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Ascensión Andina-Díaz and José A. García-Martínez

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Interpersonal comparisons and concerns for expertise^{*}

Ascensión Andina-Díaz[†] and José A. García-Martínez[‡]

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Abstract

We study the effect of introducing interpersonal comparisons on the decisions made by career concerned experts. We consider a model with two experts, a stronger and a weaker, who face common uncertainty about the state of the world. We show that whereas full transmission of the experts' private information is an equilibrium when experts care about their absolute level of expertise, this is not necessarily the case when interpersonal comparisons matter and experts care about their relative level of expertise. In this case, we obtain that there is an equilibrium in which experts' decisions follow experts' signals only when the probability of feedback is sufficiently high. Otherwise, the stronger expert benefits from discarding her private information. In equilibrium, this expert may even completely contradict her signal and the other expert's decision. We discuss the implications of this result for reaching experts' consensus and dissent.

Keywords: Interpersonal comparisons; career concerns; probability of feedback; consensus; dissent **JEL:** C72; D82; D83

1 Introduction

It's human nature to compare ourselves to others. Sometimes unconsciously, individuals tend to evaluate our own social and personal achievements based on how we stack up against others. We do it on a daily basis and across multiple dimensions, from success and intelligence, to wealth and attractiveness.

Though a regular and well established phenomenon, we do not know much about how interpersonal comparisons affect individual behavior and decision making.¹ This paper aims to contribute to this research question. More precisely, we are interested in understanding the effects that interpersonal

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[†]Dpto. Teoría e Historia Económica, Universidad de Málaga, Spain. E-mail: aandina@uma.es

[‡]Dpto. Estudios Económicos y Financieros, Universidad Miguel Hernández, Spain. E-mail: jose.garciam@umh.es

¹This phenomenon, known as the "Social Comparison Theory", was first proposed in 1954 by psychologist Leon Festinger. See Meyer and Vickers (1997), Fershtman et al. (2003, 2006), Luttmer (2005), Clark et al. (2008), Roels and Su (2014), and López-Pintado and Meléndez-Jiménez (2019) for the analysis of decision making with interpersonal comparisons under different contexts.

comparisons have on agents with a career concern for *expertise*.² This is a relevant question, as success and reputation is one of the more prominent dimensions on which individuals tend to compare ourselves with others.

To illustrate our research question, consider the following example. Suppose two policy advisors, a stronger one and a weaker one (in a reputational sense), each providing advice to a policy maker. Suppose further that policy advisors have a concern for expertise and care about how the policy maker perceives them in comparison to the other advisor. A reason for this payoff function may be that only the expert with the higher relative reputation will be listened in the future, promoted to a fixed position in the politician's cabinet, etc. Suppose further that policy advisors, who are better informed than the politician about an uncertain common variable, such as the state of the world, make recommendations at the same time. In this situation, should we expect the two policy advisors to provide the politician with the same advice? Will they differ? If so, which one will be more informative?

The results in this work show that introducing interpersonal comparisons have a important effect on the behavior of career concerned experts. In particular, it introduces an incentive for the stronger expert to discard her private information. We obtain that this incentive can be strong enough to induce the stronger expert to always contradict her signal, which results in equilibria featuring experts' dissent.

We propose a model with two experts and heterogeneous expertise, who face common uncertainty about the state of the world and who are imperfectly and asymmetrically informed about it. Each expert can be either of two types: wise or normal, the difference being the quality of the information they receive about the state of the world. Experts differ in their initial expertise, i.e., the probability that they are perceived as a wise expert. According to this, we talk about the stronger expert and the weaker one. Upon receiving information on the state, experts take simultaneous actions. The novelty of this work is to consider experts who have reputational concerns and care about interpersonal comparisons. We refer to this system as the Relative Performance Evaluation (RPE) system,³ and compare the results under this system with those under the standard system, referred to as the Absolute Performance Evaluation (APE) system, in which experts seek to maximize their individual/absolute reputation. We identify a key variable that determines whether both evaluation systems can yield the same outcomes (in terms of the experts' behavior and the information transmitted) or not: the probability of feedback, i.e., the probability that the principal learns the state. Our results show that when the probability of feedback is sufficiently high, the behavior and the information revealed by the two experts can be the same under the two systems. However, when the probability of feedback is below a certain threshold, both systems are no longer similar. In this case, we obtain that full revelation of the experts' private information is always an equilibrium under the APE system; however, it is never an equilibrium under the RPE system. In the latter case, we identify an incentive for the stronger expert

²With expertise we refer to the ability of an agent to know about an uncertain common variable, such as the state of the world. Other models of career concerns consider agents who can differ in either their ability to exert effort (Harris and Holmström (1982) and Holmström (1999)) or their preference for the implemented policy (Morris (2001)). See Lin (2015) for a recent literature review.

 $^{^{3}}$ This term was first used by Holmström (1982), who defined it for a model of effort provision in teams with career concerned agents.

to discard her informative private signal. In equilibrium, this expert may even completely contradict her signal and the other expert's decision.

Our analysis generates new insights into the effects of interpersonal comparisons on experts' consensus and dissent. We say that *consensus* exists when both experts take the same action with a probability higher than one half; there is *dissent* otherwise. Our results suggest that whereas the APE system fosters consensus, the RPE system is likely to drive dissent; the latter occurring when the probability of feedback is not very high.⁴ From our personal experience, we may probably recognize situations in which informed agents (think on tv commentators on economic, environmental or health issues, workplace colleges, friends, relatives, etc), having no different biases or preferences, choose to support different opinions and contradict each other, making consensus difficult to achieve and putting burdens on the principal's capacity to make the correct decision. This work develops a new rationale for this behavior. We show that in contrast to the APE system, in which both experts gain from following their private informative signals, which fosters consensus; under the RPE system the stronger expert can do better than sticking to her signal. The reason is that by deviating and contradicting her signal, the stronger expert also contradicts the weaker expert's action, in which case the principal will put higher weight in the informative content of the stronger expert's action than in the weaker expert's action, for the former being more likely to have a better signal.⁵ This effect explains why experts' dissent is an equilibrium outcome when experts care about interpersonal reputation comparisons and the probability that the principal learns the state of the world is not very high.

Going further with this argument, it also suggests that the more able the stronger expert is to anticipate the action of the weaker expert, the higher the incentive of the former to contradict the weaker expert's action. It is interesting to note that in models like the present one, where i) experts face common uncertainty about the state of the world and signals are correlated and ii) there is one expert that gains from following her signal, say expert j; the ability to anticipate the opponent's action depends on the quality of the signal. The reason is that under i), an increase in the quality of an expert's signal increases not only her information about the state but also her knowledge of the opponent's signal. Additionally, under ii), an increase in the quality of the signal increases the expert, as she benefits from contradicting the opponent's action. In line with this intuition, the results in this work show that the incentives of the stronger expert to discard her signal may even increase with the quality of the signal.⁶ This result has an interesting implication. It suggests that when interpersonal comparisons matter, we may expect more dissent (for career concern reasons) in contexts where signals

⁴Since signals are always informative, a profile in which experts follow their signals corresponds to a profile in which experts take the same action with a probability higher than one half, i.e., a profile in which there is experts' consensus. In contrast, a profile in which experts take different actions implies experts' dissent.

⁵This result requires signals (of the normal type experts) to be either of the same quality, independently of the expert's ex ante reputation, or to be of higher quality for the stronger expert and of lower quality for the weaker expert. For simplicity, in the paper we consider that the quality of a signal is the same for any normal type expert. However, we conjecture that the results in this paper would maintain if stronger experts received better signals.

⁶This result is obtained considering signals that are i.i.d. conditional on the state. From the argument above, we predict this result to be stronger if conditional on the state, signals were correlated.

are of good quality than when they are noisy. Coming back to the example of the policy advisors, we should expect more dissent when policy advisors have low uncertainty about the state (e.g., because they receive good quality signals) than when uncertainty is higher. If uncertainty about the state is usually higher during economic crisis and lower when the economy is doing well, our prediction is that expert's dissent (for career concern reasons) would be more likely to occur when the economy os going well.

This paper contributes to the literature about career concerns for expertise and speaks directly to the analysis of reputational concerns in the presence of competing experts (see Ottaviani and Sørensen (2001), Ottaviani and Sørensen (2006), and Bourjade and Jullien (2011)). Common to these models, we consider experts who have private information about both the state of the world (common uncertainty) and their type (private uncertainty). In contrast to these papers, we consider interpersonal comparisons and a principal who does not always learn the state of the world, i.e., the probability of feedback is positive but different from one. These are important features in our model and important distinctions from previous literature. Indeed, our results show that it is precisely when the probability of feedback is different from one that the two systems, APE and RPE, differ; otherwise, they yield the same outcome. In this sense, in our work, transparency is always beneficial to the principal. This is in contrast to Prat (2005), Levy (2007), Fox and Van Weelden (2012), and Andina-Díaz and García-Martínez (2020a,b), who have proved that transparency can have a perverse effect under different contexts.

Our results are also related to the reputational herding and anti-herding literature, initiated by Scharfstein and Stein (1990). They show that when experts make decisions in a sequential order and they do not know their type, there is an incentive for the second expert to herd on the decision of the first expert. Effinger and Polborn (2001) consider a model as in Scharfstein and Stein (1990), but assume an exogenous payoff function such that an expert is most valuable if he is the only smart expert. They show that if the value of being the only smart expert is sufficiently large, anti-herding occurs. Herding and anti-herding results have also been shown to arise when not all the states of the world are equally likely. See Levy (2004) for a model of anti-herding with a single expert and Heidhues and Lagerlöf (2003), Cummins and Nyman (2005) and Gentzkow and Shapiro (2006) for herding models with two experts. Quite intuitively, considering that one state is more likely than the others introduces and incentive to herd on the popular belief, which no longer requires competition to be sequential. In contrast to this literature, the results in this paper are derived under the assumption that all the states of the world are equally likely and competition is simultaneous. This guarantees that herding/anti-herding effects are not behind our results, as there is neither a popular belief nor a first decision to follow.

2 The model

We consider a model between two experts $i \in \{1, 2\}$ (she) with career concerns and one principal (he). There is a binary state of the world $\omega \in \{L, R\}$ and a binary set of actions $a_i \in \{\hat{l}, \hat{r}\}$. We assume that the two states are equally likely. Experts make simultaneous decisions on the actions to take. When $a_i = \omega$ we say that the action taken by expert *i* is correct; it is incorrect otherwise. Prior to taking an action, expert *i* receives a private signal $s_i \in \{l, r\}$ on the state of the world. We denote by γ the quality of a signal, with $\gamma = P(l \mid L) = P(r \mid R)$, and assume that the distribution of the quality of a signal depends on the type of the expert, which can be either wise type (W) or normal type (N). Let t_i be the type of an expert, with $t_i \in \{W, N\}$ and $i \in \{1, 2\}$. We assume that a wise type expert always receives a signal that perfectly reveals the state of the world (it has quality 1); whereas a normal type expert receives an imperfect but informative signal of quality $\gamma \in (\frac{1}{2}, 1)$. Types of experts are i.i.d. and signals are i.i.d. conditional on the state. Note that γ can be arbitrarily close to 1, i.e., normal type experts can receive signals of arbitrarily excellent quality. The type of an expert is the expert's private information. The other players (expert -i and the principal) have a common prior about the probability that expert *i* is a wise type. Let $\alpha_i \in (0, 1)$ be this common prior probability; then $1 - \alpha_i$ is the prior probability that expert *i* is normal type. We assume $\alpha_1 > \alpha_2$, i.e., it is common knowledge that, ex ante, expert 1 has a higher probability of being wise type than expert 2. Hereafter, we refer to expert 1 as the *stronger* expert and to expert 2 as the *weaker* expert.

We define the strategy of an expert as a mapping that associates with every possible type and signal of the expert a probability distribution over the space of actions. For the sake of simplicity, we denote by $\sigma_t^i(s) \in [0, 1]$ the probability that expert *i* of type *t* takes the action *a* that corresponds to her signal *s*. Thus, $\sigma_t^i(l) = P_t^i(\hat{l} \mid l)$ and $\sigma_t^i(r) = P_t^i(\hat{r} \mid r)$, for $i \in \{1, 2\}$ and $t \in \{W, N\}$. Then, $1 - \sigma_t^i(l) = P_t^i(\hat{r} \mid l)$ and $1 - \sigma_t^i(r) = P_t^i(\hat{l} \mid r)$ is the probability that expert *i* of type *t* takes the action *a* that does not correspond to expert *i*'s signal *s*.

Let $\mu > 0$ denote the probability that before forming a belief about the type of the experts, the principal receives ex-post verification of the state of the world. We refer to μ as the *probability of feedback*. We denote by $X \in \{L, R, \emptyset\}$ the feedback received by the principal, with $X = \emptyset$ indicating that there is no feedback and X = L indicating that the principal learns that the state is L (analogously for X = R). The principal observes the vector of actions (a_1, a_2) and feedback X and, based on this information, updates his beliefs about each of the experts' type. Let $\hat{\alpha}_i(a_1, a_2, X)$ denote the principal's posterior probability that expert i is type W, given (a_1, a_2) and X.

Experts have concerns for expertise and each chooses the action to take seeking to maximize her reputation for looking wise, i.e., for being perceived as type W. We consider two scenarios.

In the first scenario, each expert *i* observes signal s_i and chooses a_i seeking to maximize her (absolute) reputation for being a wise type. This scenario corresponds to the standard approach in the literature of career concerns with competing experts (see Scharfstein and Stein (1990), Ottaviani and Sørensen (2001), Ottaviani and Sørensen (2006), or Bourjade and Jullien (2011)). It illustrates a situation in which there are no interpersonal comparisons, and the evaluation of an expert is exclusively based on the expert's performance. Note, nevertheless, that even in case the action taken by expert *i*'s opponent, i.e., a_{-i} , might be useful to evaluate expert *i*'s performance. This is so when $X = \emptyset$, in which case action a_{-i} may contain information on the state; hence, it may reveal information on expert *i*'s expertise. We refer to this system of evaluation as the Absolute Performance Evaluation (APE) system and consider that the payoff function of expert i is:

$$\Pi_{i}^{A}(t; a_{i}, a_{-i}, X) = \hat{\alpha}_{i}(a_{1}, a_{2}, X).$$

In the second scenario, each expert i observes signal s_i and chooses a_i seeking to maximize her (relative) reputation. In contrast to the previous system, under this system the evaluation of an expert is based both on the own expert's performance and on the opponent's performance. We refer to this system of evaluation as the *Relative Performance Evaluation* (RPE) system and consider that the payoff function of expert i is:

$$\Pi_i^R(t; a_i, a_{-i}, X) = \frac{\hat{\alpha}_i(a_1, a_2, X)}{\hat{\alpha}_i(a_1, a_2, X) + \hat{\alpha}_{-i}(a_1, a_2, X)}$$

This evaluation system illustrates situations in which either the principal or the experts themselves evaluate an expert's performance based on a comparison with others. When it is the expert who evaluates herself this way, the expert does it through the eyes of the principal, using the principal's posterior belief that he assigns to each expert being a wise type. Note that in this case, an expert might prefer to be perceived as an expert with a small $\hat{\alpha}$ rather than a high $\hat{\alpha}$, if by so doing the distance away to the other expert is maximized.

Our equilibrium concept is Perfect Bayesian Equilibrium. We say that $(\sigma_W^i(l)^*, \sigma_W^i(r)^*; \sigma_N^i(l)^*, \sigma_N^i(r)^*)$ is an *equilibrium strategy* of expert *i* if given the equilibrium strategy of expert -i and the players' consistent beliefs, $\sigma_t^i(l)^*$ maximizes the expected payoff of expert *i* of type *t* after observing signal *l*, and $\sigma_t^i(r)^*$ does it after signal *r*. We denote an equilibrium strategy by $\{(\sigma_W^i(l)^*, \sigma_W^i(r)^*; \sigma_N^i(l)^*, \sigma_N^i(r)^*)\}_{i \in \{1,2\}}$.

3 Analysis

In this section we analyze the equilibrium behavior of the experts. Prior to presenting the results, we introduce some concepts. For a given $i \in \{1, 2\}$ and $t \in \{W, N\}$, we say that the strategy of expert i of type t is *honest* when $(\sigma_t^i(l)^*, \sigma_t^i(r)^*) = (1, 1)$, i.e., the expert takes the action that corresponds to her signal with probability one. When the two types of the two experts use an honest strategy, we say that the equilibrium is honest. Additionally, for a given $i \in \{1, 2\}$ and $t \in \{W, N\}$, we say that the strategy of expert i of type t is symmetric when $\sigma_t^i(l) = \sigma_t^i(r) = \sigma_t^i$, i.e., the expert takes the action that corresponds to her signal with the same probability across the two information sets, s = l and s = r. When the two types of the two experts use symmetric strategies, we say that the equilibrium is symmetric.

For expositional purposes, the analysis that follows considers that wise type experts always take the action that corresponds to their signal, i.e., they play an honest strategy, and focuses the attention on the behavior of the normal type experts; hereafter simply referred to as the *experts*. This assumption is relaxed in Part II of the Appendix, where we analyze the model considering that wise type experts are also strategic, and show that most of the results of the paper hold under the more general case (see Propositions 4 and 5).

Next, we state the main results of the paper, which correspond to Propositions 1 and 2. The first

result describes the equilibrium behavior under the APE system and the second result describes the equilibrium behavior under the RPE system.

Proposition 1. (APE system) Under APE there is always an equilibrium in which both the stronger and the weaker expert follow their signal, i.e., $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1, 1)$, for all $i \in \{1, 2\}$ and $\mu > 0$. Furthermore, if we restrict attention to symmetric strategies, this equilibrium is unique.

Proposition 1 presents the result in the benchmark case. It states that under the APE system there is always an honest equilibrium in which the two normal type experts, the stronger and the weaker, follow their signal. This is an equilibrium for any level of transparency $\mu > 0$. It further states that if we restrict attention to symmetric strategies, this equilibrium is unique.⁷ Proposition 2 below presents the results with interpersonal comparisons. The expressions of the thresholds and the equilibrium probability x are defined in the proof.⁸

Proposition 2. (*RPE system*) Under *RPE* there exists μ_1 and μ_2 , with $\mu_1 < \mu_2$ and $\mu_2 \in (0,1)$, such that there is an equilibrium in which $(\sigma_N^2(l)^*, \sigma_N^2(r)^*) = (1,1)$ and:

- If $\mu > \mu_2$, then $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (1, 1)$.
- If $\mu_1 < \mu < \mu_2$, then $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (x, x)$, with $x \in (0, 1)$.
- If $\mu < \mu_1$, then $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (0, 0)$, where $\mu_1 > 0$ if and only if $\alpha_1 > \bar{\alpha}$, with $\bar{\alpha} \in (\alpha_2, 1)$.

Additionally, if we restrict attention to symmetric strategies, the equilibria above are unique.

Proposition 2 identifies different scenarios according to the probability of feedback μ . For each of these scenarios, we describe the equilibrium of the game and show that they are unique if we restrict attention to the use of symmetric strategies. Note that a common feature to all the scenarios is that the weaker expert always follows her signal.

A comparison of the results in Propositions 1 and 2 show that when the probability of feedback is sufficiently high, specifically $\mu > \mu_2$, both performance evaluation systems produce the same type of incentives to the experts. However, we observe that when the probability of feedback is below threshold μ_2 , the two evaluation systems yield different equilibrium outcomes: whereas under the APE system there is always an honest equilibrium, under the RPE system this is no longer the case. This is due to the fact that RPE introduces an incentive for the stronger expert to differentiate her action from the weaker expert. This incentive increases the smaller the probability of feedback. Thus, in equilibrium, the stronger expert sticks to her signal with positive probability when $\mu_1 < \mu < \mu_2$, but she never does it when the probability of feedback is very small, i.e., $\mu < \mu_1$. Noteworthy, the

⁷Note that in our set-up, considering symmetric strategies is a natural restriction as the two information sets that correspond to the two possible states of the world are symmetric: both states are equally likely, signals are equally informative across states, and the probability of feedback is fixed and invariant across actions and/or states.

⁸The probability x is a function of the parameters in the model α_1 , α_2 , γ , and μ , and it satisfies $\Delta_r^{1,R} = 0$, with $\Delta_r^{1,R} = -\Delta_l^{1,R}$ being defined by expressions (12) and (13). In the proof of this result we also derive the explicit expressions of thresholds μ_1 , μ_2 and $\bar{\alpha}$, with μ_1 and μ_2 being a function of parameters α_1 , α_2 , and γ ; and $\bar{\alpha}$ being a function of α_2 and γ .

latter case only occurs for strong enough experts, i.e., $\alpha_1 > \bar{\alpha}$, which implies that the incentive of the stronger expert to completely discard and contradict her signal only occurs for highly reputed experts. Note that when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (x, x)$ the stronger expert uses a mixed strategy, whereas when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (0, 0)$ she uses a mirror strategy, i.e., she always chooses the action opposite to her signal. Given that in equilibrium $(\sigma_N^2(l)^*, \sigma_N^2(r)^*) = (1, 1)$ always, we say that there is an *honest equilibrium* when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (1, 1)$, there is a *mixed equilibrium* when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (x, x)$, with $x \in (0, 1)$, and there is a *mirror equilibrium* when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (0, 0)$.⁹ See Figure 1 for a graphical representation of these regions. As observed from the figure, ceteris paribus the rest of parameters, an increase in μ makes it more likely than the stronger expert follows her signal, whereas an increase in α_1 never increases this probability.

To see the intuition for the result under the RPE system, and in particular the existence of the mixed equilibrium and the mirror equilibrium, note that when the probability of feedback is sufficiently small there is no learning. In this case, whereas when experts take the same action there is not a lot that affects the principal's updating process, when they rather take different actions, by Bayes rule, the principal will put a higher weight in the action of the stronger expert than in the one of the weaker expert. The reason being that stronger experts are more likely to be wise type, hence more likely to receive a signal of better quality. This induces the stronger expert to discard her signal, in an attempt to contradict the weaker expert's action.¹⁰

The following exercise may help clarify this idea. It consider the limit case in which the probability of feedback tends to zero ($\mu \to 0$, hence $X = \emptyset$) and the weaker expert uses the honest strategy, and describes the stronger expert's payoff $\prod_{i=1}^{R} (N; a_i, a_j, X)$ when she also uses the honest strategy. In this case, the payoff to the stronger expert *i* if she sends the same action *a* than the opponent is:

$$\Pi_i^R(N; a, a, \varnothing) = \frac{\alpha_i(\alpha_j(1-\gamma)+\gamma)}{\alpha_j\gamma + \alpha_i(\gamma - 2\alpha_j(\gamma-1))},$$

whereas if she sends action $a' \neq a$, she gets:

$$\Pi_i^R(N;a',a,\varnothing) = \frac{\alpha_i(1-\alpha_j)}{\alpha_j + \alpha_i(1-2\alpha_j)}$$

It can be shown that $\Pi_i^R(N; a', a, \emptyset) > \Pi_i^R(N; a, a, \emptyset) \Leftrightarrow \alpha_i > \alpha_j$.¹¹ Noteworthy, we also obtain that $\Pi_i^R(N; a', a, \emptyset) > \Pi_i^R(N; a, a, \emptyset) \Leftrightarrow \alpha_i > \alpha_j$ when the stronger expert uses the mirror strategy and the weaker expert uses the honest strategy.¹² This result suggests a preference (and an incentive) for the stronger expert to contradict the weaker expert's action, and a preference (and an incentive) for the weaker expert to make her action coincide with that of the stronger expert. It further suggests that when the probability of feedback is very small, the RPE system shapes the nature of the game for the two players, producing very different incentives to the experts: an incentive to the stronger

 $^{^{9}}$ Note that the uniqueness result implies that the RPE system cannot produce "reversed" equilibria such as one in which the stronger expert plays the honest strategy and the weaker expert discards her signal with positive probability.

¹⁰Note that even though in a mirror equilibrium the stronger expert contradicts her signal, this is the strategy that maximizes the probability that the principal beliefs her to be a wise type expert.

¹¹ $\Pi_i^R(N; a, a, \emptyset)$ is increasing in γ , and it takes value $\Pi_i^R(N; a, a, \emptyset) = \frac{\alpha_i}{\alpha_i + \alpha_j}$ when $\gamma = 1$. This says that by sending the same message than her opponent, expert *i*'s payoff is simply her initial relative reputation.

the same message than her opponent, expert *i*'s payoff is simply her initial relative reputation. ¹²The payoffs in this case are $\Pi_i^R(N; a, a, \emptyset) = \frac{\alpha_i(\alpha_j(1-\gamma)+\gamma)}{\alpha_i\gamma+\alpha_j(\alpha_i+1-\gamma)}$ and $\Pi_i^R(N; a', a, \emptyset) = \frac{\alpha_i(\alpha_j-1)(1-\gamma)}{\alpha_i(\alpha_j+\gamma-1)-\alpha_j\gamma}$.

expert to differentiate her action from that of the weaker expert (as in strategic substitutes games), and an incentive to the weaker expert to take the same action than the stronger expert (as in strategic complements games).¹³

This exercise generates new insights into the effects of interpersonal comparisons on experts' consensus and dissent, insights which can also be derived from the results of Propositions 1 and 2. Formally, we say that there is experts' consensus when both experts take the same action with a probability higher than one half; there is dissent otherwise. The results in this paper suggest that whereas the APE system fosters experts' consensus, the RPE system is likely to drive experts' dissent; the latter occurring when the probability of feedback is not very high. The next result formally states this idea.

Corollary 1. Under the APE system experts' consensus is more likely than experts' dissent. Under the RPE system this is only the case when the probability of feedback is sufficiently high. In contrast, for a sufficiently small probability of feedback, the RPE system is likely to engender experts' dissent.

The proof of this result follows immediately from the fact that, in equilibrium, under the APE system, $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1, 1)$ for all $i \in \{1, 2\}$. In this case, it is easy to observe that the probability that experts' decisions coincide is $\gamma^2 + (1 - \gamma)^2$, which is always higher than 1/2. In contrast to this, under the RPE system, when the probability of feedback is sufficiently small, specifically $\mu < \mu_1$, in equilibrium $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (0, 0)$ and $(\sigma_N^2(l)^*, \sigma_N^2(r)^*) = (1, 1)$. In this case, the probability that the experts' decisions coincide is $2\gamma(1 - \gamma)$, which is always smaller than 1/2.

It is also worth noting that the probability of experts' dissent may even increase in the quality of the signal. The next corollary presents a comparative static exercise and, among other results, states this idea.

Corollary 2. Thresholds μ_2 and μ_1 , which delimit the regions of existence of the different type of equilibria in the RPE system, satisfy:

- 1. With respect to α_1 , we have $\frac{\partial \mu_2}{\partial \alpha_1} > 0$ and $\frac{\partial \mu_1}{\partial \alpha_1} > 0$.
- 2. With respect to γ , we have $\frac{\partial \mu_2}{\partial \gamma} > 0$ and the existence of $\hat{\alpha} \in (\bar{\alpha}, 1)$ and $\hat{\gamma} \in (\frac{1}{2}, 1)$ such that if $\alpha_1 > \hat{\alpha}$ and $\gamma < \hat{\gamma}$, then $\frac{\partial \mu_1}{\partial \gamma} > 0$. Otherwise, $\frac{\partial \mu_1}{\partial \gamma} < 0$.

Figure 1 below presents a graphical description of the results of Corollary 2, where top panels present a comparative static exercise with respect to parameter α_1 , and bottom panels a comparative static exercise with respect to parameter γ .

Figure 1 about here

¹³These ideas help also explain why there is not an equilibrium in which the stronger expert uses either the honest or the mirror strategy when $\mu \to 0$ and $\gamma \to 1$ (see the bottom-right panel of Figure 1). Note that if she were to use the honest strategy (similarly for the mirror one), the weaker expert would have an incentive to stick to her signal and make her action coincide with that of the stronger expert. However, in such a case, the stronger expert would prefer to deviate from her signal, hence contradicting the action of the weaker expert. This, of course, would lead the weaker expert to deviate from the previous strategy, and on so forth. These inconsistencies solve when the stronger expert uses a mixed strategy, in which case we have an equilibrium.



Figure 1: Top panels represent the effect of a change in α_1 on the regions where the honest (blue), the mixed (brown) and the mirror equilibrium (purple) exist. Threshold $\bar{\alpha}$ corresponds to $\mu_1(\alpha_1) = 0$. The top-left panel considers $\alpha_2 = 0.6$ and $\gamma = 0.6$ and the top-right panel considers $\alpha_2 = 0.6$ and $\gamma = 0.85$. Bottom panels represent the effect of a change in γ on the regions where the honest (blue), the mixed (brown) and the mirror equilibrium (purple) exist. The bottom-left panel considers $\alpha_2 = 0.4$ and $\alpha_1 = 0.8$ and the bottom-right panel considers $\alpha_2 = 0.4$ and $\alpha_1 = 0.98$.

As stated in Corollary 2, a look at top panels shows that ceteris paribus the rest of parameters, an increase in α_1 increases both μ_2 and μ_1 . This implies that the higher the initial reputation of the stronger expert, the smaller the range of values of parameter μ for which the honest equilibrium holds, and the higher the range of values for which the mirror equilibrium holds. We also observe that $\mu_1 \rightarrow \mu_2$ when $\alpha_1 \rightarrow 1$ (see the last part of the proof of Proposition 2 for a proof of this result), which further implies that the higher the initial reputation of the stronger expert, the smaller the region where she uses a mixed strategy. In the limit, she either sticks to her signal or contradicts it. Additionally, we observe that $\mu_2 \rightarrow 0$ when $\alpha_1 \rightarrow \alpha_2$ (see also the proof of Proposition 2 for a proof of this result), which implies that when experts are very similar in terms of their initial reputation, the slightly stronger expert reveals her signal for most of parameter values μ . In the limit, the two experts are always honest. We will come back to this idea later on.

Regarding bottom panels, we observe that the effect of γ on μ_1 is not always monotonic: whereas the left-hand side panel represents a situation in which $\frac{\partial \mu_1}{\partial \gamma} < 0$ always, the right-hand side panel represents a situation in which $\frac{\partial \mu_1}{\partial \gamma} > 0$ first and then $\frac{\partial \mu_1}{\partial \gamma} < 0$. According to Corollary 2, in the left-hand side panel we have $\alpha_1 < \hat{\alpha}$ and in the right-hand side panel we have $\alpha_1 > \hat{\alpha}$. To understand this more complex result, it is interesting to distinguish to the two effects that γ has on the behavior of the stronger expert, hence on the regions where the honest, the mixed and the mirror equilibrium exist. First, we know that when γ increases the stronger expert receives a more precise signal of the state of the world. Thus, ceteris paribus the rest of parameters, an increase in γ increases the cost of deviating from the expert's signal. Second, we also know that when γ increases the stronger expert receives a better "signal" of the action that the weaker expert will take. Thus, ceteris paribus the rest of parameters, an increase in γ increases the gain for deviating from her signal and contradicting the opponent's action. Which effect dominates explains how the equilibrium behavior changes with γ . In particular, we observe that an increase in γ always decreases the region where the honest equilibrium exists. This is due to the second effect. Possibly more interestingly, we observe that when the stronger expert has a sufficiently high initial reputation $(\alpha_1 > \hat{\alpha})$, an increase in γ can even increase the region where the mirror equilibrium exist, i.e., the region where the stronger expert completely contradicts her signal. This occurs as long as $\gamma < \hat{\gamma}$, in which case $\frac{\partial \mu_1}{\partial \gamma} > 0$ (upward sloping part of μ_1 in the right-bottom panel). The idea is that when α_1 is sufficiently high and γ sufficiently low, the gain from following the signal (seeking to match the state) may be not enough to compensate the gain from contradicting it (seeking to contradict the opponent's action). When this occurs, an increase in γ has a stronger effect on the second idea and a smaller one on the first one; hence, driving the result. A final comment regarding on the effect of parameter γ on the equilibrium regions is that $\mu_1 \rightarrow \mu_2$ when $\gamma \to 1/2$ (see also the proof of Proposition 2 for a proof of this result), which implies that the smaller γ , the smaller the region where the mixed equilibrium exists. In the limit, the stronger expert either sticks to her signal or contradicts it.

Before concluding the comparative static exercise, we would like to go back to the already discussed idea that when $\alpha_1 = \alpha_2$, i.e., the two experts have the same initial reputation, the two evaluation systems are no longer different. Quite intuitively, the reason is that when $\alpha_1 = \alpha_2$ there is neither a stronger nor a weaker expert; hence, there is no gain from contradicting the opponent's action. This result is formally stated in Proposition 3 below.

Proposition 3. Let $\alpha_1 = \alpha_2$. Under both the APE and the RPE systems, there is always an equilibrium in which $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1, 1)$, for all $i \in \{1, 2\}$ and $\mu > 0$. Additionally, the honest equilibrium is the unique symmetric equilibrium under the two evaluation systems.

Last, we would like to draw the attention of the reader to the fact that irrespective of the performance evaluation system, there is no pooling equilibrium in which the stronger expert and the weaker expert take the same action, i.e., there is not an equilibrium in which either $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1, 0)$ or $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (0, 1)$, for all $i \in \{1, 2\}$. The reason is that in a pooling equilibrium in which all the experts pool at a, deviating to a' will be interpreted by the principal as the expert being a wise type with probability 1. Then, that strategy profile cannot be an equilibrium. This result holds for any $\alpha_1 \ge \alpha_2$.

4 Conclusion

We consider a model of two experts with heterogeneous expertise (i.e., a stronger expert and a weaker one) and reputational concerns. We analyze experts' behavior under two evaluation systems. The first system is the APE system, which represents the standard system under which experts are evaluated in absolute terms. The second system is the RPE system, which describes a system where experts are evaluated in relative terms. Our results show that the two performance evaluation systems produce very different incentives for the experts. We show that under very mild conditions the unique equilibrium under the APE system is the honest equilibrium, in which both the stronger and the weaker expert always folow their signal. In contrast to this, we show that the RPE system may induce the stronger expert to contradict her signal. The reason is that by deviating and contradicting her signal, the stronger expert may also contradict the weaker expert's action, in which case the informative content of her action will receive higher weight than the one of the weaker expert, for her being the stronger one. This result has implications for the society featuring experts' consensus and/or dissent. In particular, it suggests that whereas the APE system fosters experts' consensus, the RPE system is likely to engender experts' dissent.

We further observe that although apparently similar, moving from the APE system to the RPE system completely changes the nature of the game. In fact, under the APE system both experts (even if having different ex ante expertise) have aligned incentives: they both gain by taking the same action; in particular, by taking the action that is most likely to match the state of the world. Since signals are informative, it introduces an incentive for experts to follow their signal. Introducing the RPE system completely breaks the symmetry of the game, producing very different incentives for the two experts: whereas the weaker expert gains from matching the state (hence, from following her signal), the stronger expert can now do it better than just proving her action to match the state.

The model considers some simplifying assumptions. First, we consider that signals are i.i.d. conditional on the state. This assumption does not however limit the scope of our results, and as argued in the text, we predict our results to hold (even to be stronger) if conditional on the state, signals were correlated. The idea is that with correlated signals the stronger expert will have a better "signal" of the weaker expert's action. This would make easy for the stronger expert to contradict the weaker expert's action. Hence, we predict that the probability of experts' dissent would increase in that case. This result suggests that interpersonal comparisons may yield more dissent the higher the knowledge that experts have of each other. A deeper analysis of this sort of situations and the extend to which our results may hold in more general scenarios, such as one in which experts do not face common uncertainty, are interesting questions that we plan to explore in our future work.

A Appendix

We start obtaining the consistent beliefs that the players (the principal and the other expert) place on expert *i* being a wise type $\hat{\alpha}_i(a_1, a_2, L)$. Note that when $F \neq \emptyset$, $\hat{\alpha}_i(a_1, a_2, F)$ does not depend on a_j , with $i, j \in \{1, 2\}, i \neq j$. The beliefs are:

$$\hat{\alpha}_i(l_i, a_j, R) = \hat{\alpha}_i(r_i, a_j, L) = 0, \tag{1}$$

$$\hat{\alpha}_i(l_i, a_j, L) = \frac{\alpha_i}{\alpha_i + (1 - \alpha_i)(\gamma \sigma_N^i(l) + (1 - \gamma)(1 - \sigma_N^i(r)))},\tag{2}$$

$$\hat{\alpha}_{i}(r_{i}, a_{j}, R) = \frac{\alpha_{i}}{\alpha_{i} + (1 - \alpha_{i})(\gamma \sigma_{N}^{i}(r) + (1 - \gamma)(1 - \sigma_{N}^{i}(l)))},$$

$$\hat{\alpha}_{i}(l_{i}, l_{j}, \varnothing) = \frac{\alpha_{i}}{\alpha_{i} + (1 - \alpha_{i})\left(\gamma \sigma_{N}^{i}(l) + (1 - \gamma)(1 - \sigma_{N}^{i}(r)) + (\gamma(1 - \sigma_{N}^{i}(r)) + (1 - \gamma)\sigma_{N}^{i}(l))\frac{(1 - \alpha_{j})(\gamma(1 - \sigma_{N}^{j}(r)) + (1 - \gamma)\sigma_{N}^{j}(l))}{\alpha_{j} + (1 - \alpha_{j})(\gamma \sigma_{N}^{j}(l) + (1 - \gamma)(1 - \sigma_{N}^{i}(r)))},$$
(3)

$$\hat{\alpha}_{i}(r_{i}, r_{j}, \varnothing) = \frac{\alpha_{i}}{\alpha_{i} + (1 - \alpha_{i}) \left(\gamma \sigma_{N}^{i}(r) + (1 - \gamma)(1 - \sigma_{N}^{i}(l)) + (\gamma (1 - \sigma_{N}^{i}(l)) + (1 - \gamma)\sigma_{N}^{i}(r)) \frac{(1 - \alpha_{j})(\gamma (1 - \sigma_{N}^{j}(l)) + (1 - \gamma)\sigma_{N}^{j}(r))}{\alpha_{j} + (1 - \alpha_{j})(\gamma \sigma_{N}^{j}(r) + (1 - \gamma)(1 - \sigma_{N}^{j}(l)))}\right)},$$

$$(5)$$

(4)

$$\hat{\alpha}_{i}(l_{i},r_{j},\varnothing) = \frac{\alpha_{i}}{\alpha_{i} + (1-\alpha_{i}) \left(\gamma \sigma_{N}^{i}(l) + (1-\gamma)(1-\sigma_{N}^{i}(r)) + (\gamma(1-\sigma_{N}^{i}(r)) + (1-\gamma)\sigma_{N}^{i}(l)) \frac{\alpha_{j} + (1-\alpha_{j})(\gamma \sigma_{N}^{j}(r) + (1-\gamma)(1-\sigma_{N}^{j}(l)))}{(1-\alpha_{j})(\gamma(1-\sigma_{N}^{j}(l)) + (1-\gamma)\sigma_{N}^{j}(r))}\right)},$$

$$(6)$$

$$\hat{\alpha}_{i}(r_{i}, l_{j}, \varnothing) = \frac{\alpha_{i}}{\alpha_{i} + (1 - \alpha_{i}) \left(\gamma \sigma_{N}^{i}(r) + (1 - \gamma)(1 - \sigma_{N}^{i}(l)) + (\gamma(1 - \sigma_{N}^{i}(l)) + (1 - \gamma)\sigma_{N}^{i}(r)) \frac{\alpha_{j} + (1 - \alpha_{j})(\gamma \sigma_{N}^{j}(l) + (1 - \gamma)(1 - \sigma_{N}^{j}(r)))}{(1 - \alpha_{j})(\gamma(1 - \sigma_{N}^{j}(r)) + (1 - \gamma)\sigma_{N}^{j}(l))}}\right)},$$

$$(7)$$

for all $i, j \in \{1, 2\}, a_j \in \{\hat{l}_j, \hat{r}_j\}$. Note that $\hat{\alpha}_i(l_i, a_j, L) > \hat{\alpha}_i(r_i, a_j, R)$ if and only if $\sigma_N^i(l) < \sigma_N^i(r)$.¹⁴

Let $EU_i(a_i | s_i)$ be the expected payoff (or expected reputation) to the normal type expert *i* for taking action $a_i \in \{\hat{l}_i, \hat{r}_i\}$ after signal $s_i \in \{l_i, r_i\}$:

$$EU_{i}(\hat{l}_{i} \mid s_{i}) = P(\hat{r}_{j} \mid \hat{l}_{i}, s_{i})EU_{i}(\hat{l}_{i}, \hat{r}_{j}, F \mid s_{i}) + P(\hat{l}_{j} \mid \hat{l}_{i}, s_{i})EU_{i}(\hat{l}_{i}, \hat{l}_{j}, F \mid s_{i}),$$

$$EU_{i}(\hat{r}_{i} \mid s_{i}) = P(\hat{r}_{j} \mid \hat{r}_{i}, s_{i})EU_{i}(\hat{r}_{i}, \hat{r}_{j}, F \mid s_{i}) + P(\hat{l}_{j} \mid \hat{r}_{i}, s_{i})EU_{i}(\hat{r}_{i}, \hat{l}_{j}, F \mid s_{i}).$$

with $P(a_j \mid a_i, s_i) = P(a_j \mid s_i)$ for all $a_i \in \{\hat{l}_i, \hat{r}_i\}$ being

$$P(a_j \mid s_i) = P(a_j \mid s_i, L)P(L \mid s_i) + P(a_j \mid s_i, R)P(R \mid s_i)$$

hence:

$$\begin{split} P(\hat{l}_{j} \mid l_{i}) &= (1 - \alpha_{j})(\gamma(1 - \sigma_{N}^{j}(r)) + (1 - \gamma)\sigma_{N}^{j}(l))(1 - \gamma) + (\alpha_{j} + (1 - \alpha_{j})(\gamma\sigma_{N}^{j}(l) + (1 - \gamma)(1 - \sigma_{N}^{j}(r))))\gamma, \\ P(\hat{l}_{j} \mid r_{i}) &= (1 - \alpha_{j})(\gamma(1 - \sigma_{N}^{j}(r)) + (1 - \gamma)\sigma_{N}^{j}(l))\gamma + (\alpha_{j} + (1 - \alpha_{j})(\gamma\sigma_{N}^{j}(l) + (1 - \gamma)(1 - \sigma_{N}^{j}(r))))(1 - \gamma), \\ P(\hat{r}_{j} \mid l_{i}) &= (1 - \alpha_{j})(\gamma(1 - \sigma_{N}^{j}(l)) + (1 - \gamma)\sigma_{N}^{j}(r))\gamma + (\alpha_{j} + (1 - \alpha_{j})(\gamma\sigma_{N}^{j}(r) + (1 - \gamma)(1 - \sigma_{N}^{j}(l))))(1 - \gamma), \\ P(\hat{r}_{j} \mid r_{i}) &= (1 - \alpha_{j})(\gamma(1 - \sigma_{N}^{j}(l)) + (1 - \gamma)\sigma_{N}^{j}(r))(1 - \gamma) + (\alpha_{j} + (1 - \alpha_{j})(\gamma\sigma_{N}^{j}(r) + (1 - \gamma)(1 - \sigma_{N}^{j}(l))))\gamma. \end{split}$$

Additionally, let $EU_i(a_i, a_j, F \mid s_i)$ denote the expected payoff to the normal type expert $i \in \{1, 2\}$ for taking action $a_i \in \{\hat{l}_i, \hat{r}_i\}$ after signal $s_i \in \{l_i, r_i\}$ when the other expert takes action $a_j \in \{\hat{l}_j, \hat{r}_j\}$. When payoffs are absolute (it is absolute performance that matters) we have:

$$EU_{i}(\hat{l}_{i},\hat{l}_{j},F \mid s_{i}) = (1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},\varnothing) + \mu P(L \mid s_{i},\hat{l}_{j})\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},L),$$

$$EU_{i}(\hat{l}_{i},\hat{r}_{j},F \mid s_{i}) = (1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},\varnothing) + \mu P(L \mid s_{i},\hat{r}_{j})\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},L),$$

$$EU_{i}(\hat{r}_{i},\hat{r}_{j},F \mid s_{i}) = (1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},\varnothing) + \mu P(R \mid s_{i},\hat{r}_{j})\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},R),$$

$$EU_{i}(\hat{r}_{i},\hat{l}_{j},F \mid s_{i}) = (1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},\varnothing) + \mu P(R \mid s_{i},\hat{l}_{j})\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},R).$$

¹⁴For a derivation of beliefs $\hat{\alpha}_i(a_i, a_j, \emptyset)$ see Andina-Díaz and García-Martínez (2020*b*).

with $s_i \in \{l_i, r_i\}$.

When payoffs are defined in relative terms, because experts care about relative performance, we have:

$$\begin{split} EU_{i}(\hat{l}_{i},\hat{l}_{j},F\mid s_{i}) &= (1-\mu)\frac{\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},\varnothing)}{\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},\varnothing) + \hat{\alpha}_{j}(\hat{l}_{i},\hat{l}_{j},\varnothing)} + \mu P(L\mid s_{i},\hat{l}_{j})\frac{\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},L)}{\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},L) + \hat{\alpha}_{j}(\hat{l}_{i},\hat{l}_{j},L)},\\ EU_{i}(\hat{l}_{i},\hat{r}_{j},F\mid s_{i}) &= (1-\mu)\frac{\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},\varnothing)}{\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},\varnothing) + \hat{\alpha}_{j}(\hat{l}_{i},\hat{r}_{j},\varnothing)} + \mu P(L\mid s_{i},\hat{r}_{j})\frac{\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},L)}{\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},L) + \hat{\alpha}_{j}(\hat{l}_{i},\hat{r}_{j},L)},\\ EU_{i}(\hat{r}_{i},\hat{r}_{j},F\mid s_{i}) &= (1-\mu)\frac{\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},\varnothing)}{\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},\emptyset) + \hat{\alpha}_{j}(\hat{r}_{i},\hat{r}_{j},\varnothing)}} + \mu P(R\mid s_{i},\hat{r}_{j})\frac{\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},R)}{\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},R) + \hat{\alpha}_{j}(\hat{r}_{i},\hat{r}_{j},R)},\\ EU_{i}(\hat{r}_{i},\hat{l}_{j},F\mid s_{i}) &= (1-\mu)\frac{\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},\varnothing)}{\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},\emptyset,\emptyset)} + \hat{\alpha}_{j}(\hat{r}_{i},\hat{l}_{j},\emptyset)} + \mu P(R\mid s_{i},\hat{l}_{j})\frac{\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},R)}{\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},R) + \hat{\alpha}_{j}(\hat{r}_{i},\hat{l}_{j},R)}. \end{split}$$

with $s_i \in \{l_i, r_i\}$ and $P(\omega \mid s_i, a_j)$ being

$$P(\omega \mid s_i, a_j) = \frac{P(s_i \mid a_j, \omega) P(a_j \mid \omega) P(\omega)}{P(s_i \mid a_j, L) P(a_j \mid L) P(L) + P(s_i \mid a_j, R) P(a_j \mid R) P(R)},$$

hence:

$$\begin{split} P(L \mid s_i, \hat{l}_j) &= \frac{P(s_i/L)}{P(s_i/L) + P(s_i/R) \frac{(1-\alpha_j)(\gamma(1-\sigma_N^j(r)) + (1-\gamma)\sigma_N^j(l))}{\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(l) + (1-\gamma)(1-\sigma_N^j(r)))}}, \\ P(L \mid s_i, \hat{r}_j) &= \frac{P(s_i/L)}{P(s_i/L) + P(s_i/R) \frac{\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(r) + (1-\gamma)(1-\sigma_N^j(l)))}{(1-\alpha_j)(\gamma(1-\sigma_N^j(l)) + (1-\gamma)\sigma_N^j(r))}}, \\ P(R \mid s_i, \hat{r}_j) &= \frac{P(s_i/R)}{P(s_i/R) + P(s_i/L) \frac{(1-\alpha_j)(\gamma(1-\sigma_N^j(l)) + (1-\gamma)\sigma_N^j(r))}{\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(r) + (1-\gamma)(1-\sigma_N^j(l)))}}, \\ P(R \mid s_i, \hat{l}_j) &= \frac{P(s_i/R)}{P(s_i/R) + P(s_i/L) \frac{\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(l) + (1-\gamma)(1-\sigma_N^j(r)))}{(1-\alpha_j)(\gamma(1-\sigma_N^j(r)) + (1-\gamma)\sigma_N^j(l))}}. \end{split}$$

Finally, let $\Delta_s^i(\sigma_l^1, \sigma_r^1; \sigma_l^2, \sigma_r^2)$ be the expected gain to the normal type expert *i* from taking action \hat{r}_i rather than \hat{l}_i , after observing signal $s_i \in \{l_i, r_i\}$. Then:

$$\Delta_s^i(\sigma_l^1, \sigma_r^1; \sigma_l^2, \sigma_r^2) = EU_i(\hat{r}_i \mid s_i) - EU_i(\hat{l}_i \mid s_i).$$
(8)

After some calculations we obtain:

$$\begin{aligned} \Delta_{s}^{i}(\sigma_{l}^{1},\sigma_{r}^{1};\sigma_{l}^{2},\sigma_{r}^{2}) &= P(\hat{r}_{j} \mid \hat{r}_{i},s_{i})EU_{i}(\hat{r}_{i},\hat{r}_{j},F \mid s_{i}) + P(\hat{l}_{j} \mid \hat{r}_{i},s_{i})EU_{i}(\hat{r}_{i},\hat{l}_{j},F \mid s_{i}) \\ &- \left(P(\hat{r}_{j} \mid \hat{l}_{i},s_{i})EU_{i}(\hat{l}_{i},\hat{r}_{j},F \mid s_{i}) + P(\hat{l}_{j} \mid \hat{l}_{i},s_{i})EU_{i}(\hat{l}_{i},\hat{l}_{j},F \mid s_{i})\right) \\ &= P(\hat{r}_{j} \mid s_{i}) \left(EU_{i}(\hat{r}_{i},\hat{r}_{j},F \mid s_{i}) - EU_{i}(\hat{l}_{i},\hat{r}_{j},F \mid s_{i})\right) \\ &+ P(\hat{l}_{j} \mid s_{i}) \left(EU_{i}(\hat{r}_{i},\hat{l}_{j},F \mid s_{i}) - EU_{i}(\hat{l}_{i},\hat{l}_{j},F \mid s_{i})\right). \end{aligned}$$
(9)

A.1 Part I: Results

In this section we prove the results of the text, which assume that the wise type experts always follow the honest strategy. Part II of the Appendix relaxes this assumption and considers strategic wise type experts.

First, we fully describe the expected gain described by (9) for any $s_i \in \{l_i, r_i\}$, both under the APE and the RPE system. We denote the expected gain Δ_s^i by $\Delta_r^{i,A}$ in the APE system and by $\Delta_r^{i,R}$

in the RPE system. For $i, j \in \{1, 2\}$ and $i \neq j$, in the APE system we have:

$$\begin{aligned} \Delta_{r}^{i,A} &= \\ P(\hat{r}_{j} \mid r_{i}) \left((1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},\varnothing) + \mu P(R \mid r_{i},\hat{r}_{j})\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},R) - (1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},\varnothing) - \mu P(L \mid r_{i},\hat{r}_{j})\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},L) \right) + \\ P(\hat{l}_{j} \mid r_{i}) \left((1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},\varnothing) + \mu P(R \mid r_{i},\hat{l}_{j})\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},R) - (1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},\varnothing) - \mu P(L \mid r_{i},\hat{l}_{j})\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},L) \right), \end{aligned}$$
(10)

$$\begin{aligned} \Delta_{l}^{i,A} &= \\ P(\hat{r}_{j} \mid l_{i}) \left((1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},\varnothing) + \mu P(R \mid l_{i},\hat{r}_{j})\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},R) - (1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},\varnothing) - \mu P(L \mid l_{i},\hat{r}_{j})\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},L) \right) + \\ P(\hat{l}_{j} \mid l_{i}) \left((1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},\varnothing) + \mu P(R \mid l_{i},\hat{l}_{j})\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},R) - (1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},\varnothing) - \mu P(L \mid l_{i},\hat{l}_{j})\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},L) \right). \end{aligned}$$
(11)

In the REP system the expected gain $\Delta_s^{i,R}$ is given by expressions (10) and (11) with the exception that we substitute $\hat{\alpha}_i(a_i, a_j, X)$ by $\frac{\hat{\alpha}_i(a_i, a_j, X)}{\hat{\alpha}_i(a_i, a_j, X) + \hat{\alpha}_j(a_i, a_j, X)}$. Let $\hat{\alpha}_i^R(a_i, a_j, X) = \frac{\hat{\alpha}_i(a_i, a_j, X)}{\hat{\alpha}_i(a_i, a_j, X) + \hat{\alpha}_j(a_i, a_j, X)}$. Noteworthy, the expressions for $\hat{\alpha}_i^R(a_i, a_j, X)$ greatly simplify when $a_1 \neq a_2$ and $X \neq \emptyset$, in which case $\hat{\alpha}_i^R(a_i, a_j, X) = 1$ for the expert that matches the state and it is 0 for the other expert. For $i, j \in \{1, 2\}$ and $i \neq j$, in the RPE system we have:

$$\begin{aligned} \Delta_{r}^{i,R} &= \\ P(\hat{r}_{j} \mid r_{i}) \left((1-\mu) \hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{r}_{j},\varnothing) + \mu P(R \mid r_{i},\hat{r}_{j}) \hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{r}_{j},R) - (1-\mu) \hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{r}_{j},\varnothing) - \mu P(L \mid r_{i},\hat{r}_{j}) \right) + \\ P(\hat{l}_{j} \mid r_{i}) \left((1-\mu) \hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{l}_{j},\varnothing) + \mu P(R \mid r_{i},\hat{l}_{j}) - (1-\mu) \hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{l}_{j},\varnothing) - \mu P(L \mid r_{i},\hat{l}_{j}) \hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{l}_{j},L) \right), \end{aligned}$$

$$(12)$$

$$\begin{aligned} \Delta_{l}^{i,R} &= \\ P(\hat{r}_{j} \mid l_{i}) \left((1-\mu) \hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{r}_{j},\varnothing) + \mu P(R \mid l_{i},\hat{r}_{j}) \hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{r}_{j},R) - (1-\mu) \hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{r}_{j},\varnothing) - \mu P(L \mid l_{i},\hat{r}_{j}) \right) + \\ P(\hat{l}_{j} \mid l_{i}) \left((1-\mu) \hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{l}_{j},\varnothing) + \mu P(R \mid l_{i},\hat{l}_{j}) - (1-\mu) \hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{l}_{j},\varnothing) - \mu P(L \mid l_{i},\hat{l}_{j}) \hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{l}_{j},L) \right). \end{aligned}$$

$$(13)$$

Proof of Proposition 1

Note that the strategy profile $(\sigma_N^i(l), \sigma_N^i(r)) = (1, 1)$ satisfies $\sigma_N^i(l) = \sigma_N^i(r)$, with $\sigma_N^i(l) = \sigma_N^i(r) = \sigma_N^i(r)$ denoting the symmetric strategy of the normal type expert *i*. Furthermore, note that under symmetric strategies (with $(\sigma_N^i(l), \sigma_N^i(r)) = (1, 1)$ being a case of symmetric strategies) an expert is honest with the same probability after either signal $s_i \in \{l_i, r_i\}$. This implies $EU_i(\hat{l}_i \mid l_i) = EU_i(\hat{r}_i \mid r_i)$ and $EU_i(\hat{r}_i \mid l_i) = EU_i(\hat{l}_i \mid r_i)$. Additionally, since $\Delta_r^{i,A} = EU_i(\hat{r}_i \mid r_i) - EU_i(\hat{l}_i \mid r_i)$ and $\Delta_l^{i,A} = EU_i(\hat{r}_i \mid l_i) - EU_i(\hat{l}_i \mid l_i)$, then $\Delta_r^{i,A} = -\Delta_l^{i,A}$.

Next we use σ_N^i , with $i \in \{1, 2\}$, and show that $\Delta_r^{i,A} > 0$ for any σ_N^i and σ_N^j , with $i, j \in \{1, 2\}$, which implies that $-\Delta_l^{i,A} < 0$. This proves both i) that the strategy profile $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1, 1)$ is an equilibrium strategy profile and ii) that this is the unique equilibrium under symmetric strategies.

First, given $i, j \in \{1, 2\}$ and $i \neq j$, after some algebra we obtain:

$$\Delta_r^{i,A} = \alpha_i \left(\frac{(1-2\gamma)\mu}{(\alpha_i - 1)\gamma(2\sigma_N^i - 1) - \alpha_i \sigma_N^i + \sigma_N^i - 1} + \frac{(\mu - 1)(2\gamma - 1)(2(\alpha_j - 1)(\gamma(2\sigma_N^j - 1) - \sigma_N^j) - 1)}{f_1} \right),$$
 with

$$f_1 = 2(\alpha_i - 1)(\alpha_j - 1)\gamma^2(2\sigma_N^i - 1)(2\sigma_N^j - 1) + \gamma(-2(\alpha_i - 1)\sigma_N^i(4(\alpha_j - 1)\sigma_N^j - \alpha_j + 2) + 2\alpha_i\alpha_j\sigma_N^j - 2\alpha_i\sigma_N^j + \alpha_i - 4\alpha_j\sigma_N^j + \alpha_j + 4\sigma_N^j - 2) + (\alpha_j - 1)\sigma_N^j(2(\alpha_i - 1)\sigma_N^i + 1) + (\alpha_i - 1)\sigma_N^i + 1$$

Second, note that $\Delta_r^{i,A}$ is linear in μ . Then, if $\Delta_r^{i,A}$ is greater than zero both at $\mu = 0$ and $\mu = 1$, then $\Delta_r^{i,A} > 0$. Substituting, it can be shown that:

$$\begin{split} \Delta_r^{i,A}\big|_{\mu=1} &= \alpha_i \frac{1-2\gamma}{(\alpha_i - 1)\gamma(2\sigma_N^i - 1) - \alpha_i \sigma_N^i + \sigma_N^i - 1} > 0, \\ \Delta_r^{i,A}\big|_{\mu=0} &= -\alpha_i \frac{(2\gamma - 1)(2(\alpha_j - 1)(\gamma(2\sigma_N^j - 1) - \sigma_N^j) - 1)}{f_1} > 0, \end{split}$$

which concludes the proof. \blacksquare

Proof of Proposition 2.

We focus on the use of symmetric strategies and show that, in this case, the equilibrium is unique. Note that this proves both uniqueness (under symmetric strategies) and existence of the equilibrium (for any strategy of the experts).

The proof requires Lemmas 1-5, which we prove below, and consists of the following steps.

First, by the rationale used in the proof of Proposition 1 to show $\Delta_r^{i,A} = -\Delta_l^{i,A}$, we have $\Delta_r^{i,R} = -\Delta_l^{i,R}$.

Second, by Lemma 1, we learn that the equilibria configurations 4 and 6, which we describe below, are not possible. Third, by Lemma 2, showing $\Delta_r^{1,R} > \Delta_l^{2,R}$, we learn that configurations 1, 3, 7, and 9 below can neither occur. Then, in equilibrium, only configurations 2, 5, and 8 can hold, which imply $\sigma_N^{2*} = 1$. The configurations are the following:

Fourth, by Lemma 3, showing $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$, we have $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_N^1} > 0$, as $\Delta_r^{1,R} = -\Delta_l^{1,R}$. Fifth, by Lemma 4, showing $\Delta_r^{1,R}|_{\sigma_N^1=1} < 0 \iff \mu > \mu_2$, we have that $\sigma_N^{1*} = 1$ is the unique equilibrium strategy if and only if $\mu > \mu_2$, as $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$ and $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_N^1} > 0$.

Last, Lemma 5 below consists of two points. Point 1. shows $\Delta_r^{1,R}|_{\sigma_N^1=0} < 0 \iff \alpha_1 > \bar{\alpha}$ and $\mu < \mu_1$. Since $\Delta_r^{1,R} = -\Delta_l^{1,R}$, it implies $\Delta_r^{1,R}|_{\sigma_N^1=0} < 0 \iff \Delta_l^{1,R}|_{\sigma_N^1=1} > 0$. Consequently, in the unique equilibrium strategy $\sigma_N^{1*} = 0 \iff \alpha_1 > \bar{\alpha}$ and $\mu < \mu_1$ (as $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$ and $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_N^1} > 0$). Finally, point 2. shows $\Delta_r^{1,R}|_{\sigma_N^1=0} > 0$ if and only if either $\alpha_1 < \bar{\alpha}$ or both $\alpha_1 > \bar{\alpha}$ and $\mu > \mu_1$ hold. Again, $\Delta_r^{1,R}|_{\sigma_N^1=0} > 0 \iff \Delta_l^{1,R}|_{\sigma_N^1=0} < 0$, which further implies that in the unique equilibrium strategy $0 < \sigma_N^{1*} = x < 1$ if and only if either $\alpha_1 < \bar{\alpha}$ or both $\alpha_1 > \bar{\alpha}$ and $\mu > \mu_1$ hold (as $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$).

and $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_n^1} > 0$), where x is the unique solution of the equation $\Delta_r^{1,R} = 0$, which it is also solution to $\Delta_l^{1,R} = 0$, as $\Delta_r^{1,R} = -\Delta_l^{1,R}$.

Note that, in all the previous cases the equilibrium is unique as $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$, $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_N^1} > 0$ and, in all the possible equilibria configurations, $\sigma_N^{2*} = 1$.

Next we show Lemmas 1-5.

Lemma 1. Under RPE and symmetric strategies, there is not an equilibrium in which $\sigma_N^{1*} = 1$ and $\sigma_N^{2*} < 1$.

Note that function $\Delta_r^{2,R}$ is lineal in μ . Thus, if $\Delta_r^{2,R}|_{\mu=0} > 0$ and $\Delta_r^{2,R}|_{\mu=1} > 0$, the function will be always positive.

With $\sigma_N^1 = 1$ and $\sigma_N^2 < 1$, using (12), after some calculations we obtain:

$$\begin{split} \Delta_r^{2,R}\big|_{\mu=0} &= \frac{\alpha_1 \alpha_2 (1-2\gamma) (2\alpha_1 (\gamma-1)-2\gamma+1) (\alpha_1 (\gamma-1)+\gamma (-2\alpha_2 \sigma_2+\alpha_2+2\sigma_2-2)+(\alpha_2-1)\sigma_2+1)}{(\alpha_1 (\alpha_2-1) (2\gamma-1)\sigma_2-\alpha_1 \alpha_2+\alpha_1 \gamma-\alpha_2 \gamma+\alpha_2) (\alpha_1 ((\alpha_2-1) (2\gamma-1)\sigma_2-\alpha_2+\gamma-1)-\alpha_2 \gamma)} > 0, \\ \Delta_r^{2,R}\big|_{\mu=1} &= \frac{(2\gamma-1) \left(\alpha_1^2 (\gamma-1) ((\alpha_2-1) \gamma (2\sigma_2-1)-\alpha_2 \sigma_2+\sigma_2-1)-\alpha_1 (\gamma-1) ((\alpha_2-1) (2\gamma-1)\sigma_2+\alpha_2 (-\gamma)-\alpha_2+\gamma-1)-\alpha_2 \gamma\right)}{\alpha_1 ((\alpha_2-1) (2\gamma-1)\sigma_2-\alpha_2+\gamma-1)-\alpha_2 \gamma} > 0. \end{split}$$

Since $\Delta_r^{2,R} > 0$ if $\sigma_N^1 = 1$ and $\sigma_N^2 < 1$, it cannot exist an equilibrium in which $\sigma_N^{1*} = 1$ and $\sigma_N^{2*} < 1$.

Lemma 2. Under RPE and symmetric strategies, $\Delta_r^{1,R} > \Delta_l^{2,R}$.

From (12) and (13), $\Delta_r^{1,R}$ and $\Delta_l^{2,R}$ are linear in μ . Given $\sigma_N^2 = 1$ and $\alpha_1 > \alpha_2$, after some algebra it can be shown that:

$$\begin{split} \Delta_r^{1,R} & - \Delta_l^{2,R} \Big|_{\mu=0} = \frac{2\alpha_1\alpha_2(1-2\gamma)(\alpha_1(\gamma(2\sigma_1-1)-\sigma_1)-(\alpha_2-1)(2\gamma-1)\sigma_2+\alpha_2\gamma-2\gamma\sigma_1+\sigma_1)^2}{f_2} > 0, \\ \Delta_r^{1,R} & - \Delta_l^{2,R} \Big|_{\mu=1} = \frac{f_3}{\alpha_1(\gamma(2\alpha_2(\sigma_1+\sigma_2-1)-2\sigma_2+1)-\alpha_2(\sigma_1+\sigma_2)+\sigma_2-1)+\alpha_2(-2\gamma\sigma_1+\gamma+\sigma_1-1)} > 0, \end{split}$$

with

$$f_{2} = \alpha_{1}(\gamma(2\alpha_{2}(\sigma_{1} + \sigma_{2} - 1) - 2\sigma_{2} + 1) - \alpha_{2}(\sigma_{1} + \sigma_{2}) + \sigma_{2} - 1) + \alpha_{2}(-2\gamma\sigma_{1} + \gamma + \sigma_{1} - 1) (\alpha_{1}(\gamma(2\alpha_{2}(\sigma_{1} + \sigma_{2} - 1) - 2\sigma_{2} + 1) - \alpha_{2}(\sigma_{1} + \sigma_{2}) + \sigma_{2}) + \alpha_{2}(-2\gamma\sigma_{1} + \gamma + \sigma_{1})),$$

and

$$\begin{aligned} f_3 &= (2\gamma - 1)(\alpha_1^2(\gamma(2\sigma_1 - 1) - \sigma_1)((\alpha_2 - 1)\gamma(2\sigma_2 - 1) - \alpha_2\sigma_2 + \sigma_2 - 1) \\ &+ \alpha_1(\alpha_2^2(\gamma(2\sigma_1 - 1) - \sigma_1)(\gamma(2\sigma_2 - 1) - \sigma_2) - \alpha_2(2(\gamma^2(2\sigma_1 - 1)(2\sigma_2 - 1) - (\alpha_2\sigma_2 - 1) - \alpha_2\sigma_2 + \gamma + \sigma_1\sigma_2) + \sigma_1 + \sigma_2) + (\gamma(2\sigma_1 - 1) - \sigma_1 - 1)(\gamma(2\sigma_2 - 1) - \sigma_2 + 1)) \\ &- \alpha_2(\gamma(2\sigma_1 - 1) - \sigma_1 + 1)((\alpha_2 - 1)\gamma(2\sigma_2 - 1) - \alpha_2\sigma_2 + \sigma_2 + 1)), \end{aligned}$$

which implies $\Delta_r^{1,R} > \Delta_l^{2,R}$.

Lemma 3. Let $\sigma_N^{2*} = 1$. Under RPE and symmetric strategies, $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$.

Let $\Delta_r^{1,R}\Big|_{\sigma_N^{2*}=1} = \frac{A}{B}$. Then $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} = \frac{A'B - AB'}{B^2}$, with the sing of the derivative depending on the sign of the numerator A'B - AB'.¹⁵ Note that since $\Delta_r^{1,R}$ is linear in μ , the derivative $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1}$ will also

 $^{^{15}}$ The expressions are too large to be displayed but are available from the authors upon request.

be linear in μ . Thus, if $A'B - AB'|_{\mu=0} < 0$ and $A'B - AB'|_{\mu=1} < 0$, then A'B - AB' < 0. It can be shown that:

$$\begin{split} A'B - AB'|_{\mu=0} &= \\ &- (\alpha_1 - 1)\alpha_1\alpha_2(1 - 2\gamma)^2(2\alpha_2(\gamma - 1) - 2\gamma + 1) \\ &\left(\alpha_1^2 \left(\alpha_2^2(2\gamma - 1)(\sigma_N^1 - 1)((2\gamma - 1)\sigma_N^1 - 1) + \alpha_2(\gamma - 1)(2\gamma - 1) - \gamma^2 + \gamma\right) \\ &- 2\alpha_1\alpha_2 \left(\alpha_2^2(\gamma - 1)((2\gamma - 1)\sigma_N^1 - 1) + \alpha_2(2\gamma - 1)\left((2\gamma - 1)\left(\sigma_N^1\right)^2 - 3\gamma\sigma_N^1 + \sigma_N^1 + 1\right) + (\gamma - 1)\gamma\right) \\ &+ \alpha_2^2 \left(\alpha_2(\gamma - 1)(2\gamma - 1)(2\sigma_N^1 - 1) + \gamma^2(4(\sigma_N^1 - 2)\sigma_N^1 + 3) + \gamma(-4(\sigma_N^1 - 2)\sigma_N^1 - 3) + (\sigma_N^1 - 1)^2)\right) < 0, \end{split}$$

and

$$A'B - AB'|_{\mu=1} = (\alpha_1 - 1)\alpha_1\alpha_2(1 - 2\gamma)^2(+\alpha_2 + \gamma - \alpha_2\gamma)^2((\alpha_1 - 1)\alpha_2(2\gamma - 1)\sigma_N^1 + \alpha_1(-\alpha_2) - \alpha_1\gamma + \alpha_1 + \alpha_2\gamma)^2 < 0.$$

Lemma 4. Let $\sigma_N^{2*} = 1$. Under RPE and symmetric strategies, $\Delta_r^{1,R} \Big|_{\sigma_N^1 = 1} < 0 \iff \mu > \mu_2$.

It can be shown that:

$$\Delta_{r}^{1,R}\big|_{\sigma_{N}^{1}=1} = \frac{(2\gamma-1)\big((\alpha_{2}-1)\mu\big(\alpha_{1}^{2}\big(\alpha_{2}(2\alpha_{2}-1)(\gamma-1)^{2}-\gamma\big)+\alpha_{1}\alpha_{2}\big((\gamma-2)\gamma-3\alpha_{2}(\gamma-1)^{2}\big)+\alpha_{2}^{2}(\gamma-1)\gamma\big)+\alpha_{1}\alpha_{2}(\alpha_{1}-\alpha_{2})(2\alpha_{2}(\gamma-1)-2\gamma+1)\big)}{(\alpha_{1}(2\alpha_{2}-1)-\alpha_{2})(\alpha_{1}(2\alpha_{2}(\gamma-1)-\gamma)-\alpha_{2}\gamma)}$$

which is linear in μ and $\Delta_r^{1,R}|_{\sigma_N^1=1} \leq 0$ for all $\mu \geq \mu_2$, with

$$\mu_2 = \frac{\alpha_1 \alpha_2 (\alpha_1 - \alpha_2) (2\alpha_2 (\gamma - 1) - 2\gamma + 1)}{(1 - \alpha_2) \left(\alpha_1^2 (\alpha_2 (2\alpha_2 - 1)(\gamma - 1)^2 - \gamma) + \alpha_1 \alpha_2 ((\gamma - 2)\gamma - 3\alpha_2 (\gamma - 1)^2) + \alpha_2^2 (\gamma - 1)\gamma\right)}.$$

Lemma 5. Let $\sigma_N^{2*} = 1$. Under RPE and symmetric strategies:

- 1. $\Delta_r^{1,R} \big|_{\sigma_N^1=0} < 0 \iff \alpha_1 > \bar{\alpha} \text{ and } \mu < \mu_1,$
- 2. $\Delta_r^{1,R}\big|_{\sigma_M^1=0} > 0 \iff \text{ either } \alpha_1 < \bar{\alpha} \text{ or simultaneously } \alpha_1 > \bar{\alpha} \text{ and } \mu > \mu_1.$

It can be shown that:

$$\frac{\Delta_r^{1,R} \Big|_{\sigma_N^1 = 0} = }{(2\gamma - 1) \left(-(\alpha_2 - 1)(\gamma - 1)\mu \left(\alpha_1^2 (\gamma - \alpha_2 (\gamma(\alpha_2 + \gamma + 1) - 1)) + \alpha_1 \alpha_2 \left(\alpha_2 ((\gamma - 1)\gamma + 1) + \gamma^2 + \gamma \right) - \alpha_2^2 (\gamma - 1)\gamma \right) - \alpha_1 \alpha_2 (2\alpha_2 (\gamma - 1) - 2\gamma + 1)(\gamma(\alpha_1 + \alpha_2 - 2) - \alpha_2 + 1)) \right) }{(\alpha_2 (\alpha_1 - \gamma + 1) + \alpha_1 \gamma) (\alpha_1 (\alpha_2 + \gamma - 1) - \alpha_2 \gamma)}$$

which is linear in μ , and $\Delta_r^{1,R}|_{\sigma_N^1=0} \leq 0$ for all $\mu \leq \mu_1$ with

$$\mu_1 = \frac{\alpha_1 \alpha_2 (2\alpha_2(\gamma - 1) - 2\gamma + 1)(\gamma(\alpha_1 + \alpha_2 - 2) - \alpha_2 + 1)}{(1 - \alpha_2)(\gamma - 1) \left(\alpha_1^2 (\gamma - \alpha_2(\gamma(\alpha_2 + \gamma + 1) - 1)) + \alpha_1 \alpha_2(\alpha_2((\gamma - 1)\gamma + 1) + \gamma^2 + \gamma) - \alpha_2^2(\gamma - 1)\gamma\right)}$$

Note that $\mu_1 > 0$ if and only if $\alpha_1 > \bar{\alpha}$, with

$$\bar{\alpha} = \frac{\alpha_2 \left(1 - \gamma\right) + 2\gamma - 1}{\gamma}.$$

Then, $\Delta_r^{1,R}|_{\sigma_N^1=0} > 0$ if and only if either $\alpha_1 < \bar{\alpha}$ or both $\alpha_1 > \bar{\alpha}$ and $\mu > \mu_1$ hold. In addition, it is straightforward to prove that $\bar{\alpha} = \frac{\alpha_2(1-\gamma)+2\gamma-1}{\gamma}$ is always greater than α_2 and smaller than 1; thus, $\bar{\alpha} \in (\alpha_2, 1)$. It is also directly derived that $\frac{\partial \bar{\alpha}}{\partial \alpha_2} > 0$ and $\frac{\partial \bar{\alpha}}{\partial \gamma} > 0$.

To finish the proof, we show that $\mu_2 > \mu_1$. To this aim, it suffices to obtain

$$\frac{\mu_2}{\mu_1} = \frac{(\gamma - 1)(\alpha_1 - \alpha_2) \left(\alpha_1^2 (\gamma - \alpha_2 (\gamma (\alpha_2 + \gamma + 1) - 1)) + \alpha_1 \alpha_2 \left(\alpha_2 ((\gamma - 1)\gamma + 1) + \gamma^2 + \gamma\right) - \alpha_2^2 (\gamma - 1)\gamma\right)}{(\gamma (\alpha_1 + \alpha_2 - 2) - \alpha_2 + 1) \left(\alpha_1^2 (\alpha_2 (2\alpha_2 - 1)(\gamma - 1)^2 - \gamma) + \alpha_1 \alpha_2 ((\gamma - 2)\gamma - 3\alpha_2 (\gamma - 1)^2) + \alpha_2^2 (\gamma - 1)\gamma\right)},$$

and to show that this expression is decreasing in α_1 , with $\frac{\mu_2}{\mu_1}\Big|_{\alpha_1=1} = 1$ (as for $\alpha_1 = 1$, $\mu_2 = \frac{\alpha_2(2\gamma+2\alpha_2-2\gamma\alpha_2-1)}{\gamma+\alpha_2+\alpha_2^2-\gamma\alpha_2^2} = \mu_1$). This proves $\frac{\mu_2}{\mu_1} > 1$.

Some additional results are the following. They all refer to limit cases.

If $\alpha_1 = 0$, then $\mu_2 = \mu_1 = 0$. If $\gamma = \frac{1}{2}$, then $\mu_2 = \mu_1 = \frac{4\alpha_1\alpha_2^2(\alpha_1 - \alpha_2)}{\alpha_1^2(2\alpha_2^3 - 3\alpha_2^2 - \alpha_2 + 2) - 3\alpha_1(\alpha_2^3 - \alpha_2) - \alpha_2^3 + \alpha_2^2}$. If either $\alpha_1 = \alpha_2$ or $\alpha_2 = 0$, then $\mu_2 = \mu_1 = 0$. Hence:

if $\gamma \to \frac{1}{2}$, then $\mu_2 \to \mu_1$ if $\alpha_1 \to 1$, then $\mu_2 \to \mu_1$

if $\alpha_1 \to 0$, then $\mu_2, \mu_1 \to 0$

if $\alpha_2 \to \alpha_1$, then $\mu_2, \mu_1 \to 0$

if $\alpha_2 \to 0$, then $\mu_2, \mu_1 \to 0$

This concludes the proof of Proposition 2. \blacksquare

Proof of Corollary 2

From the definition of thresholds μ_2 and μ_1 (see Lemmas 4 and 5, respectively), we obtain:

$$\begin{aligned} \frac{\partial \mu_2}{\partial \alpha_1} &= \frac{\alpha_2^2 (2\alpha_2(\gamma-1)-2\gamma+1) \left(\alpha_1^2 (2\alpha_2^2(\gamma-1)^2-4\alpha_2(\gamma-1)^2+(\gamma-3)\gamma)+2\alpha_1\alpha_2(\gamma-1)\gamma-\alpha_2^2(\gamma-1)\gamma)\right)}{(1-\alpha_2) \left(\alpha_1^2 (2\alpha_2^2(\gamma-1)^2-\alpha_2(\gamma-1)^2-\gamma)+\alpha_1\alpha_2((\gamma-2)\gamma-3\alpha_2(\gamma-1)^2)+\alpha_2^2(\gamma-1)\gamma)\right)^2} > 0 \\ \frac{\partial \mu_1}{\partial \alpha_1} &= \frac{f_4}{(\alpha_2-1)(\gamma-1) \left(\alpha_1^2 (\gamma-\alpha_2(\gamma(\alpha_2+\gamma+1)-1))+\alpha_1\alpha_2(\alpha_2((\gamma-1)\gamma+1)+\gamma^2+\gamma)-\alpha_2^2(\gamma-1)\gamma)\right)^2} > 0 \\ \frac{\partial \mu_2}{\partial \gamma} &= \frac{\alpha_1\alpha_2(\alpha_1-\alpha_2)(\alpha_1(2\alpha_2-1)-\alpha_2) \left(\alpha_1(2\alpha_2(\gamma-1)(\alpha_2(\gamma-1)-\gamma)+1)+\alpha_2 \left(-2\alpha_2(\gamma-1)^2+2\gamma(\gamma-1)+1\right)\right)}{(\alpha_2-1) \left(\alpha_1^2 (\alpha_2(2\alpha_2-1)(\gamma-1)^2-\gamma)+\alpha_1\alpha_2((\gamma-2)\gamma-3\alpha_2(\gamma-1)^2)+\alpha_2^2(\gamma-1)\gamma\right)^2} > 0, \\ \frac{\partial \mu_1}{\partial \gamma} &= \alpha_1\alpha_2 \frac{f_5}{(\alpha_2-1)(\gamma-1)^2 \left(\alpha_1^2 (\gamma-\alpha_2(\gamma(\alpha_2+\gamma+1)-1))+\alpha_1\alpha_2(\alpha_2((\gamma-1)\gamma+1)+\gamma^2+\gamma)-\alpha_2^2(\gamma-1)\gamma\right)^2}, \end{aligned}$$

with

$$f_{4} = \alpha_{2}(2\alpha_{2}(\gamma - 1) - 2\gamma + 1)\alpha_{1}^{2} \left(\alpha_{2}^{3}(-(\gamma - 1))\gamma - \alpha_{2}^{2}(\gamma - 1)^{2}(2\gamma + 1) + \alpha_{2} \left(\gamma \left(\gamma^{2} + \gamma - 4\right) + 1\right) - 2\gamma^{2} + \gamma\right) + \alpha_{2}(2\alpha_{2}(\gamma - 1) - 2\gamma + 1)(2\alpha_{1}\alpha_{2}^{2}(\gamma - 1)\gamma^{2} + \alpha_{2}^{2}(\gamma - 1)\gamma(\alpha_{2}(\gamma - 1) - 2\gamma + 1)),$$

and

$$\begin{split} f_5 &= \gamma^4 \left(-2(\alpha_1 - 1)(\alpha_2 - 1)\alpha_2 \left(\alpha_1^2 - 2\alpha_1 - (\alpha_2 - 2)\alpha_2 \right) \right) + \\ \left(\alpha_1^3 \left(\alpha_2 - 2\alpha_2^2 \right) + \alpha_1^2 (\alpha_2 - 1)^3 (2\alpha_2 + 1) + \alpha_1 \alpha_2 \left(\alpha_2^2 + \alpha_2 - 1 \right) + \alpha_2^2 \left(-2\alpha_2^2 + 3\alpha_2 - 1 \right) \right) + \\ \gamma^3 \left(2\alpha_2 \left(\alpha_1^3 (2\alpha_2 - 1) + \alpha_1^2 \left(2\alpha_2^2 - 10\alpha_2 + 5 \right) + \alpha_1 \left(-4\alpha_2^3 + 7\alpha_2^2 + 4\alpha_2 - 4 \right) + \alpha_2 \left(4\alpha_2^2 - 4\alpha_2 + 4 \right) \right) \right) + \\ \gamma^2 \left(\alpha_1^3 \left(-3\alpha_2^2 + 4\alpha_2 - 1 \right) + \alpha_1^2 \alpha_2 \left(2\alpha_2^3 - 13\alpha_2^2 + 23\alpha_2 - 10 \right) + \alpha_1 \alpha_2 \left(10\alpha_2^3 - 4\alpha_2^2 - 12\alpha_2 + 7 \right) + \alpha_2^2 \left(-12\alpha_2^2 + 21\alpha_2 - 7 \right) \right) \\ \gamma \left(2(1 - \alpha_2)(-2\alpha_1^3\alpha_2 + \alpha_1^2 (2\alpha_2^3 - 5\alpha_2^2 + 2\alpha_2 + 1) + 2\alpha_1 \alpha_2^2 (\alpha_2 + 1) + 2\alpha_2^2 (1 - 2\alpha_2)) \right). \end{split}$$

In order to obtain the sign of derivative $\frac{\partial \mu_1}{\partial \gamma}$, note that the denominator is always negative. Then, the sign of the derivative will be given by the sign of polynomial f_5 , which we write as $f_5(\gamma)$.

It can be shown that $f_5(\gamma)$ is increasing in γ , as $\frac{\partial f_5(\gamma)}{\partial \gamma} > 0$. Additionally, $f_5(\gamma = 1)$ is always greater than zero, as $f_5(\gamma = 1) = (\alpha_1 - 1)\alpha_1 (\alpha_1 (\alpha_2^2 + \alpha_2 - 1) - \alpha_2(\alpha_2 + 2)) > 0$. Then, the sing of $f_5(\gamma)$ and consequently the sign of $\frac{\partial \mu_1}{\partial \gamma}$ depends on the sign of $f_5(\gamma = \frac{1}{2})$. We obtain

$$f_5(\gamma = \frac{1}{2}) = \frac{1}{4} \left(-2\alpha_1^3 + \left(-4\alpha_1^2 + \alpha_1 + 3 \right) \alpha_2^3 - (\alpha_1 - 3)\alpha_1^2 \alpha_2 + (\alpha_1 - 1)(4\alpha_1 + 1)\alpha_2^4 + \alpha_1(4 - 3\alpha_1(\alpha_1 + 1))\alpha_2^2 \right),$$

with $f_7(\gamma = \frac{1}{2}) > 0$ if α_1 is smaller than the unique real root of polynomial $f_7(\gamma = \frac{1}{2}) = 0$ in α_1

with $f_5(\gamma = \frac{1}{2}) > 0$ if α_1 is smaller than the unique real root of polynomial $f_5(\gamma = \frac{1}{2}) = 0$ in α_1 , which we denote by $\hat{\alpha}$.

Therefore, if $\alpha_1 < \hat{\alpha}$, then $f_5(\gamma = \frac{1}{2}) > 0$, which implies $f_5(\gamma) > 0$ and $\frac{\partial \mu_1}{\partial \gamma} < 0$. However, if $\alpha_1 > \hat{\alpha}$, then $f_5(\gamma = \frac{1}{2}) < 0$, which implies there will be a threshold of γ (let us called it $\hat{\gamma}$) such that if $\gamma < \hat{\gamma}$ then $\frac{\partial \mu_1}{\partial \gamma} > 0$ and if $\gamma > \hat{\gamma}$ then $\frac{\partial \mu_1}{\partial \gamma} < 0$. This concludes the proof of Corollary 2.

Proof of Proposition 3

The case $\alpha_1 = \alpha_2$ is a particular case of Proposition 1, as the proof does not suppose anything on α_1 or α_2 . This covers the APE case. As for the RPE case, it is straightforward to show that if $\alpha_1 = \alpha_2$, then $\mu_2 = 0$. Thus, the unique equilibrium is $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1, 1)$ for all $i \in \{1, 2\}$ and $\mu > 0$.

A.2 Part II: A strategic wise type expert

In this section we show that the honest strategy is always an equilibrium strategy for the wise type expert both under the APE and the RPE systems.

Prior to the proofs, let us denote by Λ_s^i the expected gain to the *wise* type expert *i* from taking action \hat{r}_i rather than \hat{l}_i after observing signal $s_i \in \{l_i, r_i\}$. Under the APE system we denote it by $\Lambda_s^{i,A}$, and we do it by $\Lambda_s^{i,R}$ under the RPE system.

A.2.1 APE system

Proposition 4. Under APE and symmetric strategies, the honest strategy $\sigma_W^{i*} = 1$ is always an equilibrium strategy for the wise type expert, for all $i \in \{1, 2\}$. In addition, if $\sigma_N^{i*} > 0$ for all $i \in \{1, 2\}$, then the honest strategy is the unique equilibrium strategy for this type.

Proof

Suppose expert $i \in \{1, 2\}$ is a strategic wise type expert. Then $P(R \mid r_i, \hat{r}_j) = P(R \mid r_i, \hat{l}_j) = P(L \mid l_i, \hat{r}_j) = P(R \mid l_i, \hat{r}_j) = P(R \mid l_i, \hat{l}_j) = 0$. From expression (9) and the definition of $\Lambda_s^{i,A}$, the expressions for the expected gain of the wise type expert are given by:

$$\Lambda_{r}^{i,A} = P_{W}(\hat{r}_{j} \mid r_{i}) \left(((1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},\varnothing) + \mu\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},R)) - ((1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},\varnothing) \right) + P_{W}(\hat{l}_{j} \mid r_{i}) \left(((1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},\varnothing) + \mu\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},R)) - ((1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},\varnothing) \right),$$
(14)

$$\Lambda_{l}^{i,A} = P_{W}(\hat{r}_{j} \mid l_{i}) \left(((1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{r}_{j},\varnothing)) - ((1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},\varnothing) + \mu\hat{\alpha}_{i}(\hat{l}_{i},\hat{r}_{j},L)) \right) + P_{W}(\hat{l}_{j} \mid l_{i}) \left(((1-\mu)\hat{\alpha}_{i}(\hat{r}_{i},\hat{l}_{j},\varnothing)) - ((1-\mu)\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},\varnothing) + \mu\hat{\alpha}_{i}(\hat{l}_{i},\hat{l}_{j},L)) \right),$$
(15)

for $i, j \in \{1, 2\}$ and $i \neq j$, where

$$P_W(\hat{l}_j \mid l_i) = (\alpha_j + (1 - \alpha_j)(\gamma \sigma_N^j(l) + (1 - \gamma)(1 - \sigma_N^j(r)))),$$
(16)

$$P_W(\hat{l}_j \mid r_i) = (1 - \alpha_j)(\gamma(1 - \sigma_N^j(r)) + (1 - \gamma)\sigma_N^j(l)),$$
(17)

$$P_W(\hat{r}_j \mid l_i) = (1 - \alpha_j)(\gamma(1 - \sigma_N^j(l)) + (1 - \gamma)\sigma_N^j(r)),$$
(18)

$$P_W(\hat{r}_j \mid r_i) = (\alpha_j + (1 - \alpha_j)(\gamma \sigma_N^j(r) + (1 - \gamma)(1 - \sigma_N^j(l))))\gamma.$$
(19)

Now, if we compare $\Delta_r^{i,A}$ with $\Lambda_r^{i,A}$, clearly $\Lambda_r^{i,A} > \Delta_r^{i,A}$. Similarly, $\Lambda_l^{i,A} < \Delta_l^{i,A}$. This implies that a wise type expert never lies more than a normal type expert.

Therefore, for any $i, j \in \{1, 2\}$ with $i \neq j$, if in equilibrium the normal type expert optimally chooses $\sigma_N^{i*} > 0$, then $\Delta_r^{i,A} \ge 0$ and $\Delta_l^{i,A} \le 0$, which further implies $\Lambda_r^{i,A} > 0$ and $\Lambda_l^{i,A} < 0$, and hence $\sigma_W^{i*} = 1$. Then, in this case, the unique equilibrium strategy of the wise type expert is the honest strategy.

On the other hand, if $\sigma_N^{i*} = 0$ for at least one normal type expert, we can show that $\sigma_W^{i*} = 1$, for $i \in \{1, 2\}$, is an equilibrium strategy for the wise type expert. To this aim, let us assume $\sigma_W^{i*} = 1$ for $i \in \{1, 2\}$ and, without loss of generality, consider $\sigma_N^{i*} = 0$ and $\sigma_N^{j*} \ge 0$ for $i, j \in \{1, 2\}$ and $i \ne j$. Beliefs (1)-(7) apply in this case.

From (14) and for $i, j \in \{1, 2\}$, after some algebra we obtain:

$$\begin{split} \Lambda_{r}^{i} = & \frac{\alpha_{i}}{(2(\alpha_{i}-1))\gamma+1)^{2}} + (\alpha_{i}-1)\gamma \\ & \left(\frac{1-\mu}{2(\alpha_{i}-1)(\alpha_{j}-1)\gamma^{2}(2\sigma_{N}^{j}-1)-2(\alpha_{i}-2)(\alpha_{j}-1)\gamma\sigma_{N}^{j}-\gamma(\alpha_{i}+\alpha_{j}-2)-\alpha_{j}\sigma_{N}^{j}+\sigma_{N}^{j}-1} + \frac{\mu-1}{-2(\alpha_{i}-1)(\alpha_{j}-1)\gamma^{2}+(\alpha_{j}-1)(2\gamma-1)\sigma_{N}^{j}(2(\alpha_{i}-1)\gamma+1)-\gamma(\alpha_{i}+\alpha_{j}-2)} + 4\right), \end{split}$$

an expression that can be shown is always positive. Since $\Lambda_r^i > 0$, then $\Lambda_l^i < 0$ (by the same rationale used in Part I of the Appendix), and hence $\sigma_W^{i*} = 1$, for $i \in \{1, 2\}$.

A.2.2 RPE system

Proposition 5. Under RPE and symmetric strategies, the honest strategy $\sigma_W^{i*} = 1$ is always the unique equilibrium strategy for the wise type expert if $\sigma_N^{i*} > 0$ for all $i \in \{1, 2\}$.

Proof

The proof is analogous to the proof of Proposition 4.

First, we derive the expressions for $\Lambda_r^{i,R}$ and $\Lambda_l^{i,R}$:

$$\Lambda_{r}^{i,R} = P_{W}(\hat{r}_{j} \mid r_{i}) \left(\left((1-\mu)\hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{r}_{j},\varnothing) + \mu\hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{r}_{j},R) \right) - (1-\mu)\hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{r}_{j},\varnothing) \right) + \\ P_{W}(\hat{l}_{j} \mid r_{i}) \left(((1-\mu)\hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{l}_{j},\varnothing) + \mu) - (1-\mu)\hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{l}_{j},\varnothing) \right),$$

$$\begin{split} \Lambda_{l}^{i,R} &= P_{W}(\hat{r}_{j} \mid l_{i}) \left(\left((1-\mu)\hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{r}_{j},\varnothing) \right) - ((1-\mu)\hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{r}_{j},\varnothing) + \mu) \right) + \\ &P_{W}(\hat{l}_{j} \mid l_{i}) \left(((1-\mu)\hat{\alpha}_{i}^{R}(\hat{r}_{i},\hat{l}_{j},\varnothing)) - ((1-\mu)\hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{l}_{j},\varnothing) + \mu\hat{\alpha}_{i}^{R}(\hat{l}_{i},\hat{l}_{j},L)) \right), \end{split}$$

where $P_W(\hat{l}_j | l_i)$, $P_W(\hat{l}_j | r_i)$, $P_W(\hat{r}_j | l_i)$ and $P_W(\hat{r}_j | r_i)$ are defined by (16)-(19).

Second, the same argument used in the proof of Proposition 4 shows $\Lambda_r^{i,R} > \Delta_r^{i,R}$ and $\Lambda_l^{i,R} < \Delta_l^{i,R}$.

Third, for any $i, j \in \{1, 2\}$ with $i \neq j$, if in equilibrium for the normal type expert optimally chooses $\sigma_N^{i*} > 0$, then $\Delta_r^{i,R} \ge 0$ and $\Lambda_l^i \le 0$, which further implies $\Delta_r^{i,R} > 0$ and $\Delta_l^{i,R} < 0$, and hence $\sigma_W^{i*} = 1$. Then, in this case, the unique equilibrium strategy of the wise type expert is the honest strategy.

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