Notice that this function is not the expected payoff of the whole game but of the subgame starting at t, that is, if they have been competitive before, $(t-1)\lambda$ should be added to obtain the real expected payoff of the strategy.

at $T-1$:

$$W[T-1] = (1-p)2\big(p(1-\gamma_f)+(1-p)\big) + p\Bigg((1-\gamma_f) + \big(p(1-\gamma_f)+(1-p)\big)\Bigg) =$$
$$= V[T-1] + p\big((1-\gamma_f)+W[T]\big) = V[T-1] + pV[T] + (1-\gamma_f)p.$$

For $T-2$ we are going to start using the simplification $p(1-\gamma_f)+(1-p) = (1-\gamma_f p)$:

$$\begin{aligned}
W[T-2] = 3(1-p)^2(1-\gamma_f p) + p(1-p)\left(2(1-\gamma_f)+(1-\gamma_f p)\right) + \\
+ p(1-p)\left((1-\gamma_f)+2(1-\gamma_f p)\right) + p^2\left(2(1-\gamma_f)+(1-\gamma_f p)\right) = \\
= V[T-2] + p(1-p)V[T] + pV[T-1] + p^2 V[T] + \\
+ (1-\gamma_f)\left(2p(1-p)+p(1-p)+2p^2)\right) = \\
= V[T-2] + pV[T-1] + pV[T] + (1-\gamma_f)p\left(2+(1-p)\right).
\end{aligned}$$

With another term the general expression can be inferred:

$$\begin{aligned}
W[T-3] = 4(1-p)^3(1-\gamma_f p) + p(1-p)^2\left(3(1-\gamma_f)+(1-\gamma_f p)\right) + \\
+ p(1-p)^2\left(2(1-\gamma_f)+2(1-\gamma_f p)\right) + p^2(1-p)\left(3(1-\gamma_f)+(1-\gamma_f p)\right) + \\
+ p(1-p)^2\left((1-\gamma_f)+3(1-\gamma_f p)\right) + p^2(1-p)\left(3(1-\gamma_f)+(1-\gamma_f p)\right) + \\
+ p^2(1-p)\left(2(1-\gamma_f)+2(1-\gamma_f p)\right) + p^3\left(3(1-\gamma_f)+(1-\gamma_f p)\right) = \\
= V[T-3] + pV[T-2] + pV[T-1] + pV[T] + (1-\gamma_f)p\left((1-p)^2+2(1-p)+3\right),
\end{aligned}$$

Then, the general term can be written as:

$$\begin{aligned}
W[t] = V[t] + p\Bigg(\sum_{i=t+1}^{T} V[i]\Bigg) + \\
+ (1-\gamma_f)p\Bigg((T-t)(1-p)^0 + (T-t-1)(1-p) + \cdots + 1(1-p)^{T-t-1}\Bigg) = \\
V[t] + p\Bigg(\sum_{i=t+1}^{T} V[i]\Bigg) + (1-\gamma_f)p\Bigg(\sum_{i=t+1}^{T}(1-p)^{T-i}(i-t)\Bigg).
\end{aligned} \quad (2)$$

For the specific case of $t=T$, $W[t]=V[t]$

Then, the increase in expected payoffs from starting to collude in the previous period, that is $W[t]-W[t+1]$, is:

$$\Delta W[t] = W[t] - W[t+1] =$$

$$= V[t] + p\Big(\sum_{i=t+1}^{T} V[i]\Big) + (1-\gamma_f)p\Big(\sum_{i=t+1}^{T}(1-p)^{T-i}(i-t)\Big) -$$

$$- V[t+1] - p\Big(\sum_{i=t+2}^{T} V[i]\Big) - (1-\gamma_f)p\Big(\sum_{i=t+2}^{T}(1-p)^{T-i}(i-t-1)\Big) =$$

$$= V[t] - (1-p)V[t+1] + (1-\gamma_f)p\Big((1-p)^{T-t-1} + \sum_{i=t+2}^{T}(1-p)^{T-i}(i-t-i+t+1)\Big) =$$

$$= V[t] - (1-p)V[t+1] + (1-\gamma_f)p\Big(\sum_{i=t+1}^{T}(1-p)^{T-i}\Big).$$

Finally, undoing the V transformation and obtaining the sum of the geometric progression we get:

$$\Delta W[t] = (1-p)^{T-t}(1+T-t)(1-\gamma_f p) - (1-p)(1-p)^{T-t-1}(1+T-t-1)(1-\gamma_f p) +$$

$$+ p(1-\gamma_f)\frac{(1-p)^{T-t-1}(1-p)-1}{1-p-1} =$$

$$= (1-p)^{T-t}(1-\gamma_f p) + (1-\gamma_f)(1-(1-p)^{T-t}) =$$

$$= (1-\gamma_f) + (1-p)^{T-t}(1-\gamma_f p - (1-\gamma_f)) =$$

$$= 1-\gamma_f + \gamma_f(1-p)^{T-t+1}$$

Firms will collude for all periods on which $\Delta W[t] > \lambda$. Being more specific, we define τ^{**} as the number of periods which is optimal for firms to collude, that is if $t^{**} = \min t$ such as $\Delta W[t] \geq \lambda$ and $\Delta W[t-1] < \lambda$, then $\tau^{**} = T - t^{**} + 1$.[18]

To obtain the explicit expression of τ^{**} we equal the increase in payoff of colluding to the competitive profits:

$$1-\gamma_f + \gamma_f(1-p)^{T-t+1} = \lambda \Leftrightarrow (1-p)^{T-t+1} = \frac{\gamma_f + \lambda - 1}{\gamma_f},$$

taking logarithms we get the value of t:

$$(T-t+1)\ln(1-p) = ln(\gamma_f + \lambda - 1) - \ln(\gamma_f) \Leftrightarrow t = T + 1 - \frac{ln(\gamma_f+\lambda-1) - \ln(\gamma_f)}{\ln(1-p)}$$

As t^{**} is the lowest integer period where the difference is non-negative, $t^{**} = \mathrm{Ceil}(t)$, so that:

$$\tau^{**} = \mathrm{Ceil}\left(\frac{ln(\gamma_f+\lambda-1) - \ln(\gamma_f)}{\ln(1-p)}\right) \tag{3}$$

Notice that if $\gamma_f + \lambda < 1$ this value goes to infinity, which makes sense, as this inequality implies that the fine imposed by the CA is smaller than the profits for collusion, so even after fines, collusion is more profitable than competition and, consequently, firms always choose to collude rather than compete. □

[18]Notice that $\Delta W[t]$ is increasing in t, $\frac{\partial W[t]}{\partial t} = \gamma_f(1-p)(-1)(1-p)T - t + 1\ln(1-p) > 0$, which confirms that optimally firms will collude only in the last periods of the game.

Example 1. To better understand the backward induction proof in the example we solve the game in the particular case where $T = 4$.

On figure A.1 we diagram the evolution of (undiscovered) collusion and final outcomes assuming that firms collude for the whole game. Notice that the sets of final outcomes repeat themselves in a iterative pattern (highlighted by colours). This iterative pattern is the key to solve both the example and Proposition 4 in general.

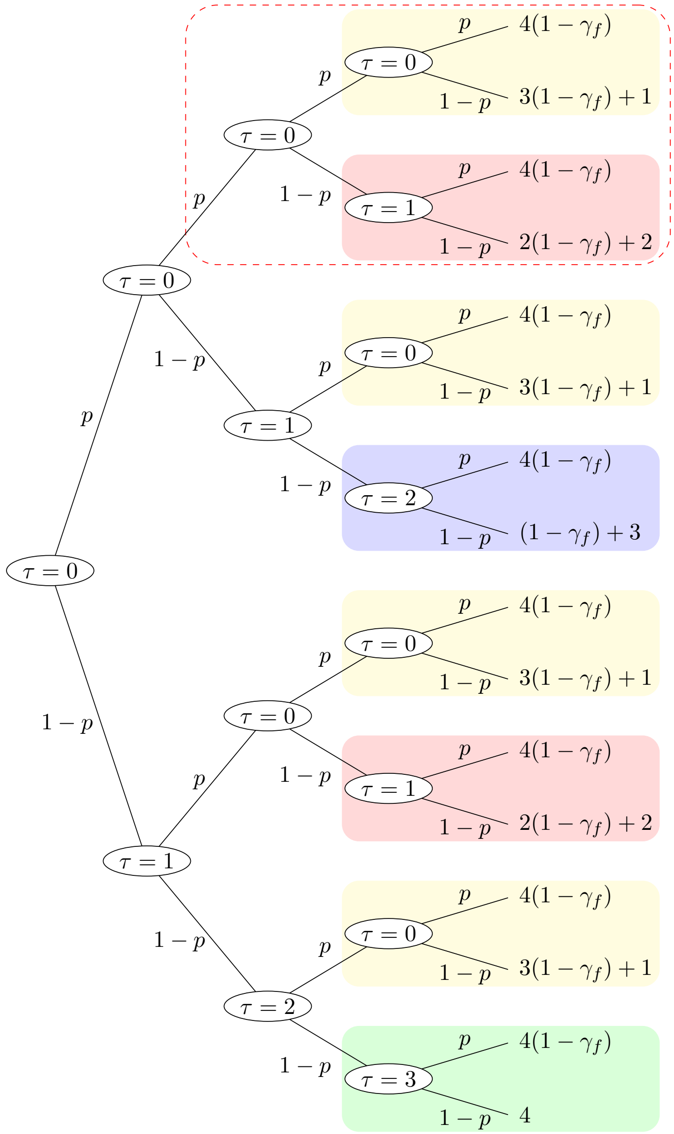

Figure A.1: Scheme of events and outcomes in the example game assuming collusion in all periods.

To solve the game recursively we compute the expected profits if firms decide to start colluding at the last period, that is $\tau_4 = 0$. Then,

$$W[t = 4] = (1-p) \cdot 1 + p(1-\gamma_f) = 1 - \gamma_f p = V[4],$$

where we have used the definition of V given in Equation (1).

A useful shortening for the rest of computations is the fact that for all terminal nodes, if a is the number of periods the cartel remains undetected $(\tau + 1)$ and b the number of detected and sanctioned periods the cartel , the expected payoff can be written as:

$$(1-p)(a + b(1-\gamma_f)) + p(a+b)(1-\gamma_f) = a(1-\gamma_f p) + b(1-\gamma_f).$$

The rest of the expected profits can be derived in the same fashion. That is if they start colluding on the third period (the red circled area in Figure A.1 represent this expectation, but subtracting the two discovered periods of collusion $2(1-\gamma_f)$):

$$\begin{aligned}W[t=3] &= 2(1-p)(1-\gamma_f p) + p\left((1-\gamma_f) + (1-\gamma_f p)\right) = \\ &= V[3] + pV[4] + p(1-\gamma_f)\end{aligned}$$

For the second period:

$$\begin{aligned}W[t=2] &= 3(1-p)^2(1-\gamma_f p) + p(1-p)\left(2(1-\gamma_f) + (1-\gamma_f p)\right) + \\ &+ p(1-p)\left((1-\gamma_f) + 2(1-\gamma_f p)\right) + p^2\left(2(1-\gamma_f) + (1-\gamma_f p)\right) = \\ &= V[2] + p(1-p)V[4] + pV[3] + p^2V[4] + \\ &+ (1-\gamma_f)\left(2p(1-p) + p(1-p) + 2p^2\right) = \\ &= V[2] + pV[3] + pV[4] + (1-\gamma_f)\left(p(1-p) + 2p\right).\end{aligned}$$

And finally, for the whole game:

$$\begin{aligned}W[t=1] &= 4(1-p)^3(1-\gamma_f p) + p(1-p)^2\left(3(1-\gamma_f) + (1-\gamma_f p)\right) + \\ &+ p(1-p)^2\left(2(1-\gamma_f) + 2(1-\gamma_f p)\right) + p^2(1-p)\left(3(1-\gamma_f) + (1-\gamma_f p)\right) + \\ &+ p(1-p)^2\left((1-\gamma_f) + 3(1-\gamma_f p)\right) + p^2(1-p)\left(3(1-\gamma_f) + (1-\gamma_f p)\right) + \\ &+ p^2(1-p)\left(2(1-\gamma_f) + 2(1-\gamma_f p)\right) + p^3\left(3(1-\gamma_f) + (1-\gamma_f p)\right) = \\ &= V[1] + pV[2] + (p(1-p) + p^2)V[3] + \\ &+ (p(1-p)^2 + p^2(1-p) + p^2(1-p) + p^3)V[4] + \\ &+ (1-\gamma_f)\Big(3p(1-p)^2 + 6p^2(1-p) + 3p^3 + 2p(1-p)^2 + \\ &+ 2p^2(1-p) + p(1-p^2)\Big) = \\ &= V[1] + pV[2] + pV[3] + pV[4] + (1-\gamma_f)\left(p(1-p)^2 + 2p(1-p) + 3p\right).\end{aligned}$$

To find the optimal decision of collusion for firms, they compare the increase in profits of colluding for another period with the competitive ones. These increases also have a neat expression:

$$\begin{aligned}\Delta W[t=3] &= W[3] - W[4] = V[3] + pV[4] + p(1-\gamma_f) - V[4] = \\ &= V[3] - (1-p)V[4] + p(1-\gamma_f) = \\ &= 2(1-p)(1-\gamma_f p) - (1-p)(1-\gamma_f p) + p(1-\gamma_f) = \\ &= (1-p)(1-\gamma_f p) + p(1-\gamma_f) = 1 - \gamma_f + \gamma_f - 2\gamma_f p + \gamma_f p^2 = \\ &= (1-\gamma_f) + \gamma_f(1-p)^2\end{aligned}$$

For the next one we will start using the sum of a geometric progression which gives a more general solution:

$$
\begin{aligned}
\Delta W[t=2] = W[2] - W[3] = & \\
&= V[2] + pV[3] + pV[4] + (1-\gamma_f)\,(p(1-p) + 2p) - \\
&\quad - \big(V[3] + pV[4] + p(1-\gamma_f)\big) = \\
&= V[2] - (1-p)V[3] + p(1-\gamma_f)\big((1-p) + 1\big) = \\
&= 3(1-p)^2(1-\gamma_f p) - (1-p)2(1-p)(1-\gamma_f p) + p(1-\gamma_f)(1 + (1-p)) = \\
&= (1-p)^2(1-\gamma_f p) + p(1-\gamma_f)\frac{(1-p)(1-p) - 1}{1-p-1} = \\
&= (1-p)^2(1-\gamma_f p) + (1-\gamma_f)(1-(1-p)^2) = \\
&= (1-\gamma_f) + (1-p)^2(1-\gamma_f p - 1 + \gamma_f) = \\
&= (1-\gamma_f) + \gamma_f(1-p)^3
\end{aligned}
$$

The last marginal increase is a bit more burdensome but follows the same logic:

$$
\begin{aligned}
\Delta W[t=1] = W[1] - W[2] = & \\
&= V[1] + pV[2] + pV[3] + pV[4] + (1-\gamma_f)\big(p(1-p)^2 + 2p(1-p) + 3p\big) - \\
&\quad - \big(V[2] + pV[3] + pV[4] + (1-\gamma_f)\,(p(1-p) + 2p)\,\big) = \\
&= V[1] - (1-p)V[2] + p(1-\gamma_f)\big((1-p)^2 + (1-p) + 1\big) = \\
&= 4(1-p)^3(1-\gamma_f p) - (1-p)3(1-p)^2(1-\gamma_f p) + p(1-\gamma_f)(1 + (1-p) + (1-p)^2) = \\
&= (1-p)^2(1-\gamma_f p) + p(1-\gamma_f)\frac{(1-p)^2(1-p) - 1}{1-p-1} = \\
&= (1-p)^3(1-\gamma_f p) + (1-\gamma_f)(1-(1-p)^3) = \\
&= (1-\gamma_f) + (1-p)^3(1-\gamma_f p - 1 + \gamma_f) = \\
&= (1-\gamma_f) + \gamma_f(1-p)^4
\end{aligned}
$$

As stated in the general proof, firms on the Authority Branch will always collude for at least one period. To increase it up to two, the following condition should hold:

$$(1-\gamma_f) + \gamma_f(1-p)^2 > \lambda \Leftrightarrow 1 - \lambda > \gamma_f(1-(1-p)^2)$$

which roughly can be interpreted as follows: firms will increase their collusion if the increase in profits from collusion $1-\lambda$ overcome the increase in the probability of being sanctioned $\gamma_f(1-(1-p)^2)$. Notice that as firms increase the length of their agreement, the exponent of $(1-p)$ increases, which decreases the right-hand term of the inequality, as the risks of being caught increase.