THE

Teoría e Historia Económica
Working Paper Series

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WP 2022-01
February 2022

Departamento de Teoría e Historia Económica
Facultad de Ciencias Económicas y Empresariales
Universidad de Málaga
ISSN 1989-6908

Evaluation and strategic manipulation*

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February 10, 2022

Abstract

We consider the problem of a group of experts who have to rank a set of candidates. Society's optimal choice relies on experts' honest judgments about the deserving ranking. However, experts' judgments are impossible to verify. Moreover, experts' judgments do not entirely determine their preferences. Then, experts might want to misreport their judgments if, by doing so, some ranking that they like best is selected. To solve this problem, we have to design a mechanism where the experts interact so that the socially optimal ranking is implemented. Whether this is possible depends on (1) how experts' judgments are aggregated to determine the socially optimal ranking and (2) how experts' preferences relate to their judgments. We state necessary and sufficient conditions on these two elements for the socially optimal ranking to be implementable in dominant strategies and Nash equilibrium. Then, we study the implementability of some widely used judgment aggregation rules, including extensions of scoring and Condorcet consistent voting rules. Finally, we propose a non-trivial judgment aggregation rule that is Nash implementable.

Keywords: Evaluation; impartiality; manipulability; ranking of candidates; mechanism design; voting rules.
J.E.L. Classification Numbers: C72, D71, D78.

*I am grateful to Salvador Barberà, Luis Corchón, Bernardo Moreno, Pietro Salmaso, and Raghul Venkatesh for helpful comments. Financial assistance from Ministerio de Ciencia e Innovación under project PID2020-114309GB-I00 and Programa Operativo FEDER Andalucía 2014-2020 under project UMA18-FEDERJA-130 is gratefully acknowledged.

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1 Introduction

Economics studies resource allocation. Resources can be allocated by markets, planners, or contests. However, on some occasions, resources are allocated by evaluation. In these cases, experts gather to agree on a specific ranking according to which resources will be allocated. Examples of this type of situation include allocation of scholarships among students, distribution of funds among universities, or distribution of prizes, medals, and awards among the participants in a contest.

The typical structure of these problems is as follows. A group of experts has to rank candidates from best to worst. Different experts may have different judgments about what the deserving ranking is. Society's optimal choice relies on those judgments, *i.e.*, there is a social choice function (SCF) that identifies the socially optimal ranking based on the experts' (honest) judgments. However, experts' judgments are impossible to verify. Moreover, experts' judgments do not entirely determine their preferences. Then, an expert might want to misreport his judgment if, by doing so, some ranking that he likes best is selected. To solve this problem, we have to design a mechanism where the experts interact so that the socially optimal ranking is implemented. It is essential to differentiate between the SCF and the mechanism in this setting. The former reflects society's objectives, while the latter is the election procedure used in practice to implement these objectives.

Whether a given SCF is implementable may depend on (1) how the experts' preferences relate to their judgments, (2) the invoked game-theoretic equilibrium concept, and (3) the properties of the SCF itself.

In order to define the relationships between an expert's preferences and his judgments, we say that an expert is *impartial* with respect to two candidates x and y if the mechanism designer knows that, when comparing any two rankings that only differ in the ranks of x and y who, moreover, are ranked consecutively, the expert prefers the ranking where x and y are ranked among them according to his judgment. We say that an expert *favors* x over y if the mechanism designer knows that, when comparing any two rankings as defined above, the expert prefers the ranking where x goes before y, regardless of his judgment.[1]

Regarding the game-theoretic equilibrium concept, we start by study-

[1] It may also happen that the mechanism designer does not have any information about the preferences of an expert with respect to x and y.

ing dominant strategy implementation. A dominant strategy is optimal for an expert regardless of the actions of others, and then, whenever possible, implementation in dominant strategies is quite robust. Our first result identifies a necessary and sufficient condition for an SCF to be implementable in dominant strategies (Theorem 1). We call this condition *unresponsiveness to partial experts* (UPE). It requires that if two candidates change their relative positions in the socially optimal ranking, then at least one expert is impartial with respect to them and changes his judgment about their relative positions in the same way that the socially optimal ranking does.

Unfortunately, most reasonable SCFs do not satisfy UPE and therefore are not implementable in dominant strategies, regardless of how the experts' preferences relate to their judgments. Theorem 2 shows that, if an SCF satisfies *unanimity* (whenever all experts honestly believe that x is better than y, then x must go before y in the socially optimal ranking) and *non-dictatorship* (there is no dictator who always determine the socially optimal ranking), then it is not implementable in dominant strategies, even in the most favorable situation where all experts are impartial with respect to all pairs of candidates. Although Theorem 2 bears a close resemblance to Gibbard-Satterthwaite Theorem (Gibbard, 1973; Satterthwaite, 1975), they are independent results (see the discussion in Section 3).

Next, we consider Nash implementation. We identify a necessary condition that we call *weak unresponsiveness to partial experts* (WUPE), which must hold for an SCF in order to be implementable in Nash equilibrium (Theorem 3). This condition requires that, for the socially optimal ranking to change, there must be at least one expert who changes his judgment about the relative position of two candidates with respect to whom he is impartial so that he goes from agreeing with the socially optimal ranking to not agreeing with it on this matter.

Whether WUPE is sufficient for Nash implementation depends on how the experts' preferences relate to their judgments. We present two results in this regard. The first one shows that, unlike dominant strategies, having experts who favor some candidates over others may facilitate Nash implementation. In particular, if at least three experts have different *friends* (candidates they would like to favor over all others) or different *enemies* (candidates they would like to harm over all others), then WUPE is sufficient for Nash implementation (Theorem 4). However, having experts with friends or enemies is not necessary for Nash implementation. For example, if all experts are impartial with respect to all pairs of candidates, then WUPE plus a no veto condition are sufficient for Nash implementation (Theorem 5).

Voting rules are some of the most widely used judgment aggregation procedures. Under the implicit assumption that individuals behave truthfully, voting rules are often used as mechanisms. However, one can think of voting rules as collective choice rules that reflect society's objectives on what candidate should win. Given this interpretation, voting rules can be extended to SCFs that rank candidates in two different ways: *extended version* (the voting rule is applied once and the candidates are ranked according to the points they get) and *recursive version* (the voting rule is applied to find the winner; this candidate gets the first position in the ranking; then, we make a new profile of judgments without the previous candidate and use it to find the winner; this candidate gets the second position in the ranking; we repeat this process until all candidates have been ranked).

We study whether the extended and recursive SCF versions of some well-known voting rules are implementable in Nash equilibrium. Specifically, we analyze three scoring voting rules (Plurality, instant-runoff, and Borda) and two Condorcet consistent voting rules (Copeland and minimax). The results are primarily negative, regardless of how the experts' preferences relate to their judgments and what tie-breaking rule is used (Propositions 1 and 2). However, there is a notable exception. The recursive versions of the two Condorcet consistent voting rules are Nash implementable when there are precisely three candidates, all experts are impartial with respect to all pairs of candidates, and the tie-breaking rule satisfies specific properties (Theorem 6).

Finally, we propose a new and non-trivial SCF that, unlike what happens with the SCF versions of the voting rules analyzed, is implementable in Nash equilibrium regardless of the number of candidates. We call it *serial pairwise comparison* SCF. The ranking selected by this SCF results from a sequence of pairwise comparisons of candidates that follows a known order and only considers the judgments of those experts who are impartial with respect to each pair. We show that the serial pairwise comparison SCF satisfies WUPE (Proposition 3) and then, it is Nash implementable under the sufficient conditions stated above.

Related literature

Amorós (2009) is probably the closest paper to ours. It also analyzes a model where a group of experts ranks a set of candidates. However, it assumes that all experts have the same judgment about the deserving ranking.

In this case, the only reasonable SCF selects the deserving ranking on which all the experts agree. The paper studies necessary and sufficient conditions for the Nash implementability of this SCF. Amorós et al. (2002) analyze the same model as Amorós (2009), assuming that each expert favors one candidate over all others while being impartial with respect to the rest. They show that, under these restrictions on experts' preferences, it is possible to Nash implement the SCF that selects the deserving ranking in which all the experts agree.

There is a branch of the literature studying the problem of aggregating judgments of experts who are imperfectly informed about the state of nature and whose preferences may be different from the planner's preferences (e.g., Austen-Smith 1993; Gerardi et al., 2009; Bhattacharya and Mukherjee, 2013). In contrast to this literature, we do not assume that there is an actual and imperfectly observed state of nature, but rather the state itself is determined by the experts' judgments.

The present paper is also connected with the literature on information transmission between informed experts and uninformed decision-makers (e.g., Krishna and Morgan, 2001; Wolinsky, 2002). In this literature, a decision-maker tries to elicit as much information as possible from several experts. The experts share the same preferences, which differ from the decision-maker's.

Our paper is also related to some literature on strategic voting (e.g., Feddersen and Pesendorfer, 1998; Duggan and Martinelli, 2001)). This literature deals with decision-making by juries composed of strategic jurors who must choose one of two alternatives. In these papers, jurors agree on the overall objective, but they may disagree on which alternative best achieves that goal based on differential information.

The remainder of the paper is structured as follows. Section 2 formally introduces the model and definitions. Sections 3 and 4 discuss the necessary and sufficient conditions for dominant strategy and Nash equilibrium implementation, respectively. Section 5 studies the Nash implementability of extended and recursive SCF versions of voting rules. Section 6 presents the serial pairwise comparison SCF. Section 7 concludes. All the proofs are in the Appendix.

2 The model

A group E of $n \geq 2$ experts must rank a set C of $m \geq 2$ candidates. A ranking of candidates is a bijection $\pi : C \rightarrow \{1, ..., m\}$ mapping each candidate x to its rank $\pi(x)$. A natural way of representing rankings is as vectors. For example, for $C = \{a, b, c\}$, the ranking π with candidate a as first, candidate c as second, and candidate b as third is represented as $\pi = acb$. Let Π denote the set of all possible rankings of candidates.

Each expert i has an (honest) *judgment* about what is the deserving ranking of candidates, $\rho_i \in \Pi$. We say that expert i honestly believes that candidate x is better than candidate y if x is ranked in a lower position than y in ρ_i, *i.e.*, $\rho_i(x) < \rho_i(y)$.

A *social choice function* (SCF) is a function $f : \Pi^n \rightarrow \Pi$ that aggregates the experts' judgments to select the ranking that is considered to be socially optimal. Abusing notation, for each profile of judgments $\rho \in \Pi^n$, let $\pi_\rho^f \equiv f(\rho)$.

Experts have preferences over rankings that may depend on their judgments. Let $\Re$ denote the class of all complete, reflexive, and transitive preference relations over Π. A *preference function* for expert i is a mapping $R_i : \Pi \rightarrow \Re$ that associates with each possible judgment ρ_i a preference relation $R_i(\rho_i)$ (the strict part of $R_i(\rho_i)$ is denoted $P_i(\rho_i)$). If all preference functions are possible, preferences may be completely unrelated to judgments. In that case, there is no interesting way of selecting the ranking of candidates based on experts' judgments. Then, we introduce some restrictions on the family of admissible preference functions.

Let $[C]^2$ denote the set of all possible pairs of candidates. An expert is *impartial* with respect to a pair of candidates $xy \in [C]^2$ if, when comparing any two rankings that only differ in the ranks of x and y who are ranked consecutively, he prefers the ranking where x and y are ranked among them according to his judgment. An expert *favors* x over y if, when comparing any two rankings as defined above, he prefers the ranking where x goes before y, regardless of his judgment.

Each expert i is characterized by two disjoint and possible empty sets $I_i, F_i \subset [C]^2$ where:

(i) I_i is the set of pairs of candidates with respect to whom the mechanism designer knows that i is impartial, and

(ii) F_i is the set of pairs candidates such that the mechanism designer knows that i favors one over the other. In what follows, when we refer to a pair of candidates in F_i, we write first the name of the "favored" candidate, *i.e.*, if i favors x over y, we write $xy \in F_i$ and if i favors y over x we write $yx \in F_i$.

A preference function $R_i : \Pi \longrightarrow \Re$ is *admissible* for expert i at (I_i, F_i) if, for every $\rho_i, \pi, \hat{\pi} \in \Pi$ such that there is $xy \in [C]^2$ with:

(1) $\pi(x) + 1 = \pi(y) = \hat{\pi}(x) = \hat{\pi}(y) + 1$,
(2) $\pi(z) = \hat{\pi}(z)$ for every $z \in C \backslash \{x, y\}$, and either
(3.1) $xy \in I_i$ and $\rho_i(x) < \rho_i(y)$, or
(3.2) $xy \in F_i$,
we have $\pi \; P_i(\rho_i) \; \hat{\pi}$.

EXAMPLE 1 *Suppose that $C = \{a, b, c\}$. Then, $[C]^2 = \{ab, ac, bc\}$ and $\Pi = \{abc, acb, bac, bca, cab, cba\}$. Suppose that the mechanism designer knows that expert i is impartial with respect to ab and favors c over b, i.e., $I_i = \{ab\}$ and $F_i = \{cb\}$. Because $ab \in I_i$, every admissible preference function for i is such that (i) whenever $\rho_i(a) < \rho_i(b)$ then abc $P_i(\rho_i)$ bac and cab $P_i(\rho_i)$ cba, and (ii) whenever $\rho_i(b) < \rho_i(a)$ then bac $P_i(\rho_i)$ abc and cba $P_i(\rho_i)$ cab. Because $cb \in F_i$, every admissible preference function for i is such that, for every $\rho_i \in \Pi$, cba $P_i(\rho_i)$ bca and acb $P_i(\rho_i)$ abc. Table 1 summarizes the requirements for an admissible preference function. Many preference functions satisfy these conditions. For example, the requirements do not determine whether abc $R_i(\rho_i)$ cba or cba $R_i(\rho_i)$ abc, whatever the judgment ρ_i is. Similarly, if $\rho_i \in \{abc, acb, cab\}$ then acb $P_i(\rho_i)$ bac but if $\rho_i \in \{bac, bca, cba\}$ the requirements do no determine whether acb $P_i(\rho_i)$ bac or bac $P_i(\rho_i)$ acb.*

$R_i : \Pi \longrightarrow \Re$

If $\rho_i \in \{abc, acb, cab\}$				If $\rho_i \in \{bac, bca, cba\}$			
$R_i(\rho_i)$ is such that				$R_i(\rho_i)$ is such that			
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
abc	cab	cba	acb	bac	cba	cba	acb
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
bac	cba	bca	abc	abc	cab	bca	abc
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 1 Admissible preference functions for expert i in Example 1.

If an expert is impartial with respect to all pairs of candidates, his most preferred ranking is the one that matches his judgment. However, several preference functions are admissible for the expert, even in this case.

EXAMPLE 2 *Suppose that $C = \{a, b, c\}$. Suppose that the mechanism designer knows that expert i is impartial with respect to all pairs of candidates, i.e., $I_i = \{ab, ac, bc\}$. Let $\rho_i = abc$. Every admissible preference function for i is such that (1) $abc\ P_i(\rho_i)\ bac$ and $cab\ P_i(\rho_i)\ cba$ (because $ab \in I_i$ and $\rho_i(a) < \rho_i(b)$), (2) $acb\ P_i(\rho_i)\ cab$ and $bac\ P_i(\rho_i)\ bca$ (because $ac \in I_i$ and and $\rho_i(a) < \rho_i(c)$), and (3) $abc\ P_i(\rho_i)\ acb$ and $bca\ P_i(\rho_i)\ cba$ (because $bc \in I_i$ and $\rho_i(b) < \rho_i(c)$). In Table 2 we show the six strict preference relations that satisfy the previous requirements.[2] A similar argument applies to the other five possible judgments of expert i.*

$R_i(abc)$					
abc	abc	abc	abc	abc	abc
acb	acb	acb	bac	bac	bac
cab	bac	bac	bca	acb	acb
bac	cab	bca	acb	bca	cab
bca	bca	cab	cab	cab	bca
cba	cba	cba	cba	cba	cba

Table 2 Admissible strict preference relations for expert i Example 2.

Let $\mathcal{R}(I_i, F_i)$ be the class of all preference functions that are admissible for i at (I_i, F_i). A *jury configuration* is a list $(I, F) = ((I_1, F_1), ..., (I_n, F_n))$ and it represents the information the mechanism designer has about the experts. Given a jury configuration (I, F) and a pair of candidates xy, let $E^I_{xy} = \{i \in E \mid xy \in I_i\}$ be the set of experts who are impartial with respect to xy and let $E^F_{xy} = \{i \in E \mid xy \in F_i\}$ be the set of experts who favor x over y.

A *mechanism* is a pair $\Gamma = (M, g)$, where $M \equiv \times_{i \in E} M_i$, M_i is a message space for expert i, and $g : M \to \Pi$ is an outcome function. Given a jury configuration (I, F), a *state* is a profile (ρ, R), where $\rho = (\rho_i)_{i \in E}$ is the profile of experts' judgments and $R = (R_i)_{i \in E}$ is the profile of experts' preference

[2]Note that, since indifference in preferences is allowed, the number of preference relations that satisfy the requirements is higher (for example, bac could be indifferent to acb, or bca could be indifferent to cab).

functions. Given a jury configuration (I, F), let $S(I, F) = \Pi^n \times \mathcal{R}(I, F)$ be the set of admissible states, where $\mathcal{R}(I, F) = \times_{i \in E} \mathcal{R}(I_i, F_i)$. Given a game-theoretic *equilibrium concept* $\mathcal{E}$, a mechanism Γ, and a state (ρ, R), let $\mathcal{E}(\Gamma, \rho, R) \subset M$ denote the set of $\mathcal{E}$-equilibrium messages of Γ at (ρ, R). The corresponding ranking of candidates is denoted $g(\mathcal{E}(\Gamma, \rho, R))$.

Given a jury configuration (I, F) and an equilibrium concept $\mathcal{E}$, a mechanism $\Gamma = (M, g)$ *implements* an SCF f in $\mathcal{E}$-equilibrium, if, for each state $(\rho, R) \in S(I, F)$, $g(\mathcal{E}(\Gamma, \rho, R)) = \pi^f_\rho$.

3 Dominant strategy implementation

Whether an SCF is implementable may depend on the jury configuration and the invoked game-theoretic equilibrium concept. Regarding the latter, the most demanding notion is that of dominant strategy equilibrium.

Given a mechanism $\Gamma = (M, g)$, $m \in M$ is a *dominant strategy equilibrium* of Γ at state (ρ, R) if, for every $i \in E$, $\hat{m}_i \in M_i$, and $\hat{m}_{-i} \in M_{-i}$, $g(m_i, \hat{m}_{-i})$ $R_i(\rho_i)$ $g(\hat{m}_i, \hat{m}_{-i})$.

Our first result identifies a necessary and sufficient condition for an SCF to be implementable in dominant strategies: If two candidates, x and y, change their relative positions in the socially optimal ranking when the profile of experts' judgments change from ρ to $\hat{\rho}$, then there is at least one expert that is impartial with respect to xy and such that, when moving from ρ to $\hat{\rho}$, he changes his judgment about the relative positions of x and y in the same way as they change in the socially optimal ranking.

DEFINITION Given a jury configuration (I, F), an SCF f satisfies *unresponsiveness to partial experts* (UPE) if, for each $\rho, \hat{\rho} \in \Pi^n$ and $xy \in [C]^2$ such that $\pi^f_\rho(x) < \pi^f_\rho(y)$ and $\pi^f_{\hat{\rho}}(y) < \pi^f_{\hat{\rho}}(x)$, there is some $i \in E$ with $xy \in I_i$, $\rho_i(x) < \rho_i(y)$, and $\hat{\rho}_i(y) < \hat{\rho}_i(x)$.

THEOREM 1 *Given a jury configuration (I, F), an SCF f is implementable in dominant strategies if and only if it satisfies UPE.*

To prove Theorem 1, we show that, when implementing in dominant strategies, we can restrict our attention to a reduced version of the direct mechanism where experts only need to announce their judgments (not their preferences).[3] If the SCF does not satisfy UPE, announcing the true judgments is not a dominant strategy equilibrium of the former mechanism and the SCF is not implementable in dominant strategies. If the SCF satisfies UPE, the reduced version of the direct mechanism implements the SCF in dominant strategies (truth-telling is a dominant strategy equilibrium and any other possible equilibrium yields the same ranking).

Whether an SCF satisfies UPE may depend on the specific jury configuration. In this regard, the most favorable jury configuration is one in which all experts are impartial with respect to all pairs of candidates (having experts who favor some candidates over others is of no help in fulfilling UPE). Unfortunately, even in this case, most reasonable SCFs do not satisfy UPE (and therefore they are not implementable in dominant strategies).

To see this point, we define two reasonable properties of SCFs. The first requires the SCF to be such that, whenever all experts honestly believe that x is better than y, x must go before y in the socially optimal ranking. The second property requires that there be no dictator who always determines the socially optimal ranking.

DEFINITION An SCF f satisfies *unanimity* if, for every $x, y \in C$ and $\rho \in \Pi^n$, whenever $\rho_i(x) < \rho_i(y)$ for every $i \in E$, then $\pi_\rho^f(x) < \pi_\rho^f(y)$.

DEFINITION An SCF f is *non-dictatorial* if, for every $i \in E$ there exist some $\rho \in \Pi^n$ such that $\pi_\rho^f \neq \rho_i$.

Our following result shows that if an SCF satisfies unanimity and non-dictatorship, it is not implementable in dominant strategies, even in the most favorable case where all experts are impartial with respect to all pairs of candidates.

THEOREM 2 *Suppose that $m \geq 3$. If an SCF f satisfies unanimity and non-dictatorship, it is not implementable in dominant strategies, regardless of the jury configuration.*

[3] The direct mechanism associated with a social choice rule is a mechanism where the message space for each agent is his space of admissible types and the outcome function is the social choice rule itself. In our model, an expert's type would be defined by his judgment and his preference relation over Π.

Relation of Theorem 2 to Arrow's impossibility Theorem

It turns out that if an SCF satisfies UPE, the relative position of any two candidates in the socially optimal ranking depends only on the relative positions of the two candidates in the experts' judgments. Let us call this property *independence of irrelevant candidates.*

Let us now reinterpret each ranking of candidates π as a preference relation over C (so that $\pi(x) < \pi(y)$ is interpreted as candidate x being preferred to candidate y according to the preference relation π). In this case, an SCF $f : \Pi^n \to \Pi$ would be an Arrovian social welfare function aggregating individuals' preference relations into a social preference relation. We can reinterpret every property defined on an SCF into a property defined on a social welfare function: (1) unanimity requires that, if everyone prefers any candidate x to any candidate y, then x is socially preferred to y; (2) non-dictatorship requires that there is no agent i such that whenever this agent prefers any candidate x to any candidate y, then x is socially preferred to y, no matter what others prefer; (3) independence of irrelevant candidates requires that whenever a pair of candidates are ranked the same way in two preference profiles, then the social preference relation must order these two candidates identically across the two profiles.

Arrow's impossibility Theorem (Arrow, 1951) states that no social welfare function satisfies the three previous properties with at least three alternatives (candidates) and unrestricted domain of preferences (judgments). Therefore, Theorem 2 can be obtained as a corollary of Theorem 1 and Arrow's impossibility Theorem.

Relation of Theorem 2 to Gibbard-Satterthwaite Theorem

Theorem 2 bears a close resemblance to Gibbard-Satterthwaite (GS) Theorem (Gibbard, 1973; Satterthwaite, 1975). However, as we explain below, it is an independent result.

In the GS Theorem setting, a social choice function selects an outcome based on a group of agents' preferences over the set of feasible outcomes. Let us call it Gibbard-Satterthwaite social choice function (GS-SCF). A GS-SCF is said to be *unanimous* if it selects the outcome that is top-ranked in the preferences of everyone whenever that outcome exists. A GS-SCF is said to be *dictatorial* if it always selects the top-ranked outcome of the same agent. A GS-SCF is said to be *strategy-proof* if, in the direct mechanism, it is a dominant strategy for each agent to report his preferences truthfully. The GS Theorem states that if there are at least three outcomes and every strict preference relation over outcomes is admissible for each agent, then no GS-SCF satisfies unanimity, non-dictatorship, and strategy-proofness. Because strategy-proofness is necessary for implementation in dominant strategies, it implies that no unanimous and non-dictatorial GS-SCF is implementable in dominant strategies.

In our model, the agents are the experts, and the set of feasible outcomes is the set of all possible rankings of candidates. Moreover, an SCF does not select an outcome based on the experts' preferences over rankings but based on their judgments. Nevertheless, given a jury configuration, the experts' preferences are related to their judgments, and then an SCF can be translated into a GS-SCF. Even so, Theorem 2 cannot be deduced from GS Theorem. To see this, consider the following example. Let $C = \{a, b, c\}$ and that the jury configuration is such that all experts are impartial with respect to all pairs of candidates. Then, the set of feasible outcomes is $\Pi = \{abc, acb, bac, bca, cab, cba\}$ and (I, F) is such that $I_i = \{ab, ac, bc\}$ for every $i \in E$. Given any expert $i \in E$, let $(\rho_i, R_i) \in \Pi \times \mathcal{R}(I_i, F_i)$ be such that $\rho_i = abc$. In this case, as shown in Example 2, only the six strict preference relations depicted in Table 2 are admissible for him. A similar argument applies to the other five possible judgments of expert i, which yields 36 different strict preference relations. Hence, although there are $6! = 720$ different strict preference relations over the elements of Π, only 36 of them are admissible for each expert. Because one of the requirements of the GS Theorem is that every strict preference relation over outcomes is admissible for each agent, this theorem has no bite here. However, Theorem 2 states that, even in this case where all experts are impartial with respect to all pairs of candidates, no SCF satisfying unanimity and non-dictatorship can be implemented in dominant strategies.

4 Nash implementation

The message delivered by Theorem 2 is that implementation in dominant strategies has minimal success when there are at least three candidates. In this section, we relax the game-theoretic equilibrium concept and analyze implementation in Nash equilibrium.

Given a mechanism $\Gamma = (M, g)$, $m \in M$ is a *Nash equilibrium* of Γ at state (ρ, R) if, for every $i \in E$ and $\hat{m}_i \in M_i$, $g(m_i, m_{-i})\ R_i(\rho_i)\ g(\hat{m}_i, m_{-i})$.

We start by identifying the following necessary condition for an SCF to be implementable in Nash equilibrium: For the optimal ranking to change when the experts change their judgments from ρ to $\hat{\rho}$, there must be at least one expert who changes his judgment about the relative position of two candidates with respect to whom he is impartial, so that he goes from agreeing with π^f_ρ to not agreeing with π^f_ρ on this matter.

DEFINITION Given a jury configuration (I, F), an SCF f satisfies *weak unresponsiveness to partial experts* (WUPE) if, for every $\rho, \hat{\rho} \in \Pi^n$ with $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$, there exist $i \in E$ and $xy \in I_i$ such that $\pi^f_\rho(x) < \pi^f_\rho(y)$, $\rho_i(x) < \rho_i(y)$, and $\hat{\rho}_i(y) < \hat{\rho}_i(x)$.[4]

THEOREM 3 *Given a jury configuration (I, F), if an SCF f is implementable in Nash equilibrium then it satisfies WUPE.*

The idea behind Theorem 3 is that, if f does not satisfy WUPE, then there are two states, (ρ, R) and $(\hat{\rho}, \hat{R})$, for which the socially optimal ranking is different, $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$, and such that, when going from (ρ, R) to $(\hat{\rho}, \hat{R})$, the desirability of π^f_ρ does not deteriorate for any expert. Then, every Nash equilibrium of a mechanism Γ at (ρ, R) resulting in π^f_ρ is also a Nash equilibrium of Γ at $(\hat{\rho}, \hat{R})$, which makes implementation in Nash equilibrium impossible.

Whether WUPE is sufficient for Nash implementation depends on the jury configuration. We present two results in this regard for the case of three or more experts. Before presenting the first result, we introduce two new concepts.

DEFINITION Given a jury configuration (I, F), we say that $x \in C$ is a *friend* of $i \in E$ if $xy \in F_i$ for every $y \in C \setminus \{x\}$.

DEFINITION Given a jury configuration (I, F), we say that $x \in C$ is an *enemy* of $i \in E$ if $yx \in F_i$ for every $y \in C \setminus \{x\}$.

Roughly speaking, candidate x is a friend of an expert if the expert favors x over all other candidates. Similarly, a candidate x is an enemy of an expert if the expert favors all other candidates over x. It turns out that if at least three experts have different friends or at least three experts have different enemies, then WUPE is sufficient for Nash implementation.

[4]Note that UPE implies WUPE. The reason is that, if $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$, then there exists a pair xy such that $\pi^f_\rho(x) < \pi^f_\rho(y)$ and $\pi^f_{\hat{\rho}}(y) < \pi^f_{\hat{\rho}}(x)$. However, WUPE does not imply UPE (we show examples in the next sections).

THEOREM 4 *Suppose that $n \geq 3$ and that the jury configuration is such that either (i) at least three experts have different friends or (ii) at least three experts have different enemies. If an SCF f is onto and satisfies WUPE, it is Nash implementable.*

This result reveals that, unlike dominant strategies, having experts who favor some candidates over others may facilitate Nash implementation. One of the greatest difficulties when implementing an SCF in Nash equilibrium is ensuring that the mechanism does not have "bad" equilibria that result in rankings other than the socially optimal. Having experts with friends or enemies is of great help in eliminating these bad equilibria.

The proof of Theorem 4 is based on the construction of a mechanism that Nash implements any SCF under the conditions assumed. Our mechanism is similar to Maskin's *canonical* mechanism for Nash implementation (Maskin, 1999). However, there is an essential difference between the two. Unlike what happens with Maskin's canonical mechanism, in our mechanism, each expert does not have to announce the entire state (ρ, R) but only the profile of judgments ρ. Therefore, each expert does not need to know the exact preference relations over rankings of others.

Although having experts who favor some candidates over others can make Nash implementation easier, it is not necessary. Our next result shows that if all experts are impartial with respect to all pairs of candidates and the SCF satisfies a "no veto" condition, then WUPE is sufficient for Nash implementation.

DEFINITION Suppose $n \geq 3$. An SCF f satisfies *no veto* if, for every $\rho \in \Pi^n$, $\pi \in \Pi$, and $j \in E$, if $\rho_i = \pi$ for every $i \neq j$ then $\pi^f_\rho = \pi$.

No veto requires that, if all experts but possibly one honestly believe that π is the deserving ranking, then π is the socially optimal ranking.

THEOREM 5 *Suppose that $n \geq 3$ and that the jury configuration is such that all experts are impartial with respect to all pairs of candidates. If an SCF f satisfies WUPE and no veto, it is Nash implementable.*

To prove Theorem 5, we use the same mechanism as in the proof of Theorem 4. WUPE and no veto are related to *monotonicity* and *no veto power*, two well-known sufficient conditions for Nash implementation in general environments (Maskin, 1999). However, while the latter properties refer to how the chosen outcome changes with the agents' preferences, the former refer to how the chosen ranking changes with the experts' judgments (not with their preferences).

5 Voting rules as SCFs

This section studies the Nash implementability of the SCF version of some well-known voting rules.[5] Voting rules are processes for aggregating individual judgments to choose one winner from a pool of candidates. They can be classified into two broad types.

Scoring voting rules: The voting rule assigns points to the candidates based on how the voters rank them, and the winner is the candidate with the highest total number of points.

Condorcet consistent voting rules: The voting rule chooses the winner based on a series of majority comparisons between the candidates and is such that, if a candidate beats every other candidate in these comparisons, he is selected.[6]

One can think of voting rules as collective choice rules that reflect society's objectives on what candidate should win.[7] Given this interpretation, voting rules can be extended to SCFs that rank candidates in two ways.

Extended voting rule SCF: The voting rule is applied once to the profile of experts' judgments, and the candidates are ranked according to the points they get.

Recursive voting rule SCF: The voting rule is applied to the profile of experts' judgments to find the winner candidate; this candidate gets the first position in the ranking; then, we make a new profile of judgments without the previous candidate and use it to find the winner; this candidate gets the

[5] All the SCFs analyzed in this section satisfy unanimity and non-dictatorship, and then, by Theorem 2, they are not implementable in dominant strategies.

[6] The Condorcet winner is a candidate who beats each opponent in a pairwise comparison. A voting rule is Condorcet consistent if it selects the Condorcet winner whenever it exists.

[7] Under the implicit assumption that individuals behave truthfully, voting rules are often used as mechanisms (if individuals had no incentive to misreport their judgments, the mechanism implementing the optimal outcome prescribed by the voting rule could be the voting rule itself).

second position in the ranking; we repeat this process until all candidates have been ranked.

Some additional notation will be useful for our analysis. For each $\rho \in \Pi^n$ and $\hat{C} \subseteq C$ let $\rho^{\hat{C}}$ denote the restriction of ρ to $\hat{C}$, *i.e.*, for each $i \in E$, $\rho_i^{\hat{C}}$ is a ranking of the candidates in $\hat{C}$ where, for each $x, y \in \hat{C}$, $\rho_i^{\hat{C}}(x) < \rho_i^{\hat{C}}(y)$ if and only if $\rho_i(x) < \rho_i(y)$. For each voting rule, there is a family of extended and recursive voting rule SCFs that differ only in the way they break ties. A *tie-breaking rule* is a function $t : 2^C \to C$ that, given a set of candidates in C, chooses one of them.

5.1 Scoring voting rules as SCFs

Plurality, instant-runoff, and Borda rules are three representative scoring voting rules.

Plurality: A candidate gets one vote for every voter who ranks him first, and the winner is the candidate with the most votes.

Instant-runoff: A candidate gets one vote for every voter who ranks him first. If a candidate gets a majority, he wins. Otherwise, the candidate with the fewest votes is eliminated, and any votes for that candidate are redistributed to the voters' next choice. This process continues until one candidate has a majority.

Borda: A candidate gets m points for every voter who ranks him first, $m-1$ points for a second-place vote, and so on. The Borda score for a candidate is the sum of the points that he gets from all voters. The winner is the candidate with the highest Borda score.

Unfortunately, the extended and recursive SCF versions of these scoring voting rules fail to be Nash implementable when $m \geq 3$, regardless of the tie-breaking rule and the jury configuration.

PROPOSITION 1 *If $m \geq 3$, the extended and recursive SCF versions of the plurality, instant-runoff, and Borda voting rules fail to be Nash implementable, regardless of the tie-breaking rule and the jury configuration.*

To prove Proposition 1, we propose examples in which WUPE, the necessary condition for Nash implementation, is not satisfied. The examples do not depend on the tie-breaking rule or the jury configuration.

5.2 Condorcet consistent rules as SCFs

Copeland and minimax rules are two characteristic Condorcet consistent voting rules.

Copeland: Each pair of candidates is compared to determine which of the two is considered better by a majority. That candidate is awarded 1 point. Each candidate is awarded $\frac{1}{2}$ point if there is a tie. The Copeland score for a candidate is the sum of the points that he gets in its pairwise comparisons with all other candidates. The winner is the candidate with the most points.

Minimax: Each candidate is pairwise compared with each other to determine which of the two is considered better by a majority. The winner is the candidate whose largest pairwise defeat is smaller than the largest pairwise defeat of any other candidate. The strength of a pairwise defeat is measured as the number of experts that honestly believe that the winning candidate is better minus the number of candidates that honestly believe that the losing candidate is better.

Similar to what happens with scoring voting rules, the extended SCF versions of these Condorcet consistent voting rules fail to be Nash implementable when $m \geq 3$, regardless of the tie-breaking rule and the jury configuration. The same negative result holds for the recursive SCF versions of these voting rules but, in this case, only when $m \geq 4$.

PROPOSITION 2

(1) If $m \geq 3$, the extended SCF versions of the Copeland and minimax voting rules fail to be Nash implementable, regardless of the tie-breaking rule and the jury configuration.

(2) If $m \geq 4$, the recursive SCF versions of the Copeland and minimax voting rules fail to be Nash implementable, regardless of the tie-breaking rule and the jury configuration.

There is a notable exception to the previous negative results. If $m = 3$, the recursive SCF versions of the Copeland and minimax voting rules are Nash implementable for particular tie-breaking rules and jury configurations. Let us define two families of tie-breaking rules that are of interest in this case.

DEFINITION A tie-breaking rule t satisfies *non-favoritism* if, for every $x \in C$ there exists some $y \in C$ such that, if $\hat{C} = \{x, y\}$, then $t(\hat{C}) = y$.

DEFINITION A tie-breaking rule t is *linear-ordered* if, for every $\hat{C}, \bar{C} \subseteq C$ and every $x, y \in \hat{C}, \bar{C}$, if $t(\hat{C}) = x$, then $t(\bar{C}) \neq y$.

Roughly speaking, t satisfies non-favoritism if there is no candidate x in favor of whom t breaks every 2-candidates tie in which x is involved. Linear-ordered tie-breaking rules are tie-breaking rules that decide upon a winner based on some linear order over candidates.

The following theorem state two positive results on the Nash implementability of the recursive SCF versions of Condorcet consistent rules when $m = 3$, $n \geq 3$, and all experts are impartial with respect to all pairs of candidates. Under these conditions, every recursive SCF version of the Copeland rule with a tie-breaking rule that satisfies non-favoritism and every recursive SCF version of the minimax rule with a linear-ordered tie-breaking rule are Nash implementable.

THEOREM 6 *Suppose that $m = 3$, $n \geq 3$, and the jury configuration is such that all experts are impartial with respect to all pairs of candidates.*

(1) *If f is a recursive SCF version of the Copeland voting rule with a tie-breaking rule that satisfies non-favoritism, it is Nash implementable.*

(2) *If f is a recursive SCF version of the minimax voting rule with a linear-ordered tie-breaking rule, it is Nash implementable.*

To prove Theorems 6 we show that, under the conditions stated there, the SCFs satisfy WUPE and no veto, the sufficient conditions for Nash implementation stated in Theorem 4.

Remark 1 *A recursive SCF version of the Copeland voting rule fail to be Nash implementable if its tie-breaking rule does not satisfy non-favoritism, even if $m = 3$, $n \geq 3$, and $E^I_{xy} = E$ for every $xy \in [C]^2$. The same is true for a recursive SCF version of the minimax voting rule if its tie-breaking rule is not linear-ordered. (See Appendix).*

Remark 2 *The fact that a recursive SCF version of a Condorcet consistent voting rule is Nash implementable does not imply that it can be implemented through the direct mechanism associated with it. The reason is that, in this mechanism, every ranking can be obtained as a Nash equilibrium result in every state. Then, in order to implement the SCF, we should use a mechanism other than the recursive version of the voting rule itself.*

6 Serial pairwise comparison SCF

Of all the SCFs analyzed in the previous section, only the recursive versions of the Condorcet consistent voting rules satisfy WUPE, the necessary condition for Nash implementation, and this only when $m = 3$. This section proposes a non-trivial SCF that satisfies WUPE for every $m \geq 2$, regardless of the jury configuration.

DEFINITION Let $\pi^* \in \Pi$ be an arbitrary ranking of candidates, interpreted as the status quo. For each $k \in \{1, ..., m\}$, let $x_k^* \in C$ be the k-th candidate in π^*, *i.e.*, $\pi^*(x_k^*) = k$. The *serial pairwise comparison* SCF, f^*, is such that, for each profile of judgments $\rho \in \Pi^n$, the socially optimal ranking $\pi_\rho^{f^*}$ is defined by the following rules:
Rule 1. x_1^* is pairwise compared with every other candidate $y \in C \backslash \{x_1^*\}$ to determine which of the two candidates is considered better by a majority among the experts who are impartial with respect to them. If at least half of the experts in $E_{x_1^* y}^I$ honestly believe that x_1^* is better than y, then x_1^* is ranked before y. Otherwise, y is ranked before x_1^*.[8]
Rule k (for each $k \in \{2, ..., m\}$). x_k^* is pairwise compared with every other candidate $y \in C \backslash \{x_1^*, ..., x_k^*\}$ whose relative ranking with respect to y has not been set by any previous Rule l (with $l < k$). If at least half of the experts who are impartial with respect to $x_k^* y$ honestly believe that x_k^* is better than y, then x_k^* is ranked before y. Otherwise, y is ranked before x_k^*.

EXAMPLE 3 *Suppose that* $C = \{a, b, c, d\}$ *and* $E = \{1, 2, 3, 4\}$. *Then* $[C]^2 = \{ab, ac, ad, bc, bd, cd\}$. *Let* (I, F) *be a jury configuration such that* $I_1 = \{bc, bd, cd\}$, $I_2 = \{ac, ad, cd\}$, $I_3 = \{ab, ad, bd\}$, *and* $I_4 = \{ab, ac, bc\}$.[9] *Suppose that the arbitrary ranking used by* f^* *is* $\pi^* = abcd$. *Consider the profile of judgments* ρ *depicted in Table 3. Let us calculate the ranking selected by* f^* *at* ρ.

[8] In particular, if no expert is impartial with respect to the pair $x_1^* y$, then x_1^* is ranked before y.

[9] This could be the case when the experts are the candidates themselves. Suppose that expert 1 is candidate a, expert 2 is candidate b, expert 3 is candidate c, and expert 4 is candidate d. Each expert is impartial with respect to every pair of candidates that do not include him. Although not relevant to this example, in this case it would be reasonable for each expert to be a friend of himself, *i.e.*, $F_1 = \{ab, ac, ad\}$, $F_2 = \{ba, bc, bd\}$, $F_3 = \{ca, cb, cd\}$, and $F_4 = \{da, db, dc\}$.

Rule 1. The first candidate in π^*, a, *is pairwise compared with the other three candidates. In each of these comparisons, we only consider the judgments of those experts who are impartial with respect to the given pair. Because* $E^I_{ab} = \{3,4\}$, $\rho_3(a) < \rho_3(b)$ *and* $\rho_4(a) < \rho_4(b)$ *(at least half of the experts in* E^I_{ab} *honestly believe that* a *is better than* b*), then* $\pi^{f^*}_\rho(a) < \pi^{f^*}_\rho(b)$. *Because* $E^I_{ac} = \{2,4\}$, $\rho_2(c) < \rho_2(a)$ *and* $\rho_4(a) < \rho_4(c)$ *(at least half of the experts in* E^I_{ac} *honestly believe that* a *is better than* c*), then* $\pi^{f^*}_\rho(a) < \pi^{f^*}_\rho(c)$.[10] *Finally, because* $E^I_{ad} = \{2,3\}$, $\rho_2(d) < \rho_2(a)$ *and* $\rho_3(d) < \rho_3(a)$ *(more than half of the experts in* E^I_{ad} *honestly believe that* d *is better than* a*), then* $\pi^{f^*}_\rho(d) < \pi^{f^*}_\rho(a)$.

Rule 2. The second candidate in π^* *is* b. *Note that* c *is the only candidate whose relative ranking with respect to* b *has not been set by Rule 1 (from Rule 1 we have* $\pi^{f^*}_\rho(d) < \pi^{f^*}_\rho(a) < \pi^{f^*}_\rho(b)$*). Then,* b *is pairwise compared with* c *for those experts who are impartial with respect to them. Because* $E^I_{bc} = \{1,4\}$, $\rho_1(b) < \rho_1(c)$ *and* $\rho_4(b) < \rho_4(c)$ *(at least half of the experts in* E^I_{bc} *honestly believe that* b *is better than* c*), then* $\pi^{f^*}_\rho(b) < \pi^{f^*}_\rho(c)$.

From Rules 1 and 2, we have $\pi^{f^*}_\rho = dabc$. *Note that all the experts who are impartial between* c *and* d *honestly believe that* c *is better than* d *(*$E^I_{cd} = \{1,2\}$, $\rho_1(c) < \rho_1(d)$ *and* $\rho_2(c) < \rho_2(d)$*). However, we do not consider this information since the relative ranking between* c *and* d *is determined by the pairwise comparisons of* a *with* c *and* d *in Rule 1. Something similar happens with the pairwise comparison between* b *and* d.

ρ

ρ_1	ρ_2	ρ_3	ρ_4
b	c	d	a
c	d	a	b
a	b	c	d
d	a	b	c

Table 3 Profile of judgments in Example 3.

The serial pairwise comparison SCF satisfies WUPE, the necessary condition for Nash implementation, regardless of the jury configuration.[11]

[10] In this case, there is a tie that breaks in favor of a because $\pi^*(a) < \pi^*(c)$.

[11] If the jury configuration satisfies the mild requirement that, for every pair of candidates, there is at least one expert who is impartial with respect to them, the serial pairwise comparison SCF satisfies unanimity and non-dictatorship. Hence, from Theorem 2, it is not implementable in dominant strategies.

PROPOSITION 3 *The serial pairwise comparison SCF f^* satisfies WUPE, regardless of the jury configuration.*

As discussed in Section 3, whether WUPE is sufficient for Nash implementation depends on the jury configuration. In this regard, we have the following two corollaries.

COROLLARY 1 *Suppose that $n \geq 3$. Suppose that the jury configuration is such that (1) for each pair of candidates, there is at least one expert who is impartial with respect to them, and either (2.1) at least three experts have different friends or (2.2) at least three experts have different enemies. Then, the serial pairwise comparison SCF f^* is Nash implementable.*

COROLLARY 2 *Suppose that $n \geq 3$. Suppose that the jury configuration is such that all experts are impartial with respect to all pairs of candidates. Then, the serial pairwise comparison SCF f^* is Nash implementable.*

Corollary 1 follows from Proposition 3, Theorem 4, and the fact that, if for every pair of candidates there is at least one expert who is impartial with respect to them, the serial pairwise comparison SCF f^* is onto. Corollary 2 follows from Proposition 3, Theorem 5, and the fact that, if $n \geq 3$ and all experts are impartial with respect to all pairs of candidates, the serial pairwise comparison SCF f^* satisfies no veto.[12]

7 Concluding remarks

We have studied the problem of implementing the socially optimal ranking that arises when a group of experts have to rank a set of candidates but may want to misreport their judgments. We have reached the following conclusions.

1. Implementation in dominant strategies is possible if and only if, whenever two candidates change their relative positions in the socially optimal ranking, there is at least one expert who is impartial with respect to them and changes his judgment about their relative positions in the same way

[12]A similar comment to that in Remark 1 may be made here, noting that the fact that f^* is Nash implementable does not imply that it can be implemented through the direct mechanism associated with it. The reason is that, in this mechanism, there may be Nash equilibria where all experts announce the same ranking regardless of their judgments.

that the socially optimal ranking does. Unfortunately, this condition is not satisfied in most reasonable cases.

2. Implementation in Nash equilibrium is possible only if, whenever the socially optimal ranking changes, there is at least one expert who changes his judgment about the relative position of two candidates with respect to whom he is impartial so that he goes from agreeing with the socially optimal ranking to not agreeing with it on this matter. If a sufficiently large number of experts want to favor some candidates over others, the previous condition is sufficient for Nash implementation. The condition is also (almost) sufficient if all experts are impartial with respect to all candidates.

3. Voting rules used in the real world to rank candidates are not implementable, with one notable exception: The recursive versions of Condorcet consistent voting rules are Nash implementable when there are precisely three candidates, all experts are impartial with respect to all pairs of candidates, and the tie-breaking rules satisfy specific properties.

4. We have proposed a new and non-trivial rule to aggregate the experts' judgments that is implementable in Nash equilibrium, regardless of the number of candidates.

Here are some suggestions for promising lines of extensions.

a. In some problems, the mechanism designer knows that the judgments of different experts cannot be too different. For example, two experts could honestly disagree about whether x is the best or the second-best candidate, but it is not possible that an expert honestly believes that x is the best candidate while another expert honestly believes that x is the worst candidate. Having this information reduces the set of admissible states, therefore facilitating the implementation of the SCF. It would be interesting to try to extend our work to this case.

b. The use of extensive form mechanisms generally facilitates the implementation problem.[13] For example, the general conditions for subgame perfect implementation are less demanding than the general conditions for Nash implementation (see Moore and Repullo, 1988). It would be interesting to study what results can be obtained when using these mechanisms.

[13] An "extensive form mechanism" is a stage mechanism in which experts make choices sequentially.

Appendix

PROOF OF THEOREM 1

The proof of this theorem is preceded by one lemma. It shows that to guarantee that an expert i prefers a ranking π to another ranking $\hat{\pi}$ when his judgment is ρ_i, it must be the case that, for every pair of candidates xy who change their relative positions when moving from π to $\hat{\pi}$, either (i) i is impartial with respect to xy and they are ranked among them according to ρ_i in π, or (ii) i favors x over y.

LEMMA 1 *Let $i \in E$ and $\rho_i, \pi, \hat{\pi} \in \Pi$. Given a jury configuration (I, F), then $[\pi \ R_i(\rho_i) \ \hat{\pi}$ for every $R_i \in R(I_i, F_i)] \Leftrightarrow$ [for every $xy \in [C]^2$ such that $\pi(x) < \pi(y)$ and $\hat{\pi}(y) < \hat{\pi}(x)$, we have either (i) $xy \in I_i$ and $\rho_i(x) < \rho_i(y)$, or (ii) $xy \in F_i$]. Moreover, in this case, $\pi \ P_i(\rho_i) \ \hat{\pi}$ for every $R_i \in R(I_i, F_i)$.*

Proof We start the proof showing that $\pi \ R_i(\rho_i) \ \hat{\pi}$ for every $R_i \in \mathcal{R}(I_i, F_i)$ if and only if there is a sequence of rankings $\pi^1, ..., \pi^s$ such that: (1) $\pi^1 = \pi$, (2) $\pi^s = \hat{\pi}$, and (3) for each $q \in \{1, ..., s-1\}$ there is $xy \in [C]^2$ such that (3.1) $\pi^q(x) + 1 = \pi^q(y) = \pi^{q+1}(x) = \pi^{q+1}(y) + 1$, (3.2) $\pi^q(z) = \pi^{q+1}(z)$ for each $z \in C \backslash \{x, y\}$, and either (3.3.1) $xy \in I_i$ and $\rho_i(x) < \rho_i(y)$, or (3.3.2) $xy \in F_i$. First, note that if there is not such a sequence of rankings then, from the definition of admissible preference function, there exists $R_i \in \mathcal{R}(I_i, F_i)$ such that and $\hat{\pi} \ P_i(\rho_i) \ \pi$. To prove the sufficient part, suppose that a sequence of rankings as defined above exists. From the definition of admissible preference function, every $R_i \in \mathcal{R}(I_i, F_i)$ is such that, for each $q \in \{1, ..., s-1\}$, $\pi^q \ P_i(\rho_i) \ \pi^{q+1}$, and therefore $\pi = \pi^1 \ P_i(\rho_i) \ \pi^2 \ P_i(\rho_i) \ ... \ \pi^{s-1} \ P_i(\rho_i) \ \pi^s = \hat{\pi}$. To conclude the proof, note that a sequence of rankings as defined above exists if and only if, for every $xy \in [C]^2$ such that $\pi(x) < \pi(y)$ and $\hat{\pi}(y) < \hat{\pi}(x)$, we have either (i) $xy \in I_i$ and $\rho_i(x) < \rho_i(y)$ or (ii) $xy \in F_i$. ■

Now, we can prove the theorem. Let $D(\Gamma, \rho, R)$ be the set of dominant strategy equilibrium messages of Γ at (ρ, R).

Step 1. If f is implementable in dominant strategies, then, for every $i \in E$, $\rho_i, \hat{\rho}_i \in \Pi$, $\rho_{-i} \in \Pi^{n-1}$, and $R_i \in \mathcal{R}(I_i, F_i)$, we have $\pi^f_{(\rho_i, \rho_{-i})} \ R_i(\rho_i) \ \pi^f_{(\hat{\rho}_i, \rho_{-i})}$.

Let $\Gamma = (M, g)$ be a mechanism implementing f in dominant strategies. For each state $(\rho, R) \in S(I, F)$, let $m^*(R(\rho)) = (m_1^*(R_1(\rho_1)), ...,$

$m_n^*(R_n(\rho_n)) \in D(\Gamma, \rho, R)$ (note that a dominant strategy for an expert i depends only on his ordinal preferences $R_i(\rho_i)$). Suppose by contradiction that there are $i \in E$, $\rho_i, \hat{\rho}_i \in \Pi$, $\rho_{-i} \in \Pi^{n-1}$, and $R_i \in \mathcal{R}(I_i, F_i)$ such that $\pi^f_{(\hat{\rho}_i,\rho_{-i})}\ P_i(\rho_i)\ \pi^f_{(\rho_i,\rho_{-i})}$. Let $\hat{R}_i \in \mathcal{R}(I_i, F_i)$ and $R_{-i} \in \mathcal{R}(I_{-i}, F_{-i})$. Let $\rho = (\rho_i, \rho_{-i})$, $\hat{\rho} = (\hat{\rho}_i, \rho_{-i})$, $R = (R_i, R_{-i})$, and $\hat{R} = (\hat{R}_i, R_{-i})$. Because Γ implements f in dominant strategy equilibrium, there exists $m^*(R(\rho)) = (m_i^*(R_i(\rho_i)), m_{-i}^*(R_{-i}(\rho_{-i}))) \in D(\Gamma, \rho, R)$ such that $g(m^*(R(\rho))) = \pi^f_\rho$. Similarly, there exists $m^*(\hat{R}(\hat{\rho})) = (m_i^*(\hat{R}_i(\hat{\rho}_i)), m_{-i}^*(R_{-i}(\rho_{-i}))) \in D(\Gamma, \hat{\rho}, \hat{R})$ such that $g(m^*(\hat{R}(\hat{\rho}))) = \pi^f_{\hat{\rho}}$. Because $\pi^f_{(\hat{\rho}_i,\rho_{-i})}\ P_i(\rho_i)\ \pi^f_{(\rho_i,\rho_{-i})}$, then $g(m_i^*(\hat{R}_i(\hat{\rho}_i)),$ $m_{-i}^*(R_{-i}(\rho_{-i})))\ P_i(\rho_i)\ g(m_i^*(R_i(\rho_i)), m_{-i}^*(R_{-i}(\rho_{-i})))$, which contradicts that $m_i^*(R_i(\rho_i))$ is a dominant strategy for expert i at state (ρ, R).

Step 2. f is implementable in dominant strategies if and only if, for each $i \in E$, $\rho_i, \hat{\rho}_i \in \Pi$, $\rho_{-i} \in \Pi^{n-1}$, and $xy \in [C]^2$ such that $\pi^f_{(\rho_i,\rho_{-i})}(x) < \pi^f_{(\rho_i,\rho_{-i})}(y)$ and $\pi^f_{(\hat{\rho}_i,\rho_{-i})}(y) < \pi^f_{(\hat{\rho}_i,\rho_{-i})}(x)$, we have $\rho_i(x) < \rho_i(y)$, $\hat{\rho}_i(y) < \hat{\rho}_i(x)$, and $xy \in I_i$.

First, we prove the necessity part. Let $i \in E$, $\rho_i, \hat{\rho}_i \in \Pi$, and $\rho_{-i} \in \Pi^{n-1}$. If f is implementable in dominant strategies, by Step 1 we have (1) $\pi^f_{(\rho_i,\rho_{-i})}$ $R_i(\rho_i)\ \pi^f_{(\hat{\rho}_i,\rho_{-i})}$ for every $R_i \in \mathcal{R}(I_i, F_i)$ and (2) $\pi^f_{(\hat{\rho}_i,\rho_{-i})}\ R_i(\hat{\rho}_i)\ \pi^f_{(\rho_i,\rho_{-i})}$ for every $R_i \in \mathcal{R}(I_i, F_i)$. By (1), from Lemma 1, for every $xy \in [C]^2$ such that $\pi^f_{(\rho_i,\rho_{-i})}(x) < \pi^f_{(\rho_i,\rho_{-i})}(y)$ and $\pi^f_{(\hat{\rho}_i,\rho_{-i})}(y) < \pi^f_{(\hat{\rho}_i,\rho_{-i})}(x)$, we have either (1.1) $xy \in I_i$ and $\rho_i(x) < \rho_i(y)$ or (1.2) $xy \in F_i$. Similarly, by (2), for every $xy \in [C]^2$ such that $\pi^f_{(\hat{\rho}_i,\rho_{-i})}(y) < \pi^f_{(\hat{\rho}_i,\rho_{-i})}(x)$ and $\pi^f_{(\rho_i,\rho_{-i})}(x) < \pi^f_{(\rho_i,\rho_{-i})}(y)$, we have either (2.1) $xy \in I_i$ and $\hat{\rho}_i(y) < \hat{\rho}_i(x)$ or (2.2) $yx \in F_i$. Clearly, if $xy \in F_i$ then $yx \notin F_i$. Therefore, for every $xy \in [C]^2$ such that $\pi^f_{(\rho_i,\rho_{-i})}(x) < \pi^f_{(\rho_i,\rho_{-i})}(y)$ and $\pi^f_{(\hat{\rho}_i,\rho_{-i})}(y) < \pi^f_{(\hat{\rho}_i,\rho_{-i})}(x)$, we have $xy \in I_i$, $\rho_i(x) < \rho_i(y)$, and $\hat{\rho}_i(y) < \hat{\rho}_i(x)$.

Now we prove the sufficient part. Suppose that, for each $i \in E$, $\rho_i, \hat{\rho}_i \in \Pi$, $\rho_{-i} \in \Pi^{n-1}$, and $xy \in [C]^2$ such that $\pi^f_{(\rho_i,\rho_{-i})}(x) < \pi^f_{(\rho_i,\rho_{-i})}(y)$ and $\pi^f_{(\hat{\rho}_i,\rho_{-i})}(y) < \pi^f_{(\hat{\rho}_i,\rho_{-i})}(x)$, we have $xy \in I_i$ and $\rho_i(x) < \rho_i(y)$. By Lemma 1, $\pi^f_{(\rho_i,\rho_{-i})}\ P_i(\rho_i)\ \pi^f_{(\hat{\rho}_i,\rho_{-i})}$ for every $R_i \in \mathcal{R}(I_i, F_i)$. Therefore, for every $i \in E$, $\rho_i, \hat{\rho}_i \in \Pi$, $\rho_{-i} \in \Pi^{n-1}$, and $R_i \in \mathcal{R}(I_i, F_i)$ such that $\pi^f_{(\rho_i,\rho_{-i})} \neq \pi^f_{(\hat{\rho}_i,\rho_{-i})}$ we have $\pi^f_{(\rho_i,\rho_{-i})}\ P_i(\rho_i)\ \pi^f_{(\hat{\rho}_i,\rho_{-i})}$. Then, the direct mechanism associated with f implements it in dominant strategies. To see this, note that, for every

$(\rho, R) \in S(I, F)$ and $i \in E$, either (1) $\pi^f_{(\rho_i,\hat{\rho}_{-i})} \neq \pi^f_{(\hat{\rho}_i,\hat{\rho}_{-i})}$ for some $\hat{\rho}_i \in \Pi$ and $\hat{\rho}_{-i} \in \Pi^{n-1}$, and then ρ_i is the only weakly dominant strategy for expert i, or (2) $\pi^f_{(\rho_i,\hat{\rho}_{-i})} = \pi^f_{(\hat{\rho}_i,\hat{\rho}_{-i})}$ for every $\hat{\rho}_i \in \Pi$ and $\hat{\rho}_{-i} \in \Pi^{n-1}$, and then every $\hat{\rho}_i \in \Pi$ is a weakly dominant strategy for expert i. Therefore, for every $(\rho, R) \in S(I, F)$, there is a dominant strategy equilibrium $\rho^* \in \Pi^n$ such that, for each $i \in E$, either (i) $\rho^*_i = \rho_i$, or (ii) $\rho^*_i \neq \rho_i$ and $\pi^f_{(\rho_i,\hat{\rho}_{-i})} = \pi^f_{(\rho^*_i,\hat{\rho}_{-i})}$ for every $\hat{\rho}_{-i} \in \Pi^{n-1}$. Suppose $\rho^* \neq \rho$. By (ii), $\pi^f_{(\rho^*_1,\rho^*_2,...,\rho^*_n)} = \pi^f_{(\rho_1,\rho^*_2,...,\rho^*_n)} = \pi^f_{(\rho_1,\rho_2,...,\rho^*_n)} = ... = \pi^f_{(\rho_1,\rho_2,...,\rho_n)}$. Hence, every dominant strategy equilibrium of the direct mechanism associated with f at state (ρ, R) results in π^f_ρ.

Step 3. f is implementable in dominant strategies if and only if it satisfies UPE.

Clearly, UPE implies the necessary and sufficient condition for dominant strategy implementation stated in Step 2. Next, we prove the necessity part. Suppose by contradiction that there exist $\rho, \hat{\rho} \in \Pi^n$ and $xy \in [C]^2$ such that $\pi^f_\rho(x) < \pi^f_\rho(y)$, $\pi^f_{\hat{\rho}}(y) < \pi^f_{\hat{\rho}}(x)$, and, for every $i \in E$ with $\rho_i(x) < \rho_i(y)$ and $\hat{\rho}_i(y) < \hat{\rho}_i(x)$, we have $xy \notin I_i$. Let us assign a number to each expert so that $E = \{1, ..., n\}$. Let $\rho^0, \rho^1, ..., \rho^n \in \Pi$ be a sequence of profiles of experts' judgments that goes from ρ to $\hat{\rho}$ where, for each $q \in \{0, 1, ..., n\}$, ρ^q is such that (i) $\rho^q_i = \rho_i$ for every $i > q$ and (ii) $\rho^q_i = \hat{\rho}_i$ for every $i \leq q$. Note that $\rho^0 = \rho = (\rho_1, \rho_{-1})$ and $\rho^1 = (\hat{\rho}_1, \rho_{-1})$. Moreover, $\pi^f_{(\rho_1,\rho_{-1})}(x) < \pi^f_{(\rho_1,\rho_{-1})}(y)$ and, in case $\rho_1(x) < \rho_1(y)$ and $\hat{\rho}_1(y) < \hat{\rho}_1(x)$, then $xy \notin I_i$. Hence, because f is implementable in dominant strategies, by Step 2, $\pi^f_{\rho^1}(x) < \pi^f_{\rho^1}(y)$. Similarly, $\rho^1 = (\rho_2, \rho^1_{-2})$ and $\rho^2 = (\hat{\rho}_2, \rho^1_{-2})$. Moreover, $\pi^f_{\rho^1}(x) < \pi^f_{\rho^1}(y)$ and, in case $\rho_2(x) < \rho_2(y)$ and $\hat{\rho}_2(y) < \hat{\rho}_2(x)$, then $xy \notin I_2$. Hence, because f is implementable in dominant strategies, by Step 2, $\pi^f_{\rho^2}(x) < \pi^f_{\rho^2}(y)$. Repeating this argument, we have $\pi^f_{\rho^n}(x) < \pi^f_{\rho^n}(y)$. Because $\rho^n = \hat{\rho}$, this contradicts that $\pi^f_{\hat{\rho}}(y) < \pi^f_{\hat{\rho}}(x)$. ■

PROOF OF THEOREM 2

We say that f satisfies *independence of irrelevant candidates* if for each $\rho, \hat{\rho} \in \Pi^n$ and $x, y \in C$, $[\rho_i(x) < \rho_i(y) \Leftrightarrow \hat{\rho}_i(x) < \hat{\rho}_i(y)]$ for every $i \in E$ implies that $\left[\pi^f_\rho(x) < \pi^f_\rho(y) \Leftrightarrow \pi^f_{\hat{\rho}}(x) < \pi^f_{\hat{\rho}}(y)\right]$ (*i.e.*, the relative position of two candidates in the socially optimal ranking depends only on the relative positions of the two candidates in the experts' judgments).

Claim 1. If an SCF f is implementable in dominant strategies, it satisfies independence of irrelevant candidates.

Suppose by contradiction that there exist $\rho, \hat{\rho} \in \Pi^n$ and $x, y \in C$ such that, (1) for every $i \in E$, $\rho_i(x) < \rho_i(y)$ if and only if $\hat{\rho}_i(x) < \hat{\rho}_i(y)$, but (2) $\pi^f_\rho(x) < \pi^f_\rho(y)$ and $\pi^f_{\hat{\rho}}(y) < \pi^f_{\hat{\rho}}(x)$. Because f is implementable in dominant strategies, by Theorem 1, it satisfies UPE. Then, by (2), there exists some $i \in E$ with $xy \in I_i$, $\rho_i(x) < \rho_i(y)$, and $\hat{\rho}_i(y) < \hat{\rho}_i(x)$, which contradicts (1).

Claim 2. If $m \geq 3$ and the SCF f satisfies unanimity and non-dictatorship, it is not implementable in dominant strategies.

A well-known result in voting theory is Arrow's impossibility Theorem (Arrow, 1951). In terms of our model, this theorem states that no SCF satisfies unanimity, non-dictatorship, and independence of irrelevant alternatives with at least three candidates and unrestricted expert judgments. Hence, by Step 1, no SCF satisfying unanimity and non-dictatorship is implementable in dominant strategies if $m \geq 3$. ■

PROOF OF THEOREM 3

Let $N(\Gamma, \rho, R)$ be the set of Nash equilibrium messages of Γ at (ρ, R).

Claim 1. Let $i \in E$ and $\rho_i, \hat{\rho}_i, \pi \in \Pi$ be such that, for every $xy \in I_i$ with $\pi(x) < \pi(y)$ and $\rho_i(x) < \rho_i(y)$, we have $\hat{\rho}_i(x) < \hat{\rho}_i(y)$. Then there exist $R_i, \hat{R}_i \in \mathcal{R}(I_i, F_i)$ such that for every $\hat{\pi} \in \Pi$ with $\pi \; R_i(\rho_i) \; \hat{\pi}$ we have $\pi \; \hat{R}_i(\hat{\rho}_i) \; \hat{\pi}$.

It follows from the definition of $\mathcal{R}(I_i, F_i)$.

Claim 2. For every $\rho, \hat{\rho} \in \Pi^n$ with $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$ and every $R, \hat{R} \in \mathcal{R}(I, F)$, *there exist $i \in E$ and $\pi \in \Pi$ such that $\pi^f_\rho \; R_i(\rho_i) \; \pi$ and $\pi \; \hat{P}_i(\hat{\rho}_i) \; \pi^f_\rho$.*

Let $\Gamma = (M, g)$ be a mechanism implementing f in Nash equilibrium. Suppose by contradiction that there exist $\rho, \hat{\rho} \in \Pi^n$ *with* $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$ and $R, \hat{R} \in \mathcal{R}(I, F)$ such that, for every $i \in E$ and $\pi \in \Pi$, if $\pi^f_\rho \; R_i(\rho_i) \; \pi$ then $\pi^f_\rho \; \hat{R}_i(\hat{\rho}_i) \; \pi$. Because Γ implements f in Nash equilibrium, there exists $m \in N(\Gamma, \rho, R)$ such that $g(m) = \pi^f_\rho$. Then, for every $i \in E$ and every $\hat{m}_i \in M_i$, $\pi^f_\rho = g(m_i, m_{-i}) \; R_i(\rho_i) \; g(\hat{m}_i, m_{-i})$. Hence, for every $i \in E$ and every $\hat{m}_i \in M_i$, $\pi^f_\rho = g(m_i, m_{-i}) \; \hat{R}_i(\hat{\rho}_i) \; g(\hat{m}_i, m_{-i})$. Therefore, $m \in N(\Gamma, \hat{\rho}, \hat{R})$, which contradicts that Γ implements f in Nash equilibrium because $g(m) = \pi_\rho^f \neq \pi_{\hat{\rho}}^f$.

Claim 3. For every $\rho, \hat{\rho} \in \Pi^n$ with $\pi_\rho^f \neq \pi_{\hat{\rho}}^f$, there exist $i \in E$ and $xy \in I_i$ such that $\pi_\rho^f(x) < \pi_\rho^f(y)$, $\rho_i(x) < \rho_i(y)$, and $\hat{\rho}_i(y) < \hat{\rho}_i(x)$.

Suppose on the contrary that there exist $\rho, \hat{\rho} \in \Pi^n$ with $\pi_\rho^f \neq \pi_{\hat{\rho}}^f$ such that, for every $i \in E$ and every $xy \in I_i$ such that $\pi_\rho^f(x) < \pi_\rho^f(y)$ and $\rho_i(x) < \rho_i(y)$, we have $\hat{\rho}_i(x) < \hat{\rho}_i(y)$. Then, by Claim 1, for every $i \in E$ there exist $R_i, \hat{R}_i \in \mathcal{R}(I_i, F_i)$ such that for every $\pi \in \Pi$ with $\pi_\rho^f \; R_i(\rho_i) \; \pi$ we have $\pi_\rho^f \; \hat{R}_i(\hat{\rho}_i) \; \pi$. Let $R = (R_i)_{i \in E}$ and $\hat{R} = (\hat{R}_i)_{i \in E}$. Then, (ρ, R) and $(\hat{\rho}, \hat{R})$ are such that $\pi_\rho^f \neq \pi_{\hat{\rho}}^f$ and, for every $i \in E$ and every $\pi \in \Pi$ such that $\pi_\rho^f \; R_i(\rho_i) \; \pi$, we have $\pi_\rho^f \; \hat{R}_i(\hat{\rho}_i) \; \pi$, which contradicts Claim 2. ■

PROOF OF THEOREM 4

Claim 1. For every $i \in E$, $\rho_i \in \Pi$, $R_i \in \mathcal{R}(I_i, F_i)$, and $\pi \in \Pi \backslash \{\rho_i\}$, we have $\rho_i \; P_i(\rho_i) \; \pi$.

If $\pi \neq \rho_i$, there is a sequence of rankings $\pi^1, ..., \pi^s$ such that (1) $\pi^1 = \rho_i$, (2) $\pi^s = \pi$, and (3) for each $q \in \{1, ..., s-1\}$ there is $xy \in [C]^2$ such that (3.1) $\pi^q(x) + 1 = \pi^q(y) = \pi^{q+1}(x) = \pi^{q+1}(y) + 1$, (3.2) $\rho_i(x) < \rho_i(y)$, and (3.3) $\pi^q(z) = \pi^{q+1}(z)$ for every $z \in C \backslash \{x, y\}$. Because $E_{xy}^I = E$ for every $xy \in [C]^2$, every $R_i \in \mathcal{R}(I_i, F_i)$ is such that, for each $q \in \{1, ..., s-1\}$, $\pi^q \; P_i(\rho_i) \; \pi^{q+1}$, and therefore $\rho_i = \pi^1 \; P_i(\rho_i) \; \pi^2 \; P_i(\rho_i) \; ... \; \pi^{s-1} \; P_i(\rho_i) \; \pi^s = \pi$.

Claim 2. f is Nash implementable.

The proof is constructive. Consider the following mechanism $\Gamma = (M, g)$, where expert i's message set is $M_i = \Pi^n \times \boldsymbol{N}_+$, with typical message $m_i = (\rho^i, \lambda^i) \in M_i$ (*i.e.*, each expert announces a profile of judgments and a positive integer). The outcome function g is as follows:

Rule 1. If $m_i = (\rho, 1)$ for all $i \in E$, then $g(m) = \pi_\rho^f$..

Rule 2. If there exists $j \in E$ such that $m_i = (\rho, 1)$ for every $i \neq j$ but $m_j = (\rho^j, \lambda^j) \neq (\rho, 1)$, then:

$$g(m) = \begin{cases} \pi_{\rho^j}^f; & \text{if, for every } xy \in [C]^2 \text{ such that } \pi_\rho^f(x) < \pi_\rho^f(y) \\ & \text{and } \pi_{\rho^j}^f(y) < \pi_{\rho^j}^f(x), \text{ we have either (i) } xy \in I_j \\ & \text{and } \rho_j(x) < \rho_j(y) \text{ or (ii) } xy \in F_j \\ \pi_\rho^f; & \text{otherwise.} \end{cases}$$

Rule 3. In all other cases, $g(m) = \pi^f_{\rho^j}$ where j is the first expert in alphabetical order among those who announce the highest integer.

We now show that mechanism Γ implements f in Nash equilibrium.

Step 1. For each $(\rho, R) \in S(I, F)$, there is $m \in N(\Gamma, \rho, R)$ with $g(m) = \pi^f_\rho$.

Let $m_i = (\rho, 1)$ for every $i \in E$. Then $g(m) = \pi^f_\rho$. By Rule 2 and Lemma 1, for each $i \in E$ and $\hat{m}_i \in M_i$ such that $g(\hat{m}_i, m_{-i}) \neq \pi^f_\rho$ we have $\pi^f_\rho \ P_i(\rho_i)$ $g(\hat{m}_i, m_{-i})$. Therefore, $m \in N(\Gamma, \rho, R)$.

Step 2. For each $(\rho, R) \in S(I, F)$, if $m \in M$ is such that $g(m) \neq \pi^f_\rho$ then $m \notin N(\Gamma, \rho, R)$.

Suppose on the contrary that $g(m) = \pi^f_{\hat{\rho}}$ for some $\hat{\rho} \in \Pi^n$ with $\pi^f_{\hat{\rho}} \neq \pi^f_\rho$ but $m \in N(\Gamma, \rho, R)$.

Case 2.1 Rule 1 applies to m.

Then, $m_i = (\hat{\rho}, 1)$ for every $i \in E$. Because $\pi^f_{\hat{\rho}} \neq \pi^f_\rho$, by WUPE, there exist $i \in E$ and $xy \in I_i$ such that $\pi^f_{\hat{\rho}}(x) < \pi^f_{\hat{\rho}}(y)$, $\hat{\rho}_i(x) < \hat{\rho}_i(y)$, and $\rho_i(y) < \rho_i(x)$. Let $\tilde{\pi} \in \Pi$ be such that $\tilde{\pi}(x) = \pi^f_{\hat{\rho}}(y)$, $\tilde{\pi}(y) = \pi^f_{\hat{\rho}}(x)$, and $\tilde{\pi}(z) = \pi^f_{\hat{\rho}}(z)$ for every $z \in C \backslash x, y\}$. By Lemma 1, $\tilde{\pi} \ P_i(\rho_i) \ \pi^f_{\hat{\rho}}$.

Because f satisfies no veto it is also onto, and then there exists $\rho \in \Pi$ such that $\pi^f_\rho = \tilde{\pi}$. Let $\tilde{m}_i = (\rho, .)$. By Rule 2, $g(\tilde{m}_i, m_{-i}) = \pi^f_\rho$. Hence, $g(\tilde{m}_i, m_{-i}) \ P_i(\rho_i) \ g(m)$, which contradicts that $m \in N(\Gamma, \rho, R)$.

Case 2.2 Either Rule 2 or Rule 3 applies to m.

Then, there is $j \in E$ such that, for every $\rho \in \Pi^n$, every expert $i \neq j$ can get π^f_ρ via Rule 3, by announcing a high enough integer. Because f satisfies no veto and $\pi^f_{\hat{\rho}} \neq \pi^f_\rho$, there is $i \neq j$ such that $\rho_i \neq \pi^f_{\hat{\rho}}$. Then, by Claim 1, ρ_i $P_i(\rho_i) \ \pi^f_{\hat{\rho}}$. Because f is onto, there exists $\rho \in \Pi$ such that $\pi^f_\rho = \rho_i$. Hence, expert i can improve by deviating unilaterally from m, which contradicts that $m \in N(\Gamma, \rho, R)$. ■

PROOF OF THEOREM 5

Claim 1. For every $\pi \in \Pi$ and every $j \in E$ there exist $i \neq j$ and $\tilde{\pi} \in \Pi$ such that $\tilde{\pi} \ P_i(\rho_i) \ \pi$ for every $(\rho_i, R_i) \in \Pi \times \mathcal{R}_i(I, F)$.

Suppose first and w.l.o.g. that $a, b, c \in C$ are such that a is a friend of expert 1, b is a friend of expert 2, and c is a friend of expert 3. The friends of at least two of these experts are not ranked first in π. Suppose w.l.o.g. that $\pi(a) \neq 1$ and $\pi(b) \neq 1$. Suppose $i \neq 1$. Let $x \in C$ be such that $\pi(x) + 1 = \pi(a)$. Let $\tilde{\pi} \in \Pi$ be such that $\tilde{\pi}(a) = \pi(x)$, $\tilde{\pi}(x) = \pi(a)$, and $\tilde{\pi}(y) = \pi(y)$ for every $y \in C \backslash \{a, x\}$. Because a is a friend of 1 and by definition of $\mathcal{R}_1(I, F)$, $\tilde{\pi}\ P_1(\rho_1)\ \pi$ for every $(\rho_1, R_1) \in \Pi \times \mathcal{R}_1(I, F)$. Suppose $i = 1$. Let $x \in C$ be such that $\pi(x) + 1 = \pi(b)$. Let $\tilde{\pi} \in \Pi$ be such that $\tilde{\pi}(b) = \pi(x)$, $\tilde{\pi}(x) = \pi(b)$, and $\tilde{\pi}(y) = \pi(y)$ for every $y \in C \backslash \{b, x\}$. Because b is a friend of 2 and by definition of $\mathcal{R}_2(I, F)$, $\tilde{\pi}\ P_2(\rho_2)\ \pi$ for every $(\rho_2, R_2) \in \Pi \times \mathcal{R}_2(I, F)$.

Suppose now that, w.l.o.g., that $a, b, c \in C$ are such that a is an enemy of expert 1, b is an enemy of expert 2, and c is an enemy of expert 3. The enemies of at least two of these experts are not ranked last in π. Suppose w.l.o.g. that $\pi(a) \neq m$ and $\pi(b) \neq m$. Suppose $i \neq 1$. Let $x \in C$ be such that $\pi(a) + 1 = \pi(x)$. Let $\tilde{\pi} \in \Pi$ be such that $\tilde{\pi}(a) = \pi(x)$, $\tilde{\pi}(x) = \pi(a)$, and $\tilde{\pi}(y) = \pi(y)$ for every $y \in C \backslash \{a, x\}$. Because a is an enemy of 1 and by definition of $\mathcal{R}_1(I, F)$, $\tilde{\pi}\ P_1(\rho_1)\ \pi$ for every $(\rho_1, R_1) \in \Pi \times \mathcal{R}_1(I, F)$. Suppose $i = 1$. Let $x \in C$ be such that $\pi(b) + 1 = \pi(x)$. Let $\tilde{\pi} \in \Pi$ be such that $\tilde{\pi}(b) = \pi(x)$, $\tilde{\pi}(x) = \pi(b)$, and $\tilde{\pi}(y) = \pi(y)$ for every $y \in C \backslash \{b, x\}$. Because b is an enemy of 2 and by definition of $\mathcal{R}_2(I, F)$, $\tilde{\pi}\ P_2(\rho_2)\ \pi$ for every $(\rho_2, R_2) \in \Pi \times \mathcal{R}_2(I, F)$.

Claim 2. f is Nash implementable.

The proof is practically the same as the proof of Claim 2 in Theorem 4: The same mechanism $\Gamma = (M, g)$ proposed there works in this case. The only difference occurs in the proof of Case 2.2. Suppose by contradiction that there exist $(\rho, R) \in S(I, F)$ and $m \in N(\Gamma, \rho, R)$ such that $g(m) = \pi^f_{\hat{\rho}}$ for some $\hat{\rho} \in \Pi^n$ with $\pi^f_{\hat{\rho}} \neq \pi^f_\rho$ and either Rule 2 or Rule 3 applies to m. Then, there is $j \in E$ such that, for every $\rho \in \Pi^n$, every expert $i \neq j$ can get π^f_ρ via Rule 3, by announcing a high enough integer. By Claim 1, there exist $i \neq j$ and $\tilde{\pi} \in \Pi$ such that $\tilde{\pi}\ P_i(\rho_i)\ \pi^f_{\hat{\rho}}$. Because f is onto, there exists $\rho \in \Pi$ such that $\pi^f_\rho = \tilde{\pi}$. Hence, expert i can improve by deviating unilaterally from m, which contradicts that $m \in N(\Gamma, \rho, R)$. ■

PROOF OF PROPOSITION 1

Claim 1. Extended and recursive plurality SCFs may fail Nash implementability when $m \geq 3$.

Suppose w.l.o.g. that $C = \{a, b, c\}$.[14] Let $n = 9$. Let f be an extended plurality SCF. Let $\rho, \hat{\rho} \in \Pi^n$ be as depicted in Table 4 ($E_1, E_2, E_3 \subset E$ is a partition of E such that all the experts in each set have the same judgment; the second row indicates the number of experts in each group; the judgment of the experts in each group is represented in the corresponding column). Because ρ is such that a, b, and c are ranked in the first position by 4, 3, and 2 experts respectively, then $\pi^f_\rho = abc$. Similarly, because $\hat{\rho}$ is such that a, b, and c are ranked in the first position by 4, 5, and 0 experts respectively, then $\pi^f_{\hat{\rho}} = bac$. Therefore, $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$. Moreover, the only difference between ρ and $\hat{\rho}$ is that the experts in E_3 change their judgments about the relative positions of b and c from $\rho_i(c) < \rho_i(b)$ to $\hat{\rho}_i(b) < \hat{\rho}_i(c)$. However, $\pi^f_\rho(b) < \pi^f_\rho(c)$. Then, f does not satisfy WUPE regardless of the jury configuration (even if all experts are impartial with respect to all pairs of candidates) and, by Theorem 3, it is not Nash implementable.

Let $\hat{f}$ be a recursive plurality SCF. Clearly, $\pi^{\hat{f}}_\rho(a) = 1$. Given $\hat{C} = C \backslash \{a\}$, $\rho^{\hat{C}}$ is such that b and c are ranked in the first position by 7 and 2 experts, respectively, and then $\pi^{\hat{f}}_\rho(b) = 2$. Therefore, $\pi^{\hat{f}}_\rho = abc$. Consider now the profile $\hat{\rho}$. Clearly, $\pi^{\hat{f}}_{\hat{\rho}}(b) = 1$. Given $\hat{C} = C \backslash \{b\}$, $\hat{\rho}^{\hat{C}}$ is such that a and c are ranked in the first position by 4 and 5 experts, respectively, and then $\pi^{\hat{f}}_{\hat{\rho}}(c) = 2$. Therefore, $\pi^{\hat{f}}_{\hat{\rho}} = bca \neq \pi^{\hat{f}}_\rho$. Using the same argument as with f, we conclude that $\hat{f}$ is not Nash implementable.

ρ

E_1	E_2	E_3
4	3	2
a	b	**c**
b	c	**b**
c	a	a

$\hat{\rho}$

E_1	E_2 and E_3
4	5
a	**b**
b	**c**
c	a

Table 4 Profiles of judgments in the proof of Proposition 1, Claim 1.

Claim 2. Extended and recursive instant-runoff SCFs may fail Nash implementability when $m \geq 3$.

Suppose w.l.o.g. that $C = \{a, b, c\}$. Let $n = 29$. Let f be an extended instant-runoff SCF. Let $\rho, \hat{\rho} \in \Pi^n$ be as depicted in Table 5. Note that ρ

[14] Although the examples in the proof of this proposition are for the case $m = 3$, they are easily generalizable to the case $m \geq 3$.

is such that a, b, and c are ranked in the first position by 11, 8, and 10 experts, respectively. We eliminate b because no candidate has a majority and b has the fewest first positions in ρ. The judgment profile resulting after eliminating b in ρ is such that a and c are ranked in the first position by 11 and 18 experts, respectively, and then $\pi^f_\rho = cab$. The profile of judgments $\hat{\rho}$ is such that a, b, and c are ranked in the first position by 7, 8, and 14 experts, respectively. Then, no candidate has a majority, and a has the fewest first positions in $\hat{\rho}$. The judgment profile resulting after eliminating a in $\hat{\rho}$ is such that b and c are ranked in the first position by 15 and 14 experts, respectively, and then $\pi^f_{\hat{\rho}} = bca$. Therefore, $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$. Moreover, the only difference between ρ and $\hat{\rho}$ is that the experts in E_4 change their judgments about the relative positions of a and c from $\rho_i(a) < \rho_i(c)$ to $\hat{\rho}_i(c) < \hat{\rho}_i(a)$. However, $\pi^f_\rho(c) < \pi^f_\rho(a)$. Then, f does not satisfy WUPE, regardless of the jury configuration.

Let $\hat{f}$ be a recursive instant-runoff SCF. Because every extended instant-runoff SCF f is such that $\pi^f_\rho = cab$, then $\pi^{\hat{f}}_\rho(c) = 1$. Given $\hat{C} = C \backslash \{c\}$, $\rho^{\hat{C}}$ is such that a and b are ranked in the first position by 21 and 8 experts, respectively, and then $\pi^{\hat{f}}_\rho = cab$. Consider now the profile $\hat{\rho}$. Because every extended instant-runoff SCF f is such that $\pi^f_{\hat{\rho}} = bca$, then $\pi^{\hat{f}}_\rho(b) = 1$. Given $\hat{C} = C \backslash \{b\}$, $\hat{\rho}^{\hat{C}}$ is such that a and c are ranked in the first position by 21 and 8 experts respectively, and then $\pi^{\hat{f}}_{\hat{\rho}} = bac \neq \pi^{\hat{f}}_\rho$. Using the same argument above, we conclude that $\hat{f}$ is not Nash implementable.

ρ

E_1	E_2	E_3	E_4
7	8	10	4
a	b	c	**a**
b	c	a	**c**
c	a	b	b

$\hat{\rho}$

E_1	E_2	E_3 and E_4
7	8	14
a	b	**c**
b	c	**a**
c	a	b

Table 5 Profiles of judgments in the proof of Proposition 1, Claim 2.

Claim 3. Extended and recursive Borda SCFs may fail Nash implementability when $m \geq 3$.

Suppose w.l.o.g. that $C = \{a, b, c\}$. Let $n = 5$. Let f be an extended Borda SCF. Let $\rho, \hat{\rho} \in \Pi^n$ be as depicted in Table 6. The Borda scores in ρ for a, b, and c are $3 \times 3 + 1 \times 2 = 11$, $2 \times 3 + 3 \times 2 = 12$, and $1 \times 3 + 2 \times 2 = 7$, respectively, and then $\pi^f_\rho = bac$. The Borda scores in $\hat{\rho}$ for a, b, and c are $3\times3+2\times2 = 13$, $2\times3+3\times2 = 12$, and $1\times3+1\times2 = 5$, respectively, and then $\pi^f_{\hat{\rho}} = abc$. Therefore, $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$. Moreover, the only difference between ρ and $\hat{\rho}$ is that the experts in E_2 change their judgments about the relative positions of a and c from $\rho_i(c) < \rho_i(a)$ to $\hat{\rho}_i(a) < \hat{\rho}_i(c)$. However, $\pi^f_\rho(a) < \pi^f_\rho(c)$. Then, f does not satisfy WUPE, regardless of the jury configuration.

Let $\hat{f}$ be a recursive Borda SCF. Because the Borda scores in ρ for a, b, and c are 11, 12, and 7, respectively, then $\pi^{\hat{f}}_\rho(b) = 1$. Given $\hat{C} = C\backslash\{b\}$, the Borda scores in $\rho^{\hat{C}}$ for a and c are 8 and 7 respectively, and then $\pi^{\hat{f}}_\rho(a) = 2$. Therefore, $\pi^{\hat{f}}_\rho = bac$. Consider now the profile $\hat{\rho}$. Because the Borda scores in $\hat{\rho}$ for a, b, and c are 13, 12, and 5, respectively, then $\pi^{\hat{f}}_{\hat{\rho}}(a) = 1$. Given $\hat{C} = C\backslash\{a\}$, the Borda scores in $\hat{\rho}^{\hat{C}}$ for b and c are 10 and 5 respectively, and then $\pi^{\hat{f}}_{\hat{\rho}}(b) = 2$. Therefore, $\pi^{\hat{f}}_{\hat{\rho}} = abc$. Using the same argument as with f, we conclude that $\hat{f}$ is not Nash implementable. ■

ρ			$\hat{\rho}$	
E_1	E_2		E_1	E_2
3	2		3	2
a	b		a	b
b	**c**		b	**a**
c	**a**		c	**c**

Table 6 Profiles of judgments in the proof of Proposition 1, Claim 3.

PROOF OF PROPOSITION 2

Claim 1. Extended Copeland SCF may fail Nash implementability when $m \geq 3$.

Suppose w.l.o.g. that $C = \{a, b, c\}$.[15] Let $n = 6$. Let f be an extended Copeland SCF. Suppose w.l.o.g. that the tie-breaking rule breaks the ties between b and c in favor of b. Let $\rho, \hat{\rho} \in \Pi^n$ be as depicted in Table 7. The majority pairwise comparisons based on ρ are such that: a defeats b; a and

[15] The examples in the proof of Claims 1 and 2 in this proposition are for the case $m = 3$, but they are easily generalizable to the case $m \geq 3$. Similarly, the examples in the proof of Claims 3 and 4 are for the case $m = 4$, but they are easily generalizable to the case $m \geq 4$.

c are involved in a tie; b and c are involved in a tie.[16] Then, the Copeland scores in ρ for a, b, and c are 1.5, 0.5, and 1 respectively, and then $\pi^f_\rho = acb$. The majority pairwise comparisons based on $\hat{\rho}$ are such that: a defeats b and c; b and c are involved in a tie. Then, the Copeland scores in $\hat{\rho}$ for a, b, and c are 2, 0.5, and 0.5, respectively. Hence, there is tie in Copeland Scores between b and c and $\pi^f_{\hat{\rho}} = abc$. Therefore, $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$. Moreover, the only difference between ρ and $\hat{\rho}$ is that the experts in E_3 change their judgments about the relative positions of a and c from $\rho_i(c) < \rho_i(a)$ to $\hat{\rho}_i(a) < \hat{\rho}_i(c)$. However, $\pi^f_\rho(a) < \pi^f_\rho(c)$. Then, f does not satisfy WUPE, regardless of the jury configuration.

ρ

E_1	E_2	E_3
3	2	1
a	c	**c**
b	b	**a**
c	a	b

$\hat{\rho}$

E_1	E_2	E_3
3	2	1
a	c	**a**
b	b	**c**
c	a	b

Table 7 Profiles of judgments in the proof of Proposition 2, Claim 1.

Claim 2. Extended minimax SCF may fail Nash implementability when $m \geq 3$.

Consider the example in the proof of Claim 2 in Proposition 1. Let f be an extended minimax SCF. Note that ρ is such that (1) 21 experts rank a before b and 8 experts rank b before a, (2) 11 experts rank a before c and 18 experts rank c before a, and (3) 15 experts rank b before c and 14 experts rank c before b. Then, a's largest pairwise defeat in ρ is 7 (against c), b's largest pairwise defeat in ρ is 13 (against a), and c's largest pairwise defeat in ρ is 1 (against b). Therefore, $\pi^f_\rho = cab$. Similarly, $\hat{\rho}$ is such that (1) 21 experts rank a before b and 8 experts rank b before a, (2) 7 experts rank a before c and 22 experts rank c before a, and (3) 15 experts rank b before c and 14 experts rank c before b. Then, a's largest pairwise defeat in $\hat{\rho}$ is 15 (against c), b's largest pairwise defeat in $\hat{\rho}$ is 13 (against a), and c's largest pairwise defeat in $\hat{\rho}$ is 1 (against b). Therefore, $\pi^f_{\hat{\rho}} = cba$. Therefore $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$ and, using the same argument as in the proof of Claim 2 in Proposition 1, it

[16] For example, in a pairwise comparison between a and b in o, the four experts in $E_1 \cup E_3$ honestly believe that a is better than b, while the two experts in E_2 honestly believe that b is better than a, and then a defeats b.

can be shown that f is not implementable in Nash equilibrium, regardless of the jury configuration.

Claim 3. Recursive Copeland SCF may fail Nash implementability when $m \geq 4$.

Suppose w.l.o.g. that $C = \{a, b, c, d\}$. Let $n = 6$. Let f be a recursive Copeland SCF. Suppose, w.l.o.g., that the tie-breaking rule t is such that, if $\hat{C} = \{a, b\}$, $t(\hat{C}) = b$. Let $\rho, \hat{\rho} \in \Pi^n$ be as depicted in Table 8. The majority pairwise comparisons based on ρ are such that: a defeats c and d; b defeats c; c defeats d; a and b are involved in a tie; b and d are involved in a tie. Then, the Copeland scores in ρ for a, b, c, and d are 2.5, 2, 1, and 0.5, respectively. Therefore, $\pi^f_\rho(a) = 1$. Given $\hat{C} = C \backslash \{a\}$, the Copeland scores in $\rho^{\hat{C}}$ for b, c, and d are 1.5, 1, and 0.5, respectively. Therefore, $\pi^f_\rho(b) = 2$. Finally, given $\tilde{C} = C \backslash \{a, b\}$, the Copeland scores in $\rho^{\tilde{C}}$ for c and d are 1 and 0, respectively. Therefore, $\pi^f_\rho(c) = 3$. Hence, $\pi^f_\rho = abcd$. Majority pairwise comparisons based on $\hat{\rho}$ are such that: a defeats c and d; b defeats c and d; c defeats d; a and b are involved in a tie. Then, the Copeland scores in ρ for a, b, c, and d are 2.5, 2.5, 1, and 0, respectively. Because, $t(\hat{C}) = b$ when $\hat{C} = \{a, b\}$, then $\pi^f_{\hat{\rho}}(b) = 1$. Given $\hat{C} = C \backslash \{b\}$, the Copeland scores in $\hat{\rho}^{\hat{C}}$ for a, c, and d are 2, 1, and 0, respectively. Therefore, $\pi^f_{\hat{\rho}}(a) = 2$. Finally, given $\tilde{C} = C \backslash \{a, b\}$, the Copeland scores in $\hat{\rho}^{\tilde{C}}$ for c and d are 1 and 0 respectively. Therefore, $\pi^f_{\hat{\rho}}(c) = 3$. Hence, $\pi^f_{\hat{\rho}} = bacd$, and $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$. Moreover, the only difference between ρ and $\hat{\rho}$ is that the experts in E_2 change their judgments about the relative positions of b and d from $\rho_i(d) < \rho_i(b)$ to $\hat{\rho}_i(b) < \hat{\rho}_i(d)$. However, $\pi^f_\rho(b) < \pi^f_\rho(d)$. Then, f does not satisfy WUPE, regardless of the jury configuration.

ρ

E_1	E_2	E_3
3	2	1
b	a	d
a	c	a
c	**d**	b
d	**b**	c

$\hat{\rho}$

E_1	E_2	E_3
3	2	1
b	a	d
a	c	a
c	**b**	b
d	**d**	c

Table 8 Profiles of judgments in the proof of Proposition 2, Claim 3.

Claim 4. Recursive minimax SCF may fail Nash implementability when $m \geq 4$.

Suppose w.l.o.g. that $C = \{a, b, c, d\}$. Let $n = 20$. Let f be a recursive minimax SCF. Let $\rho, \hat{\rho} \in \Pi^n$ be as depicted in Table 9. Note that ρ is such that (1) 10 experts rank a before b and 10 experts rank b before a, (2) 11 experts rank a before c and 9 experts rank c before a, (3) 9 experts rank a before d and 11 experts rank d before a, (4) 8 experts rank b before c and 12 experts rank c before b, (5) 13 experts rank b before d and 7 experts rank d before b, and (6) 7 experts rank c before d and 13 experts rank d before c. Then, a's largest pairwise defeat in ρ is 2 (against d), b's largest pairwise defeat in ρ is 4 (against c), c's largest pairwise defeat in ρ is 6 (against d), and d's largest pairwise defeat in ρ is 6 (against b), Therefore, $\pi^f_\rho(a) = 1$. Given $\hat{C} = C \backslash \{a\}$, b's largest pairwise defeat in $\rho^{\hat{C}}$ is 4 (against c), c's largest pairwise defeat in $\rho^{\hat{C}}$ is 6 (against d), and d's largest pairwise defeat in $\rho^{\hat{C}}$ is 6 (against b). Then $\pi^f_\rho(b) = 2$. Finally, given $\tilde{C} = C \backslash \{a, b\}$, $\rho^{\tilde{C}}$ is such that d defeats c in a majority pairwise comparison, and then $\pi^f_\rho(d) = 3$. Therefore, $\pi^f_\rho = abdc$. The majority pairwise comparisons in $\hat{\rho}$ are the same as in ρ with the only exception of pair bc: $\hat{\rho}$ is such that 12 experts rank b before c and 8 experts rank c before b. Then, a's largest pairwise defeat in $\hat{\rho}$ is 2 (against d), b's largest pairwise defeat in $\hat{\rho}$ is 0 (against a), c's largest pairwise defeat in ρ is 6 (against d), and d's largest pairwise defeat in ρ is 6 (against b), Therefore, $\pi^f_{\hat{\rho}}(b) = 1$ and $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$. Moreover, the only difference between ρ and $\hat{\rho}$ is that the experts in E_3 change their judgments about the relative positions of b and c from $\rho_i(c) < \rho_i(b)$ to $\hat{\rho}_i(b) < \hat{\rho}_i(c)$. However, $\pi^f_\rho(b) < \pi^f_\rho(c)$. Then, f does not satisfy WUPE, regardless of the jury configuration. ■

ρ

E_1	E_2	E_3	E_4	E_5	E_6
7	5	4	2	1	1
b	c	a	c	b	d
d	a	d	d	d	c
a	b	**c**	b	c	a
c	d	**b**	a	a	b

$\hat{\rho}$

E_1	E_2	E_3	E_4	E_5	E_6
7	5	4	2	1	1
b	c	a	c	b	d
d	a	d	d	d	c
a	b	**b**	b	c	a
c	d	**c**	a	a	b

Table 9 Profiles of judgments in the proof of Proposition 2, Claim 4.

PROOF OF THEOREM 6

Suppose that $C = \{a, b, c\}$, $n \geq 3$, and the jury configuration (I, F) is such that $E^I_{xy} = E$ for every $xy \in [C]^2$.

Claim 1. If f is a recursive SCF version of the Copeland voting rule with a tie-breaking rule that satisfies non-favoritism, it is Nash implementable.

Because $m = 3$, f involves two rounds: In the first round, each candidate is pairwise compared with each other, and the candidate with the largest Copeland score gets the first position; in the second round, the two remaining candidates are compared, the winner gets the second position, and the loser gets the third position. A tie-breaking rule that satisfies non-favoritism is used in the event of a tie in either of these two rounds.

Step 1.1. f satisfies WUPE.

Suppose by contradiction that there exist $\rho, \hat{\rho} \in \Pi^n$ with $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$ and such that, for every $i \in E$ and $xy \in [C]^2$ with $\rho_i(x) < \rho_i(y)$ and $\hat{\rho}_i(y) < \hat{\rho}_i(x)$ we have $\pi^f_\rho(y) < \pi^f_\rho(x)$. Suppose w.l.o.g. that $\pi^f_\rho = abc$. Then, majority pairwise comparisons based on ρ are such that one of the following cases occurs: (1) a defeats b, a defeats c, and b defeats c; (2) a defeats b, a defeats c, b and c are involved in a tie, and the tie-breaking rule is such that, if $\hat{C} = \{b, c\}$, then $t(\hat{C}) = b$; (3) a and b are involved in a tie, a defeats c, b defeats c, and the tie-breaking rule is such that if $\hat{C} = \{a, b\}$, then $t(\hat{C}) = a$; (4) a defeats b, a and c are involved in a tie, and b defeats c; (5) a and b are involved in a tie, a defeats c, b and c are involved in a tie, and the tie-breaking rule is such that if $\hat{C} = \{b, c\}$, then $t(\hat{C}) = b$; (6) a defeats b, a and c are involved in a tie, b and c are involved in a tie, and the tie-breaking rule is such that if $\hat{C} = \{b, c\}$, then $t(\hat{C}) = b$; (7) a defeats b, c defeats a, b defeats c, and the tie-breaking rule is such that, if $\hat{C} = \{a, b, c\}$, then $t(\hat{C}) = a$.[17]

Step 1.1.1. $\pi^f_{\hat{\rho}}(a) \neq 1$.

Suppose on the contrary that $\pi^f_{\hat{\rho}}(a) = 1$. Because $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$, then $\pi^f_{\hat{\rho}} = acb$. Therefore, c wins or ties against b in $\hat{\rho}$. Similarly, because $\pi^f_\rho = abc$, b wins or ties against c in ρ. Moreover, if b and c are tied in ρ, then they are not tied in $\hat{\rho}$ (otherwise, their relative positions in π^f_ρ and $\pi^f_{\hat{\rho}}$ should be the same). Hence, there is some $i \in E$ such that $\rho_i(b) < \rho_i(c)$ and $\hat{\rho}_i(c) < \hat{\rho}_i(b)$. Then, $\pi^f_\rho(c) < \pi^f_\rho(b)$, which is a contradiction.

Step 1.1.2. The Copeland score of a does not decrease from ρ to $\hat{\rho}$.

Suppose not. Then, there is some $i \in E$ and some $x \in \{b, c\}$ such that $\rho_i(a) < \rho_i(x)$ and $\hat{\rho}_i(x) < \hat{\rho}_i(a)$. This implies that $\pi^f_\rho(x) < \pi^f_\rho(a)$, which is a contradiction.

[17] The case where b defeats a, a defeats c, c defeats b, and the tie-breaking rule is such that $t(abc) = a$ is not possible. The reason is that, in that case, $\pi^f_\rho(c) = 2$.

Step 1.1.3. The Copeland score of c does not increase from ρ to $\hat{\rho}$.

Suppose not. Then, there is some $i \in E$ and some $x \in \{a, b\}$ such that $\rho_i(x) < \rho_i(c)$ and $\hat{\rho}_i(c) < \hat{\rho}_i(x)$. This implies that $\pi^f_\rho(c) < \pi^f_\rho(x)$, which is a contradiction.

Step 1.1.4. If b defeats c in ρ, the Copeland score of b does not increase from ρ to $\hat{\rho}$.

Suppose not. Then, there is some $i \in E$ such that $\rho_i(a) < \rho_i(b)$ and $\hat{\rho}_i(b) < \hat{\rho}_i(a)$. This implies that $\pi^f_\rho(b) < \pi^f_\rho(a)$, which is a contradiction.

Step 1.1.5. Neither Case (1) nor Case (2) occurs.

Suppose on the contrary that Case (1) or Case (2) occurs. Then, the Copeland score of a in ρ is 2 (the highest possible one). Because $\pi^f_{\hat{\rho}}(a) \neq 1$, the Copeland score of a decreases from ρ to $\hat{\rho}$, which contradicts Step 1.1.2.

Step 1.1.6. Neither Case (3) nor Case (4) occurs.

Suppose on the contrary that Case (3) or Case (4) occurs. Then, the Copeland scores of a, b, and c in ρ are 1.5, 1.5, and 0, or 1.5, 1, and 0.5, respectively. Because $\pi^f_{\hat{\rho}}(a) \neq 1$, some of the following cases occurs: (i) the Copeland score of a decreases from ρ to $\hat{\rho}$; (ii) the Copeland score of b increases from ρ to $\hat{\rho}$; (iii) the Copeland score of c increases from ρ to $\hat{\rho}$. By Steps 1.1.2 and 1.1.3, neither (i) nor (iii) can occur. Moreover, because b defeats c in ρ, by Step 1.1.4, (ii) cannot occur either, which is a contradiction.

Step 1.1.7. Neither Case (5) nor Case (6) occurs.

Suppose on the contrary that Case (5) or Case (6) occurs. Because the tie-breaking rule satisfies non-favoritism, if $\hat{C} = \{a, b\}$, then $t(\hat{C}) = a$. The Copeland scores of a, b, and c in ρ are 1.5, 1, and 0.5, or 1.5, 0.5, and 1, respectively. Because $\pi^f_{\hat{\rho}}(a) \neq 1$, by Steps 1.1.2 and 1.1.3, the only possibility is that the Copeland scores of a, b, and c in $\hat{\rho}$ are 1.5, 1.5, and 0, respectively. Hence, $\hat{\rho}$ is such that a and b are involved in a tie in the first round of the recursive Copeland SCF f and then $\pi^f_{\hat{\rho}}(a) = 1$, which is a contradiction.

Step 1.1.8. Case (7) does not occur.

Suppose on the contrary that Case (6) occurs. The Copeland scores of a, b, and c in ρ are 1. Because $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$, the Copeland scores of at least two candidates change from ρ to $\hat{\rho}$. Because $\pi^f_{\hat{\rho}}(a) \neq 1$, the Copeland score of a does not increase from ρ to $\hat{\rho}$ (if the Copeland score of a increases to 2, then $\pi^f_{\hat{\rho}}(a) = 1$; if the Copeland score of a increases to 1.5, because $\pi^f_{\hat{\rho}}(a) \neq 1$, either the Copeland score of b or the Copeland score of c increases, which is a contradiction with Step 1.1.4 or Step 1.1.3, respectively). Then, when going from ρ to $\hat{\rho}$, either the Copeland score of b increases and the Copeland score of c decreases or the Copeland score of b decreases and the Copeland score of c increases, which again is a contradiction with Step 1.1.4 or Step 1.1.3, respectively.

Steps 1.1.5-1.1.8 imply that $\pi_{\rho}^{f} \neq abc$, a contradiction.

Step 1.2. f satisfies no veto.

Let $\rho \in \Pi^n$, $\pi \in \Pi$, and $j \in E$, be such that $\rho_i = \pi$ for every $i \neq j$. Suppose, w.l.o.g., that $\pi = abc$. Then, ρ is such that a beats every other candidate in pairwise comparisons. Hence, because f is a recursive Copeland SCF, $\pi_{\rho}^{f}(a) = 1$. Given $\hat{C} = C \backslash \{a\}$, $\rho^{\hat{C}}$ is such that b beats c in $\hat{C}$ and then, because f is a recursive Copeland SCF, $\pi_{\rho}^{f}(b) = 2$. Then, $\pi_{\rho}^{f} = abc = \pi$.

Because $n \geq 3$ and $E_{xy}^{I} = E$ for every $xy \in [C]^2$, Steps 1.1 and 1.2 and Theorem 4 prove that f is Nash implementable.

Claim 2. If f is a recursive SCF version of the minimax voting rule with a linear-ordered tie-breaking rule, it is Nash implementable.

Because $m = 3$, f involves two rounds: In the first round, each candidate is pairwise compared with each other, and the candidate with the smallest maximum pairwise defeat gets the first position; in the second round, the two remaining candidates are compared, the winner gets the second position, and the loser gets the third position. A linear-ordered tie-breaking rule is used in the event of a tie in either of these two rounds.

Step 2.1. f satisfies WUPE.

Suppose by contradiction that there exist $\rho, \hat{\rho} \in \Pi^n$ with $\pi_{\rho}^{f} \neq \pi_{\hat{\rho}}^{f}$ and such that, for every $i \in E$ and $xy \in [C]^2$ with $\rho_i(x) < \rho_i(y)$ and $\hat{\rho}_i(y) < \hat{\rho}_i(x)$ we have $\pi_{\rho}^{f}(y) < \pi_{\rho}^{f}(x)$. Suppose w.l.o.g. that $\pi_{\rho}^{f} = abc$.

Step 2.1.1. $\pi_{\hat{\rho}}^{f}(a) \neq 1$.

Identical to Step 1.1.1.

Step 2.1.2. Either (1) the largest pairwise defeat of a increases when going from ρ to $\hat{\rho}$ or (2) the largest pairwise defeat of some $x \in \{b, c\}$ decreases when going from ρ to $\hat{\rho}$.

Suppose not. Then, because $\pi_{\rho}^{f}(a) = 1$ and $\pi_{\hat{\rho}}^{f}(a) \neq 1$, the largest pairwise defeats of a, b, and c in ρ are equal, the largest pairwise defeat of some $x \in \{b, c\}$ increases when going from ρ to $\hat{\rho}$, and the tie-breaking rule t is such that $t(C) = a$ and, when $\hat{C} = \{a, y\}$ (where $y \in C \backslash \{a, x\}$), $t(\hat{C}) = y$. This contradicts that t is linear-ordered.

Step 2.1.3. The largest pairwise defeat of b in ρ is smaller than the largest pairwise defeat of b in $\hat{\rho}$. Moreover, the largest defeat of b in ρ is against c.

Case (1) of Step 2.1.2 cannot occur because it would imply that there is some $i \in E$ and some $y \in C \backslash \{a\}$ such that $\rho_i(a) < \rho_i(y)$ and $\hat{\rho}_i(y) < \hat{\rho}_i(a)$, and then, $\pi^f_\rho(y) < \pi^f_\rho(a)$, which is a contradiction. Therefore, Case (2) of Step 2.1.2 holds. Let $y \in C \backslash \{x\}$ be the candidate against whom the largest defeat of x in ρ occurs. Then, $x \in \{b, c\}$ and $y \in C \backslash \{x\}$ are such that there is some $i \in E$ with $\rho_i(y) < \rho_i(x)$ and $\hat{\rho}_i(x) < \hat{\rho}_i(y)$. Note that then $\pi^f_\rho(x) < \pi^f_\rho(y)$. If $y = a$, then $\pi^f_\rho(x) < \pi^f_\rho(a)$, which is a contradiction. If $y = b$, then $x = c$ and $\pi^f_\rho(c) < \pi^f_\rho(b)$, which is a contradiction. Therefore, $y = c$ and $x = b$.

Step 2.1.4. The pairwise comparisons in ρ are such that a ties against b, a wins or ties against c, and b ties against c.

Because $\pi^f_\rho = abc$, b wins or ties against c. Moreover, because the largest defeat of b in ρ is against c, then b does not win against c (otherwise $\pi^f_\rho(b) = 1$). Therefore, b ties against c. Because the largest defeat of b in ρ is against c, then b wins or ties against a. Moreover, because b ties against c, then b does not win against a (otherwise $\pi^f_\rho(a) \neq 1$). Therefore, b ties against a. Finally, because c ties against b, then c does not win against a (otherwise $\pi^f_\rho(a) \neq 1$).

Step 2.1.5. The pairwise comparisons in $\hat{\rho}$ are such that a ties against b, a wins or ties against c, and b wins against c.

Because a wins or ties against c in ρ, then a wins or ties against c in $\hat{\rho}$ (otherwise there is $i \in E$ such that $\rho_i(a) < \rho_i(c)$ and $\hat{\rho}_i(c) < \hat{\rho}_i(b)$, and then $\pi^f_\rho(c) < \pi^f_\rho(b)$, which is a contradiction). Because the largest pairwise defeat of b in ρ is against c, b ties against c in ρ, and the largest pairwise defeat of b in ρ is smaller than the largest pairwise defeat of b in $\hat{\rho}$, then b wins against c in $\hat{\rho}$. Because, a ties against b in ρ, then a wins or ties against b in $\hat{\rho}$ (otherwise there is $i \in E$ such that $\rho_i(a) < \rho_i(b)$ and $\hat{\rho}_i(b) < \hat{\rho}_i(a)$, and then $\pi^f_\rho(b) < \pi^f_\rho(a)$, which is a contradiction). If a wins against b in $\hat{\rho}$, then the largest pairwise defeat of a in $\hat{\rho}$ is zero, while the largest pairwise defeats of b and c in $\hat{\rho}$ are larger than zero. Therefore, $\pi^f_{\hat{\rho}}(a) = 1$, which by Step 2.1.1 is not possible. Then, a ties against b in $\hat{\rho}$.

Step 2.1.6. The pairwise comparisons in ρ are such that a does not win against c.

Suppose on the contrary that a wins against c in ρ. Then, a also wins against c in $\hat{\rho}$ (otherwise, there is some $i \in E$ such that $\rho_i(a) < \rho_i(c)$ and $\hat{\rho}_i(c) < \hat{\rho}_i(b)$, and then $\pi^f_\rho(c) < \pi^f_\rho(b)$, which is a contradiction). Then, by Step 2.1.4, the largest pairwise defeats of a and b in ρ are zero, while the largest pairwise defeat of c in ρ is larger than zero. Hence, in the first round of the recursive process in ρ there is a tie between a and b. Because $\pi^f_\rho = abc$, the tie-breaking rule used by f breaks this tie in favor of a. Similarly, by Step 2.1.5, the largest pairwise defeats of a and b in $\hat{\rho}$ are zero, while the largest pairwise defeat of c in $\hat{\rho}$ is larger than zero. Then, in the first round of the recursive process in $\hat{\rho}$ there is a tie between a and b. Because the tie-breaking rule used by f breaks this tie in favor of a, $\pi^f_{\hat{\rho}}(a) = 1$, which by Step 2.1.1 is not possible.

Step 2.1.7. The pairwise comparisons in ρ are such that a does not tie against c.

Suppose on the contrary that a ties against c in ρ. Then, by Step 2.1.4, the largest pairwise defeats of a, b, and c in ρ are zero. Therefore, in the first round of the of the recursive process in ρ there is a tie between a, b, and c. Because $\pi^f_\rho = abc$, the tie-breaking rule used by f resolves this tie in favor of a. By Step 2.1.5, the largest pairwise defeats of a and b in $\hat{\rho}$ are zero, while the largest pairwise defeat of c in $\hat{\rho}$ is larger than zero. Then, in the first round of the recursive process in $\hat{\rho}$ there is a tie between a and b. Because the tie-breaking rule used by f is linear-ordered and it resolves a tie among a, b, and c in favor of a, then it also resolves a tie between a and b in favor of a. Then, $\pi^f_{\hat{\rho}}(a) = 1$, which by Step 2.1.1 is not possible.

Steps 2.1.6 and 2.1.7 contradict Step 2.1.5.

Step 2.2. f satisfies no veto.

Identical to Step 1.2.

Because $n \geq 3$ and $E^I_{xy} = E$ for every $xy \in [C]^2$, Steps 2.1 and 2.2 and Theorem 4 prove that f is Nash implementable. ■

PROOF OF REMARK 1

Suppose that $C = \{a, b, c\}$, $n \geq 3$, and the jury configuration (I, F) is such that $E^I_{xy} = E$ for every $xy \in [C]^2$.

Claim 1. A recursive SCF version of the Copeland rule whose tie-breaking rule does not satisfy non-favoritism may fail to be Nash implementable.

Suppose that $n = 6$. Let f be a recursive Copeland SCF with a tie-breaking rule t that does not satisfy non-favoritism. In particular, suppose w.l.o.g. that, if $\hat{C} \subset C$ is such that $\left|\hat{C}\right| = 2$ and $b \in \hat{C}$, then $t(\hat{C}) = b$. Let $\rho, \hat{\rho} \in \Pi^n$ be as depicted in Table 10. The majority pairwise comparisons based on ρ are such that: a and b are involved in a tie; a defeats c; b and c are involved in a tie. Then, the Copeland scores in ρ for a, b, and c are 1.5, 1, and 0.5, respectively. Therefore, $\pi^f_\rho(a) = 1$. Moreover, because b and c are involved in a tie in the second round and $t(\hat{C}) = b$ when $\hat{C} = \{b, c\}$, then we have $\pi^f_\rho(b) = 2$. Hence $\pi^f_\rho = abc$. The majority pairwise comparisons based on $\hat{\rho}$ are such that: a and b are involved in a tie; a defeats c; b defeats c. Then, the Copeland scores in $\hat{\rho}$ for a, b, and c are 1.5, 1.5, and 0, respectively. Because a and b are involved in a tie in the first round and $t(\hat{C}) = b$ when $\hat{C} = \{a, b\}$, then we have $\pi^f_{\hat{\rho}}(b) = 1$. Moreover, because a defeats c in a majority pairwise comparison based on $\hat{\rho}$, we have $\pi^f_{\hat{\rho}}(a) = 2$. Hence $\pi^f_{\hat{\rho}} = bac$. Then, $\pi^f_\rho \neq \pi^f_{\hat{\rho}}$. Moreover, the only difference between ρ and $\hat{\rho}$ is that the experts in E_1 change their judgments about the relative positions of b and c from $\rho_i(c) < \rho_i(b)$ to $\hat{\rho}_i(b) < \hat{\rho}_i(c)$. However, $\pi^f_\rho(b) < \pi^f_\rho(c)$. Then, f does not satisfy WUPE, regardless of the jury configuration. ■

ρ

E_1	E_2	E_4
3	2	1
a	b	b
c	c	a
b	a	c

$\hat{\rho}$

E_1	E_2	E_4
3	2	1
a	b	b
b	c	a
c	a	c

Table 10 Profiles of judgments in the proof of Remark 1, Claim 1.

Claim 2. A recursive SCF version of the minimax rule whose tie-breaking rule is not linear-ordered may fail to be Nash implementable.

Suppose that $n = 5$. Let f be a recursive minimax SCF with a tie-breaking rule t that is not linear-ordered. In particular, suppose w.l.o.g. that (i) $t(C) = a$ and (ii) if $\hat{C} = \{a, b\}$, $t(\hat{C}) = b$. Let $\rho, \hat{\rho} \in \Pi^n$ be as depicted in Table 11. Note that ρ is such that (1) 3 experts rank a before b and 2 experts rank b before a, (2) 2 experts rank a before c and 3 experts rank c before a, and (3) 3 experts rank b before c and 2 experts rank c before b. Then, a's largest pairwise defeat in ρ is 1 (against c), b's largest pairwise defeat in ρ is 1 (against a), and c's largest pairwise defeat in ρ is 1 (against b).

Because all candidates are involved in a tie in the first round and $t(C) = a$, we have $\pi_\rho^f(a) = 1$. Moreover, because b defeats c in a majority pairwise comparison based on ρ, we have $\pi_\rho^f(b) = 2$. Hence $\pi_\rho^f = abc$. Similarly, $\hat{\rho}$ is such that (1) 3 experts rank a before b and 2 experts rank b before a, (2) 2 experts rank a before c and 3 experts rank c before a, and (3) 4 experts rank b before c and 1 expert rank c before b. Then, a's largest pairwise defeat in $\hat{\rho}$ is 1 (against c), b's largest pairwise defeat in $\hat{\rho}$ is 1 (against a), and c's largest pairwise defeat in $\hat{\rho}$ is 3 (against b). Because a and b are involved in a tie in the first round and, if $\hat{C} = \{a, b\}$, $t(\hat{C}) = b$, we have $\pi_{\hat{\rho}}^f(b) = 1$. Moreover, because c defeats a in a majority pairwise comparison based on $\hat{\rho}$, we have $\pi_{\hat{\rho}}^f(c) = 2$. Hence $\pi_{\hat{\rho}}^f = bca$. Then, $\pi_\rho^f \neq \pi_{\hat{\rho}}^f$. Moreover, the only difference between ρ and $\hat{\rho}$ is that the experts in E_2 change their judgments about the relative positions of b and c from $\rho_i(c) < \rho_i(b)$ to $\hat{\rho}_i(b) < \hat{\rho}_i(c)$. However, $\pi_\rho^f(b) < \pi_\rho^f(c)$. Then, f does not satisfy WUPE, regardless of the jury configuration. ■

ρ

E_1	E_2	E_3	E_4
2	1	1	1
a	**c**	c	b
b	**b**	a	c
c	a	b	a

$\hat{\rho}$

E_1	E_2	E_3	E_4
2	1	1	1
a	**b**	c	b
b	**c**	a	c
c	a	b	a

Table 11 Profiles of judgments in the proof of Remark 1, Claim 2.

PROOF OF PROPOSITION 3

Suppose by contradiction that f^* does not satisfy WUPE. Then, there are $\rho, \hat{\rho} \in \Pi^n$ such that (i) $\pi_\rho^{f^*} \neq \pi_{\hat{\rho}}^{f^*}$ and (ii) for every $i \in E$ and $xy \in I_i$ such that $\pi_\rho^{f^*}(x) < \pi_\rho^{f^*}(y)$ and $\rho_i(x) < \rho_i(y)$, we have $\hat{\rho}_i(x) < \hat{\rho}_i(y)$. By (i), there are $x, y \in C$ such that $\pi_\rho^{f^*}(x) < \pi_\rho^{f^*}(y)$ and $\pi_{\hat{\rho}}^{f^*}(y) < \pi_{\hat{\rho}}^{f^*}(x)$. Let $k \in \{1, ..., m\}$ be the number of the rule in the definition of f^* that determines the relative ranking between x and y in $\pi_\rho^{f^*}$ (*i.e.*, the fact that $\pi_\rho^{f^*}(x) < \pi_\rho^{f^*}(y)$ is determined by Rule k of f^*). Note that then $\pi^*(x) \geq k$ and $\pi^*(y) \geq k$ (where π^* is the arbitrary ranking used by f^*).

Claim 1. $\pi^*(x) \neq k$.

Suppose that $\pi^*(x) = k$. Then $\pi^*(y) > k$ and, since $\pi^{f^*}_\rho(x) < \pi^{f^*}_\rho(y)$, $\left|\{i \in E^I_{xy} \mid \rho_i(x) < \rho_i(y)\}\right| \geq \frac{\left|E^I_{xy}\right|}{2}$. Then, by (ii), $\left|\{i \in E^I_{xy} \mid \hat{\rho}_i(x) < \hat{\rho}_i(y)\}\right| \geq \frac{\left|E^I_{xy}\right|}{2}$ and, if the relative ranking between x and y in $\pi^{f^*}_{\hat{\rho}}$ is not determined by any Rule $l < k$, we have $\pi^{f^*}_{\hat{\rho}}(x) < \pi^{f^*}_{\hat{\rho}}(y)$, which is a contradiction. Therefore, the relative ranking between x and y in $\pi^{f^*}_{\hat{\rho}}$ is determined by some Rule $l < k$. In particular, because $\pi^{f^*}_{\hat{\rho}}(y) < \pi^{f^*}_{\hat{\rho}}(x)$, we have $\pi^{f^*}_{\hat{\rho}}(y) < \pi^{f^*}_\rho(x^*_l) < \pi^{f^*}_{\hat{\rho}}(x)$, where x^*_l is the l-th candidate in π^*. Moreover, since the relative ranking between x and y in $\pi^{f^*}_\rho$ is determined by Rule $k > l$, then either (1) $\pi^{f^*}_\rho(x) < \pi^{f^*}_\rho(x^*_l)$ and $\pi^{f^*}_\rho(y) < \pi^{f^*}_\rho(x^*_l)$ or (2) $\pi^{f^*}_\rho(x^*_l) < \pi^{f^*}_\rho(x)$ and $\pi^{f^*}_\rho(x^*_l) < \pi^{f^*}_\rho(y)$ (otherwise, the relative ranking between x and y in $\pi^{f^*}_\rho$ would be determined by Rule l). If (1) happens, then $\left|\{i \in E^I_{x^*_l x} \mid \rho_i(x) < \rho_i(x^*_l)\}\right| > \frac{\left|E^I_{x^*_l x}\right|}{2}$ and then, by (ii), $\left|\{i \in E^I_{x^*_l x} \mid \hat{\rho}_i(x) < \hat{\rho}_i(x^*_l)\}\right| > \frac{\left|E^I_{x^*_l x}\right|}{2}$, which contradicts that $\pi^{f^*}_\rho(x^*_l) < \pi^{f^*}_{\hat{\rho}}(x)$. If (2) happens, then $\left|\{i \in E^I_{x^*_l y} \mid \rho_i(x^*_l) < \rho_i(y)\}\right| \geq \frac{\left|E^I_{x^*_l y}\right|}{2}$ and then, by (ii), $\left|\{i \in E^I_{x^*_l y} \mid \hat{\rho}_i(x^*_l) < \hat{\rho}_i(y)\}\right| \geq \frac{\left|E^I_{x^*_l y}\right|}{2}$, which contradicts that $\pi^{f^*}_{\hat{\rho}}(y) < \pi^{f^*}_\rho(x^*_l)$.

Claim 2. $\pi^*(y) \neq k$.

Suppose that $\pi^*(y) = k$. Then $\pi^*(x) > k$ and, since $\pi^{f^*}_\rho(x) < \pi^{f^*}_\rho(y)$, $\left|\{i \in E^I_{xy} \mid \rho_i(x) < \rho_i(y)\}\right| > \frac{\left|E^I_{xy}\right|}{2}$. Then, by (ii), $\left|\{i \in E^I_{xy} \mid \hat{\rho}_i(x) < \hat{\rho}_i(y)\}\right| > \frac{\left|E^I_{xy}\right|}{2}$ and, if the relative ranking between x and y in $\pi^{f^*}_{\hat{\rho}}$ is not determined by any Rule $l < k$, we have $\pi^{f^*}_{\hat{\rho}}(x) < \pi^{f^*}_{\hat{\rho}}(y)$, which is a contradiction. Therefore, the relative ranking between x and y in $\pi^{f^*}_{\hat{\rho}}$ is determined by some Rule $l < k$. The rest of the proof of this claim is analogous to that of Claim 1.

Claim 3. Either $\pi^*(x) \leq k$ or $\pi^*(y) \leq k$.

Suppose that $\pi^*(x) > k$ *and* $\pi^*(y) > k$. Because the relative ranking between x and y in $\pi^{f^*}_\rho$ is determined by Rule k of f^* and $\pi^{f^*}_\rho(x) < \pi^{f^*}_\rho(y)$, then $\pi^{f^*}_\rho(x) < \pi^{f^*}_\rho(x^*_k) < \pi^{f^*}_\rho(y)$, where x^*_k is the k-th candidate in π^*. The fact that $\pi^{f^*}_\rho(x) < \pi^{f^*}_\rho(x^*_k)$ implies that $\left|\{i \in E^I_{x^*_k x} \mid \rho_i(x) < \rho_i(x^*_k)\}\right| > \frac{\left|E^I_{x^*_k x}\right|}{2}$

and then, by (ii), $\left|\{i \in E^I_{x^*_k x} \mid \hat{\rho}_i(x) < \hat{\rho}_i(x^*_k)\}\right| > \frac{\left|E^I_{x^*_k x}\right|}{2}$. Similarly, the fact that $\pi^{f^*}_{\rho}(x^*_k) < \pi^{f^*}_{\rho}(y)$ implies that $\left|\{i \in E^I_{x^*_k y} \mid \rho_i(x^*_k) < \rho_i(y)\}\right| \geq \frac{\left|E^I_{x^*_k y}\right|}{2}$ and then, by (ii), $\left|\{i \in E^I_{x^*_k y} \mid \hat{\rho}_i(x^*_k) < \hat{\rho}_i(y)\}\right| \geq \frac{\left|E^I_{x^*_k y}\right|}{2}$. Then, in case the relative ranking between x and y in $\pi^{f^*}_{\hat{\rho}}$ has not been determined by any Rule $l < k$ of f^*, we have $\pi^{f^*}_{\hat{\rho}}(x) < \pi^{f^*}_{\hat{\rho}}(x^*_k) < \pi^{f^*}_{\hat{\rho}}(y)$, which contradicts that $\pi^{f^*}_{\hat{\rho}}(y) < \pi^{f^*}_{\hat{\rho}}(x)$. Therefore, the relative ranking between x and y in $\pi^{f^*}_{\hat{\rho}}$ is determined by some Rule $l < k$. The rest of the proof of this claim is analogous to that of Claim 1.

From Claims 1, 2, and 3, we have that either $\pi^*(x) < k$ or $\pi^*(y) < k$, which contradicts that the relative ranking between x and y in $\pi^{f^*}_{\rho}$ is determined by Rule k of f^*. ■

References

Amorós, P., Corchón, L.C., and Moreno, B. (2002), "The scholarship assignment problem," *Games and Economic Behavior*, 38, 1-18.

Amorós, P. (2009): "Eliciting socially optimal rankings from unfair jurors," *Journal of Economic Theory*, 144, 1211-1226.

Austen-Smith, D. (1993): "Interested experts and policy advice: Multiple referrals under open rule," *Games and Economic Behavior*, 5, 3–43.

Arrow, K. J. (1951): "Social choice and individual values," Hoboken: John Wiley and Sons.

Bhattacharya, S. and Mukherjee, A. (2013): "Strategic information revelation when experts compete to influence," *The RAND Journal of Economics*, 44, 522-544.

Duggan, J. and Martinelli, C. (2001): "A Bayesian model of voting in juries," *Games and Economic Behavior*, 37, 259–294.

Feddersen, T. and Pesendorfer, W. (1998): "Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting," *American Political Science Review*, 92, 23-35.

Gerardi, D., McLean, R., and Postlewaite, A. (2009): "Aggregation of expert opinions," *Games and Economic Behavior*, 65, 339-371.

Gibbard, A. (1973): "Manipulation of voting schemes: A general result," *Econometrica*, 41, 587-601.

Krishna, V. and Morgan, J. (2001): "A model of expertise," *Quaterly Journal of Economics*, 116, 747-775.

Maskin, E. (1999): "Nash equilibrium and welfare optimality," *Review of Economic Studies*, 66, 23-38.

Moore, J. and Repullo, R. (1988): "Subgame perfect implementation," *Econometrica*, 56, 1191-1220.

Satterthwaite, M. A. (1975): "Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions," *Journal of Economic Theory*, 10, 187-217.

Wolinsky, A. (2002): "Eliciting information from multiple experts," *Games and Economic Behavior*, 41, 141-60.