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Energy mix, technological change, and the environment

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Energy mix, technological change, and the environment[☆]

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Abstract

This paper studies the relationship between the energy mix and the environment using a theoretical framework in which two alternative energy sources are considered: fossil fuels (dirty energy) and renewable energy (clean energy). We find that a positive aggregate productivity shock increases energy consumption and emissions but reduces energy intensity and emissions per unit of output as renewable energy consumption increases, that is, carbon emissions are procyclical but emissions per unit of output are countercyclical. Second, an energy efficiency improvement provokes a "rebound effect" above 100% (the backfire effect), resulting in a rise of pollutant emissions by increasing energy use. Third, a technological improvement in emissions leads to a reduction in emissions per unit of fossil fuel but also implies a slow-down in the adoption of renewable energy sources. Finally, we also study the effects of a price shock to the pollutant energy, resulting in a substitution of the "dirty" by the "clean" energy, leading to a decline in energy consumption and emissions but at the cost of decreasing output.

Keywords: Energy mix; Emissions; Fossil fuels; Renewable energy; Technological change.

JEL Classification: Q41, Q42, Q43, Q52, Q55.

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1. Introduction

The energy mix used in the economy is arguably one of the key factors in explaining the dynamic relationship among output, energy consumption, carbon emissions, and the environment. However, existing environmental-economic models have focused on a variety of environmental policies, including Pigouvian taxes, abatement instruments, promotion of energy efficiency, limits to emissions, etc., with little attention to the implications of the energy mix and energy transition in linking economic activities with damages to the environment. Production activities requires the use of energy as an additional input to physical capital and labor. Pollutant emissions are not a direct by-product of production activities which would imply constant energy intensity, but they depend on the particular energy mix of the economy, where each type of energy source has a different impact on the environment. Emissions from alternatives energy sources are very heterogenous and therefore attention must be paid to the composition of the primary energy consumption. In general, we can distinguish between two types of energy sources: Renewable and non-renewable. Renewable energy source (hydroelectric power, geothermal, solar, wind and biomass), are considered "clean" energies sources, producing no direct greenhouse gases emissions. On the other side, non-renewable or fossil fuels (oil, natural gas, and coal), are "dirty" energies as they produce direct gas emissions although at different rates (coal produces more emissions than oil and natural gas).[1]

This paper contributes to the literature by studying how technological change and energy prices shocks affect the relationship between the environment and the energy mix, using an Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model. The dynamic relationship between economic growth and environmental protection remains central for sustainable development, where environmental problems are generated by the economic activity can be an impediment for future economic growth (World Bank, 2012). Figure 1 plots the energy intensity for the U.S. for the period 1950-2018, measured as the primary energy consumption (thousand BTU: British Thermal Unit) to GDP ratio. During the full period energy intensity declined from 15.12 to 5.45, that is, a reduction of 63.96%. During the same period, carbon emissions to GDP ratio (measured in metric tons carbon dioxide per million dollars) drops from 1040 to 284 units, a reduction of 72.69%. Hence, not all decline in carbon emissions is explained by energy intensity decline. Reduction in energy intensity can be explained by sectorial change toward less energy intensity industries, and

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[1] Nuclear energy power is not considered. This energy source does not produce gas emissions to the atmosphere but it produces other residuals that can damage the environment. As our focus here is on carbon emissions, we exclude nuclear energy (representing a 8.8% of total primary energy consumption in the U.S. in 2018), from our analysis.

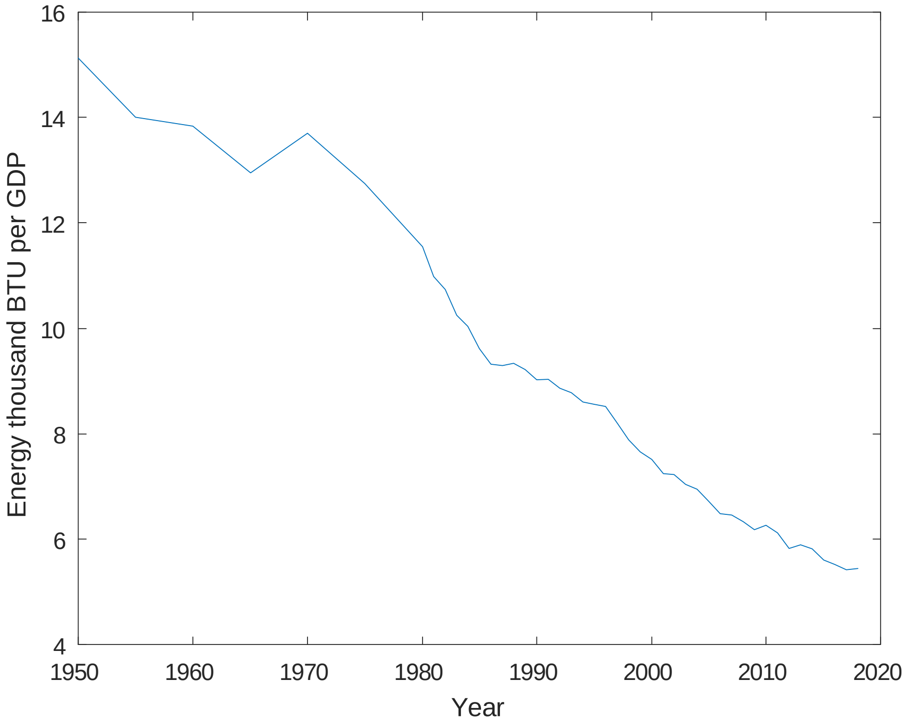

Figure 1: Energy intensity in the US. Source: U.S. Energy Information Administration (EIA).

by energy efficiency technologies. Reduction in carbon emissions not accounted by energy intensity decline (about 12% for the U.S.) is explained by emissions technological change and by changes in the energy mix toward cleaner energy resources.

Figure 2 plots the proportion of clean (renewable) energy with respect to fossil fuel energy consumption for the U.S. Whereas renewable energy represents a small fraction of total energy consumption, the impact on emissions is large, measured in term of forgone pollutant emissions by the replaced fossil fuels in the total energy consumption. Prior to 1970, the ratio of renewable to fossil fuel energy declined, not as a consequence of a decrease in the use of renewable energy but to a higher expansion in fossil fuels consumption. During the decade of 1970s, it is observed a positive trend in the ratio, with a rapid expansion in renewable energy consumption, with a stagnation during the 1980s and even a decline in renewable energy ratio in the first years of the XXI Century. However, in the last years renewable energy has been gaining positions with respect to fossil fuels.

The analysis done in this paper highlights the importance of the energy mix and energy policies implemented on the alternative energy sources in explaining environmental damage and the relationship between output and the environment. Whereas environmental policies

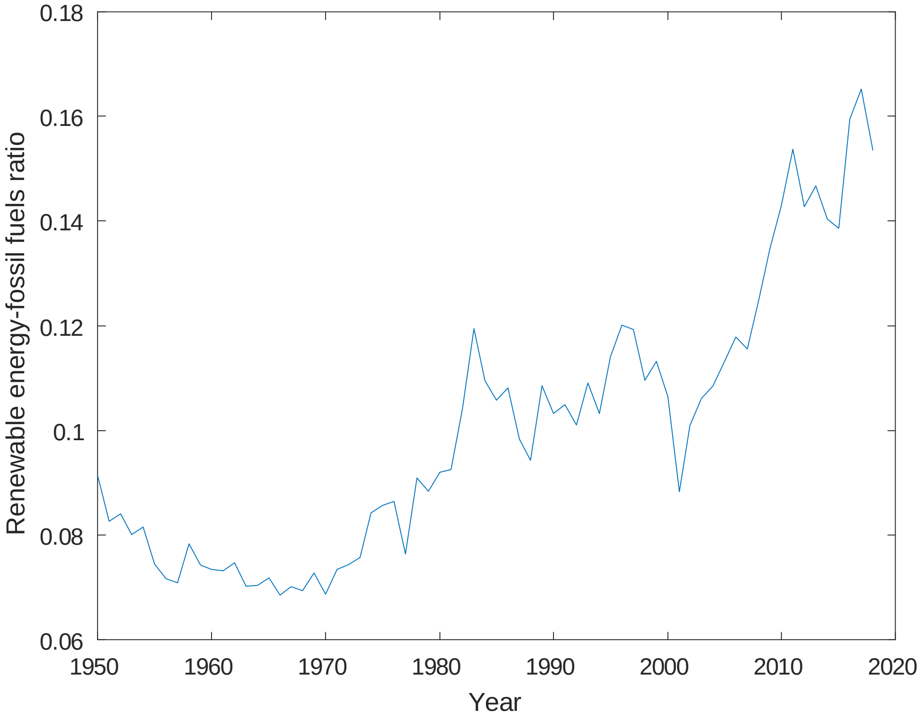

Figure 2: Renewable to fossil fuels energy ratio. Source: EIA.

considering a number of instruments have been widely studied in the literature, little focus has been placed in the implications of such policies on the energy mix and their impact on pollution. As pointed out by Atalla, Blazquez, Hunt and Manzano (2017) energy mix is the result of the interaction of fuel prices, technology, and energy policies. First, the energy mix can be a policy driving decision by strategic reasons, mainly in economies without fossil fuels resources and as a way to diversify energy sources. Second, energy mix is also determined by environmental concerns. This is evident in the case of nuclear and coal electric power. Finally, the energy mix depends on the relative prices of the alternative energy sources which are mainly driven by technological factors. Tahvonen and Salo (2001) studied the transitions between non-renewable and renewable energy depending on the development stage of an economy. They obtain an inverted-U relationship between fossil fuel and the income level. Atalla *et al.* (2017) studied the role of fossil fuel prices relative to energy policy in driving the primary fossil fuel mix, and found that relative fossil fuel prices are the main source explaining the fossil energy mix in the U.S., Germany and the UK. However, the question remains open when not only fossil fuels are considered and also other primary non-fossil energy is taken into account. We depart from Atalla *et al.* (2017) study, by considering also the role of non-fossil fuel energy sources, and focusing on the consequences of technological shocks and fuel price shocks in explaining the energy mix and the impact on the environment.

This paper contributes to the literature by studying the relationship between production and the environment in a Dynamic Stochastic General Equilibrium (DSGE) model with alternative energy sources and endogenous energy transition. In particular, we propose an economy where two alternative energy sources can be used in the production sector: one energy that produce emissions (i.e. fossil fuels), and another clean energy (i.e., renewable energy). In this framework emissions do not depend on final output, as it has been considered previously in the literature (see Fischer and Springborn, 2011; Angelopolous *et al.*, 2010, 2013; Heutel, 2012; Annicchiarico and Di Dio, 2015), but on the consumption of fossil fuel energy. Our model considers a three inputs production function: physical capital, labor, and energy. Energy used in the production function is a composite of fossil fuels and renewable energy, and emissions depend on the quantity of fossil fuels used in the final energy mix. The stock of pollutants is an externality affecting negatively to final output (see Nordhaus, 2008; Heutel, 2012). Two types of technologies related to energy and emissions are considered: a technology that improves energy use efficiency, and a technology that reduces the quantity of emissions as a function of the quantity of fossil fuels.

We use the model to study the implications for the economy, the energy mix, and the environment of four shocks: an aggregate productivity shock, an energy use efficiency technological shock, a clean energy technological shock, and a fossil fuel price shock. First, a positive neutral technological shock produces two opposite effects on output. First, the increase in aggregate productivity also increases output, as expected, but also increases the demand of the two types of energy, resulting in an increase of carbon emissions and in the accumulation of CO2 in the atmosphere. The higher level of CO2 concentration in the atmosphere has a negative impact on productivity, limiting the positive effects of the productivity shock on final output. We find that energy consumption is procyclical, as expected, but that the productivity shock reduces energy intensity and emissions per unit of output, consistent with empirical evidence. This is because the expansion in economic activity following the productivity shock increases the demand of both fossil fuels and renewable energy. As also the demand for renewable energy increases, emissions per energy unit decrease. However, it is also true that as a consequence of this productivity shock the renewable to fossil fuel energy ratio falls. The main results is that carbon emissions are procyclical but carbon emissions per unit of output is countercyclical.

Second, we study the implications of an energy efficiency technological shock common to the two energy sources. Energy efficiency technology provokes an increase in the quantity of energy used in the production process, increasing the level of emissions, which implies that the positive initial effect of a technological improvement in energy efficiency leads to an increase in energy consumption and in CO2 concentration in the atmosphere. Energy efficiency technology not only provokes the well-known "rebound effect" (Frondel, Ritter and Colin, 2012; Gillingham, Rapson and Wagner, 2016), which implies that the positive initial effect of a technological improvement in energy efficiency leads to save less energy than initially expected, but we find that energy efficiency improvement increases energy consumption (a "rebound effect" above 100%), the so-called “backfire effect” (Sorrel, 2009; Gillingham *et al.*, 2016), not as a consequence of the optimal response of households who does not internalize the cost of pollution, but as the optimal decision by a central planner to maximize social welfare. Importantly, the energy efficiency shock leads to a decline in the renewable-fossil fuels ratio, resulting in a technology that hinders the adoption of cleaner energy sources.

Third, we consider the case of a technological improvement in emissions (i.e., cleaner technologies as particulate filters and catalytic converters). This is an example of an asymmetric specific technological shock affecting only one of the energy sources: the "dirty" energy, as we assume that renewable energy does not produce carbon emissions. As one would expect, this technological improvement reduces emissions per fossil fuel unit, and hence, also emissions per output reduces. However, surprisingly, this technological change provokes an increase in the quantity of "dirty" energy used in production and reduces the use of renewable energy. Therefore, technological change associated to emissions promotes the use of "dirty" energy sources as the negative externality produced by this energy declines. These results show that environmental policies promoting investment in energy efficiency and emissions efficiency technologies have different effects on the stock of CO2 concentration in the atmosphere; whereas the former increases the stock of CO2, the latter reduces the stock of CO2. Nevertheless, both policies are an obstacle to energy transition from non-renewable to renewable energy sources.

Finally, we study the effects of a fossil fuel price shock, which also represents an asymmetric shock resulting in a change in the relative prices of the alternative energy sources. The effect of this shock on the economy have been widely studied in the literature (Balke and Brown, 2018; Punzi, 2019), resulting in an output contraction. We find that an increase in the price of the fossil fuels energy provokes a substitution of the "dirty" energy by the "clean" energy, having a positive effect on the environment as the level of emissions decline. However, we also find that this shocks generates a output downturn, consistent with the literature. His downturn in output indicates that the price effects is higher than the substitution effect, resulting in a lower consumption of energy.

The rest of the paper is structured as follows. Section 2 presents an E-DSGE model including non-renewable and renewable energy sources as an additional input factor to capital and labor. Section 3 presents the calibration of the parameter of the model. Section 4 studies the dynamic properties of the model to different technological shocks. Section 5 simulates the model for a fossil fuel price shock. Finally, Section 6 presents some conclusions.

2. An E-DSGE model with energy mix

In this section, we develop an E-DSGE model with a three-inputs production function: physical capital, labor and energy. We consider two types of energy sources: Fossil fuels and renewable energy. We assume that for production some energy source must be used as an additional input to capital and labor, and that burning fossil fuels releases greenhouse gases (CO2) into the atmosphere. Renewable energy is a clean energy as it does not produce emissions. The stock of pollution is a negative externality that will negatively affect aggregate productivity. The model includes three technological shocks: an aggregate productivity shock, an energy efficiency technological shock, and an emission efficiency technological shock. Additionally, the model considers an oil price shock.

2.1. Household utility function

The economy is populated by an infinitely-lived representative agent who maximizes the expected value of her lifetime utility. Households obtain utility from consumption and leisure. Household utility function is defined as:

$$U(C_t, L_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \omega \frac{L_t^{1+\frac{1}{\upsilon}}}{1+\frac{1}{\upsilon}} \tag{1}$$

where C_t is the consumption, L_t and is working hours, γ is a aversion-risk parameter, υ is the Frisch elasticity of labor supply, and $\omega > 0$ represents the willingness to work. We consider a centralized economy where a central planner maximizes social welfare. The budget constraint is defined as:

$$C_t + I_t + P_{o,t}O_t + P_{s,t}S_t = Y_t \tag{2}$$

where I_t is investment in physical capital, Y_t is final output (total income), O_t is the quantity of fossil fuel, S_t is the quantity of renewable energy, $P_{o,t}$ is the price of fossil fuels and $P_{s,t}$ is the price of renewable energy. The two energy prices are assumed to be exogenous and no restriction on the extraction of non-renewable energy is considered.

In the literature, we find two alternative ways to introduce the negative externality produced by damages to the environment. The first is the introduction of this externality in the aggregate production function. This is the case, for instance, in Heutel (2012) and Golosov *et al.* (2014). It is assumed that climate change damages the environment and hence production, by reducing productivity. Pollution, defined as the CO2 concentration in the atmosphere, is considered as a stock variable that accumulate with carbon emissions. Therefore, atmospheric carbon concentration has a negative economic impact reducing final output. The second way to consider externalities from pollution is by assuming that it is can be either a flow or a stock variable, negatively affecting households' utility function. Examples of this modeling are John and Pecchenino (1994), Jones and Manuelli (1995), and Stokey (1988). As pointed out by John and Pecchenino (1994), in general, environmental externalities could arise from production or consumption and could affect welfare or productivity. Following Nordhaus (2007) and Heutel (2012), our model only consider a pollution externality in the production.

Investment accumulates into physical capital. Physical capital stock accumulation equation is defined as:

$$K_{t+1} = (1 - \delta_k)K_t + I_t \tag{3}$$

where K_t is the capital stock and δ_k $(0 < \delta_k < 1)$ is the depreciation rate of physical capital.

2.2. Emissions and the stock of pollution

In the environmental-economic literature, a number of works assumes that emissions are a function of final output. However, this assumption neglects the possibility of declines in emissions when output increases. In this context, a negative relationship between emissions and output can only be obtained under technological change affecting abatement and/or emissions. A more realistic assumption is that carbon emissions depends on the energy mix combining both non-renewable dirty energy with renewable clean energy sources. To consider that possibility, in our model, carbon emissions are related with the energy source used in the final production. That is, carbon emission is assumed to be generated by the use of fossil fuels, whereas we assume that renewable energy sources do not produce emissions. In particular, we assume that damages are proportional to the quantity of fossil energy.

$$X_t = \eta B_t O_t \tag{4}$$

where $\eta > 0$ represents the carbon content of fossil fuel or carbon emission per fossil fuel unit, and B_t is an exogenous technology for emission (emissions efficiency), representing fossil fuels consumption technologies that reduce gas emissions. Hence, we are assuming that the change in emissions is equal to the change in fossil fuel consumption. Hence a technological improvement that reduces emissions by fossil fuels energy unit is represented by a decrease in that exogenous shock (i.e. catalytic converter technology). We abstract from the fact that the level of emissions of the fossil fuel mix are different depending on the share of oil, coal, and natural gas, where emissions produced by natural gas are lower than emissions from coal. In fact, for instance, if the share of gas increases at the expense of coal, this results in a "cleaner" energy mix.

Emissions accumulate into a stock of pollutants, Z_t, where the atmospheric carbon accumulation process is given by,

$$Z_{t+1} = (1 - \delta_z)Z_t + X_t \tag{5}$$

where δ_z ($0 < \delta_z < 1$) is the stock of pollutants natural decay rate.[2] Emissions efficiency technology is assumed to be exogenous and follows a first-order autorregresive process:

$$\log B_t = (1 - \rho_B) \log \overline{B} + \rho_B \log B_{t-1} + \varepsilon_t^B \tag{6}$$

where $\overline{B}$ is the steady state value for the emission technology, $\rho_B < 1$, and ε_t^B is a i.i.d. innovation in the stochastic process. Emission efficiency technological progress is represented by a negative shock to B_t.

2.3. Production technology

The model considers a three-factor aggregate production function: physical capital, labor, and energy. We assume the following aggregate production function, that exhibits constant returns to scale on all factors, represented by a Cobb-Douglas production function:

$$Y_t = A_t \exp(-\phi Z_t) K_t^{\alpha_1} \left(D_t E_t\right)^{\alpha_2} L_t^{1-\alpha_1-\alpha_2} \tag{7}$$

where the term $\exp(-\phi Z_t)$ represents the cost of the damage of pollutants measured as forgone output, and $\phi > 0$ is a parameter governing the elasticity of aggregate productivity with respect to the stock of pollutants.[3] Final output is influenced by a neutral technology component A_t (total factor productivity, TFP) and by an externality due to emissions. This externality may be included in the economy by affecting the utility function, instead of the production function. However, the literature considers that this alternative is more appropriate for pollutants that affect health directly and that the stock of pollution is expected to affect the production possibilities of the world economy (Nordhaus, 2008).

Energy is a Armington aggregator of fossil fuel and renewable energy:

$$E_t = \left[\mu O_t^{\frac{\sigma-1}{\sigma}} + (1-\mu) S_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{8}$$

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[2] We do not study the role of technological progress in Carbon Dioxide Removal (CDR). CDR comprises a set of chemical and biological instruments that can reduce the amount of CO2 already in the atmosphere. CDR technologies includes capture and storage of CO2, biochar (a pyrolysis of biomass process), ocean fertilization, and enhanced weathering. These technologies change the accumulation process of the stock of pollution but without affecting emissions.

[3] Alternatively, the negative effect of CO2 concentration in the atmosphere on productivity can be modeled as a function of the temperature. See Nordhaus (2007).

where $\sigma > 1$ is the elasticity of substitution between both types of energies and μ is a parameter representing the weight of each type of energy in the final energy mix. The model assumes that both types of energies are imperfect substitutes. The amount of energy used, E_t, is influenced by an energy-augmenting technological change of the economy, denoted by D_t. The higher is D_t, the more energy efficient is the production sector. Total Factor Productivity is assumed to be exogenous and follows a first-order autorregresive process:

$$\log A_t = (1-\rho_A)\log \overline{A} + \rho_A \log A_{t-1} + \varepsilon_t^A \tag{9}$$

where $\overline{A}$ is the steady state value for TFP, $\rho_A < 1$, and ε_t^A is a i.i.d. innovation in the stochastic process. A similar stochastic process is assumed for D_t:

$$\log D_t = (1-\rho_D)\log \overline{D} + \rho_D \log D_{t-1} + \varepsilon_t^D \tag{10}$$

2.4. Centralized equilibrium

Given the existence of a negative externality on the environment, we consider a centralized economy. The central planner solution is derived by choosing the path for consumption, labor, capital, fossil fuels, renewable energy, and stock of pollution, to maximize the sum of the discounted utility subject to resource, technology, and carbon emissions constraints. From the first order conditions for the centralized problem, we obtain the following equilibrium conditions (see Appendix A for details):

$$L_t^{\frac{1}{v}+1} = \frac{(1-\alpha_1-\alpha_2)Y_t}{\omega C_t^{\gamma}} \tag{11}$$

$$\frac{C_{t+1}^{\gamma}}{C_t^{\gamma}} = \beta\left[(1-\delta_k) + \alpha_1\frac{Y_{t+1}}{K_{t+1}}\right] \tag{12}$$

$$P_{s,t} = \alpha_2(1-\mu)\frac{Y_t S_t^{\frac{-1}{\sigma}}}{\mu O_t^{\frac{\sigma-1}{\sigma}} + (1-\mu)S_t^{\frac{\sigma-1}{\sigma}}} \tag{13}$$

$$Y_{t+1} = \frac{\left[P_{0,t+1} - \alpha_2\mu\frac{Y_{t+1}O_{t+1}^{\frac{-1}{\sigma}}}{\mu O_{t+1}^{\frac{\sigma-1}{\sigma}} + (1-\mu)S_{t+1}^{\frac{\sigma-1}{\sigma}}}\right](1-\delta_z)}{\phi\eta B_{t+1}} - \frac{C_t^{-\gamma}\left[P_{0,t} - \alpha_2\mu\frac{Y_t O_t^{\frac{-1}{\sigma}}}{\mu O_t^{\frac{\sigma-1}{\sigma}} + (1-\mu)S_t^{\frac{\sigma-1}{\sigma}}}\right]}{\beta C_{t+1}^{-\gamma}\phi\eta B_t} \tag{14}$$

where β is the discount factor. Expression (11) is the optimal labor supply. Expression (12) is the optimal consumption path. Expression (13) is the equilibrium condition for renewable energy consumption, whereas expression (14) indicates the optimal stock of pollution. Notice that the price for the fossil fuels includes the pollution externality cost. Finally, to close the model, energy prices are assumed to be exogenous and driving by the following stochastic processes:

$$\log P_{s,t} = (1-\rho_s)\log \overline{P}_s + \rho_s \log P_{s,t-1} + \varepsilon_t^s \tag{15}$$

$$\log P_{o,t} = (1-\rho_o)\log \overline{P}_o + \rho_o \log P_{o,t-1} + \varepsilon_t^o \tag{16}$$

where $\overline{P}_s$ and $\overline{P}_o$ are the steady state values for the price of fossil energy and renewable energy, respectively.

3. Data and Calibration

This section presents the calibration of the parameters of the model. Since the model is composed by macroeconomic parameters and also parameters related to emissions, we use different sources for its calibration. Macroeconomic parameters are calibrated from the Real Business Cycle literature, while energy and emissions parameters are taken from studies related to environment and climate change, mostly from Stern (2012), Heutel (2012). We use data for the U.S. The discount factor (for annual data) is fixed to 0.975, whereas the relative risk aversion parameter is equal to 1.2, values that are standard in the literature. Parameter values for labor supply are selected just to replicate the observed fraction of time devoted to working activities of 0.33, as reported by the BLS (Bureau of Labor Statistics). Production function technological parameters are taken from the EIA (U.S. Energy Information Administration) and the BEA (Bureau of Economic Analysis). We assume that the fraction of labor compensation over total income is 0.65. As the production function assumes the existence of constant returns to scale, the sum of the technological parameters for the other two inputs, capital and energy, must be 0.35. The technological parameter governing the elasticity of output with respect to energy is obtained from the proportion of energy consumption over GDP and is estimated to be 0.0982. Therefore, the elasticity of output with respect to physical capital is 0.2518.

The parameter representing the proportion of fossil fuels on total energy mix is fixed at 0.84, accounting the rest 0.16 for renewable energy. The parameter governing the elasticity of substitution between fossil fuels energy and renewable energy is fixed at 1.5. Finally, environmental parameters are taken from Nordhaus (2008) and calibrated simultaneously to produce a loss of productivity of 1% in the steady state. The pollution decay rate is fixed at 0.012, as it is standard in the literature, which corresponds to a half-life of carbon concentration of around 58 years. Heutel (2012) estimates a elasticity of emissions with respect of output of 0.696, whereas the productivity loss from pollution is estimated to be of 0.26%. We fix the emission parameter to be 0.1, resulting in a pollution damage parameter of 0.0875 to reduces productivity a 1% in steady state. For all exogenous shocks, the autoregressive parameter is fixed to 0.9, and the standard deviation to 0.01. A summary of the calibration of the parameters is presented in Table 1.

Table 1: Calibration of the parameters

	Parameter	Definition	Value
Preferences	β	Discount factor	0.97
	γ	Risk aversion	1.2
	ω	Labor weight	15.60
	ν	Frisch elasticity parameter	0.72
Technology	α_1	Output-capital elasticity	0.2518
	α_2	Output-energy elasticity	0.0982
	δ_k	Physical capital depreciation rate	0.06
Energy	μ	Weight of fossil fuels	0.83
	σ	Energy sources substitution	1.50
Environment	η	Emissions parameter	0.1
	δ_z	Pollutant stock decay rate	0.012
	ϕ	Pollution damage parameter	0.0875

4. Technological shocks

The calibrated model is used to study how the economy, the energy mix, the level of emissions, and the environment, respond to different shocks. In particular, we are interested in studying the impact of different technological shocks on the energy mix and emissions, and their implications on the shift from fossil fuels to renewable energy sources that can increases output without further damages to the environment. We simulate three technological shocks: a total factor productivity shock (i. e., a neutral technological shock), an energy efficiency technological shock (energy-augmented shock), and an emissions efficiency technological shock.

4.1. Aggregate productivity shock

First, we present some simulations to show the dynamics of the model economy via impulse-response functions to an aggregate productivity shock. This first exercise considers the case of an exogenous idiosyncratic positive neutral shock to the economy, represented by an increase in Total Factor Productivity (TFP), A_t. Expansion of economic activity following the productivity shock is expected to increase energy consumption, but its effects on CO2 emissions will depend on how the energy mix will change. Empirical evidence shows that total energy consumption is procyclical, a result which is also consistent with the model. The key question here is, given the existence of two alternative energy sources with a separate effect on emissions, how the shock affects the energy mix and damages to the environment.

Figure 3 plots the impulse-response for the main aggregate variables of the economy, as percentage deviations from the steady state. As expected, the rise in total factor productivity increases output. This rise in output is distributed between consumption and investment, similar to the response observed in a standard DSGE model. The amount of inputs also increases, including energy, given that marginal productivity of each input is now higher, as this is the case of a neutral technological shock. The demand of both types of energy increases, being higher the response of fossil fuels to the shock compared to that of renewable energy. As a consequence of the increase in fossil fuels consumption, the level of emissions also increases. Therefore, our model produces, using our benchmark calibration, a positive relationship between output and environmental deterioration when an aggregate productivity shock occurs. Emissions are procyclical as the neutral technological shock increases the demand for all energy sources. Importantly, the effects of the positive aggregate productivity shock on economic activity are mitigated due to the counter-effect of the pollution externality by reducing productivity gains following the shock. The higher the cost of the pollution externality, the less the output increases following a positive TFP shock. These results are consistent with the findings of Heutel (2012), as he found that carbon emissions are significantly procyclical with an elasticity with respect to GDP between 0.5 and 0.9. However, that result emerges directly from the modelling assumption that emissions is a proportion of output. In the model presented in this paper, emissions are not proportional to output but to the consumption of fossil fuel, but we also find that carbon emissions are procyclical, as the neutral technological shock increases the demand of both renewable and fossil fuels energy sources. Our estimated value is an elasticity of 0.7835, a value in the range estimated by Heutel (2012).

This simulation exercise illustrates that a substitution effect of "dirty" by "clean" energy does not happen endogenously for the benchmark calibration, and hence, expansion of the economic activity does not produces endogenous energy transition from fossil fuels to renewable energy sources. Only when the cost in foregone output is higher enough, would changes in the energy mix reduces the use of fossil fuels to mitigate environmental damage. But this is not the case in our simulated economy, where the profits from increasing fossil fuels consumption are higher than the cost of damages to the environment. The other important result we obtain is that the productivity shock reduces energy intensity. In response to the shock both output and energy consumption increase, but where the increase in the

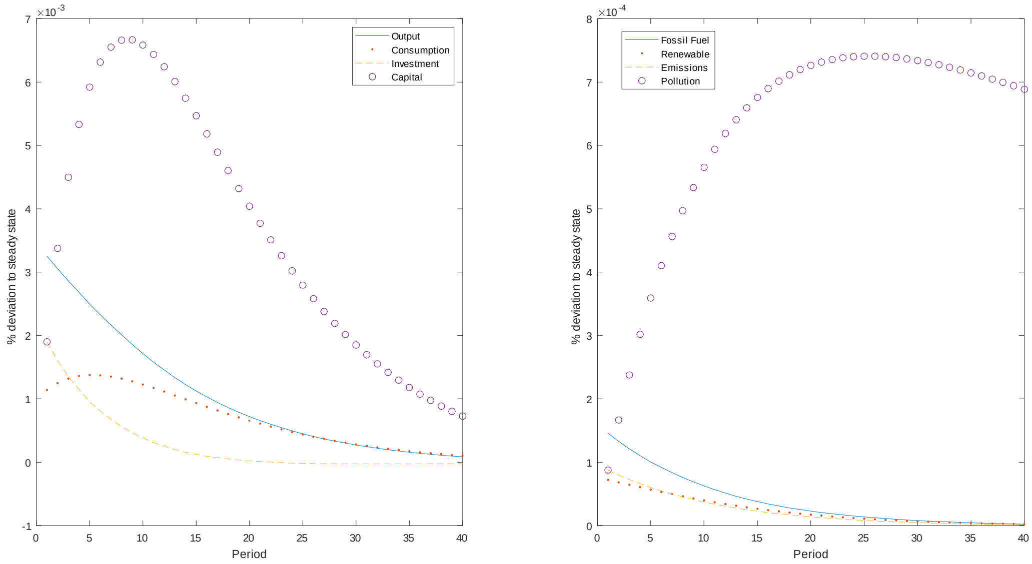

Figure 3: Impulse-response functions to a positive Total Factor Productivity shock

first is higher than in the second. Therefore, we show that TFP shocks are an important source in explaining the decline in energy intensity observed in the data. However, from the benchmark simulation it is clear that energy intensity decline does not necessarily implies a lower level of emissions when output growths. Finally, another outstanding result is that the productivity shock reduces the level of carbon emissions per unit of output as a consequence of an expansion in renewable energy sources. Nevertheless, total carbon emissions increases as a consequence of the higher fossil fuels consumption, and hence, the accumulation of CO2 in the atmosphere accelerates.

4.2. Energy efficiency shock

Second, we study the response of the economy to a shock that increases energy efficiency, D_t. Given that most anthropogenic emissions of greenhouse gases released into the atmosphere are generated by energy consumption, environmental policies have focused on promoting energy efficiency as an instrument to reduce emissions. Energy efficiency refers to technological changes that reduce the amount of energy needed to produce a given quantity of goods and services in combination with the other inputs, resulting in a decline in energy intensity (an energy-augmenting technical change). This shock is general to the consumption of energy per output unit, and hence, affects symmetrically to the two energy sources. The implications of energy-saving technological changes on the economy have been widely studied in the literature, for instance, in Newell, Jaffe and Stavins (1999). They obtain the energy price changes are the main driving force for energy-efficiency technological change. Here, we pay attention to how energy efficiency changes the energy mix. We find that energy efficiency technology not only provokes the well-known "rebound effect" (Frondel, Ritter and Colin, 2012; Gillingham, Rapson and Wagner, 2016), which implies that the positive initial effect of a technological improvement in energy efficiency leads to save less energy than initially expected, but we find that energy efficiency improvement increases energy consumption, resulting in the so-called “backfire effect” (Sorrel, 2009; Gillingham *et al.*, 2016), not as a consequence of the optimal response of households who does not internalize the cost of pollution, but as the optimal decision by a central planner to maximize social welfare. Energy intensity reduces as energy efficiency increases, but surprisingly also the level of emissions increases and hence, energy efficiency policies have harmful consequences for the environment as they incentive energy consumption.

Figure 4 plots the impulse-response functions of the main variables of the model to an energy-efficiency technological shock. As expected, the response of output is positive, as the energy-saving shock increases productivity of one of the inputs: energy. As a consequence, the response of consumption and investment is also positive, indicating that this efficiency technology shock increases physical capital accumulation. What is more important is that it is observed that gains in energy efficiency lead to an increase in the demand of energy. As the shock is common to the two energy sources, both the demand of non-renewable and the demand renewable energy increase, where the increase in renewable energy is larger than the rise in fossil fuels energy. This larger response of the renewable energy is consequence of the increase in the relative price of fossil fuels energy, as the pollution externality cost rises the total cost of fossil fuels. However, the increase in the user cost of fossil fuels caused by the pollution externality cost is lower than the reduction in the user cost of this energy source due to the improvement in energy-efficiency, resulting in a final rise in the quantity of energy used in production, and resulting in more emissions.

These results are consistent with the so-called "rebound effect" or "take-back effect" described in the literature on energy-efficiency, consisting in a reduction in the expected gains or, even in a loss, from new technologies that increase the efficiency of energy use. That effect is derived from the optimal response of economic agents to a technological improvement in energy efficiency, leading to a rise in energy consumption. This is the mechanism that we observe in our model, where this technological shock provokes a rise in the quantity of energy used in the production activity, as the rise in energy-efficiency is equivalent to a reduction of the energy price. For the benchmark calibration of the model, we obtain a "rebound effect" higher than 100%, the so-called backfire effect, which generates a negative effect on the environment, as the technological improvement in energy efficiency implies a rise in the emission of pollutants. Given that energy is a normal and, also an ordinary good,

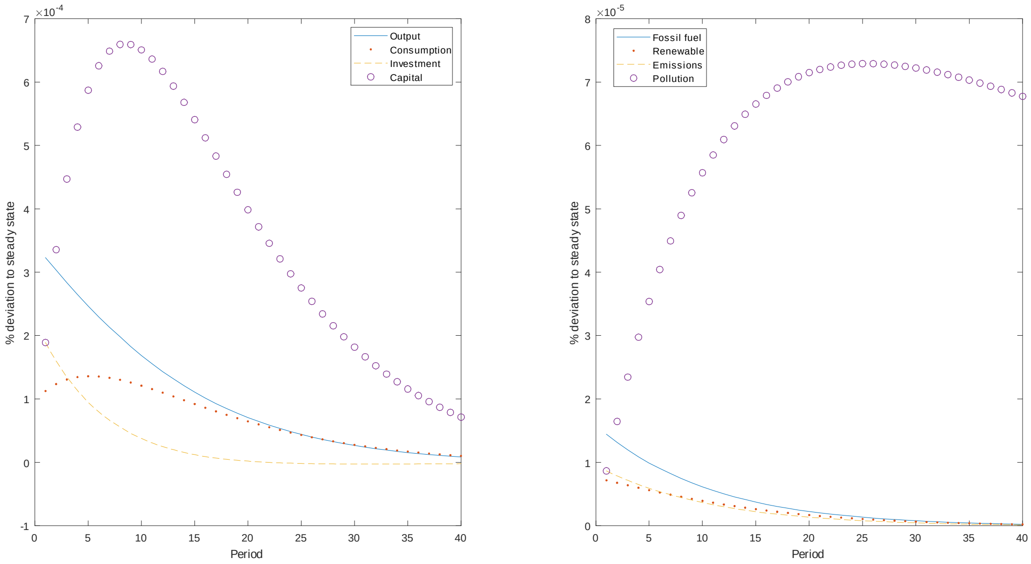

Figure 4: Impulse-response functions to an energy efficiency technological shock

the rebound effect can be explained according to an income and a substitution effects. In our theoretical framework this technological shock increases productivity of energy in producing the final good, increasing the demand of this input and reducing the demand for the other inputs. This is the substitution effect. On the other hand, the rise in the productivity of one of the factors, increases aggregate productivity, increasing the demand for all factors. This is the income effect. Both effects contribute to the observed backfire effect and the consequent increase in the carbon concentration in the atmosphere.

4.3. Emissions efficiency technology shock

Finally, we study the implications of a technological change that increases emissions efficiency, i.e., a negative shock to B_t. This shock implies a reduction in the level of emissions per fossil fuel unit used in the production process. For instance, this is the case of an improvement in catalytic converter technology. This shock can also be interpreted as an improving in abatement technology. In this case, the technological change does not affect energy efficiency but emissions efficiency specific to fossil fuels consumption, resulting in an asymmetric shock depending on the type of energy source consumed. Therefore, this shock will reduce the level of emissions per fossil fuel unit, but it does not directly affect to the use of renewable energy sources, as the later is a "clean" energy, and thus, not related to emissions. However, given the general equilibrium effects generated by our model economy, this specific shock to the use of non-renewable energy will also change renewable energy consumption.

As shown in Figure 5, output increases in response to this technological shock. This change in total output results in a rise in consumption and investment. Investment shows a positive response in the following periods, increasing the stock of physical capital. Importantly, the amount of energy used in production increases but, in balance, produces a lower level of emissions. The economic intuition behind these effects is the following. This shock is equivalent to a reduction the cost of the pollution externality, increasing productivity and reducing the user cost of fossil fuels energy. The positive effect on economic activity is explained by two forces. First, the shock reduces damages to the environment and relieving its harmful effects on productivity and expanding output. This initial expansion in output results in higher investment, increasing capital stock. Therefore, the effects of an emission efficiency shock on the economy are, qualitatively, similar to those of an aggregate productivity shock.

Investing in cleaner technologies has two opposite effects. First, the shock means that less carbon emissions are generated by unit of fossil fuel consumption. However, total carbon emissions will depends not only on the direct effect of the shock on emissions per fossil fuel unit but on the indirect effect of the shock on fossil fuels consumption. Indeed, it is observed an increase in energy consumption, at the same time that the level of emissions declines. Second, there is an increase in the use of fossil fuels and a reduction in the renewable energy use. These two different results are consequence of the different impact of this technological shock over the two energy sources. The shock directly affects to the use of the fossil fuels energy source. The shock reduces the level of emissions per unit of energy, reducing the externality cost of using the "dirty" energy. As an indirect effect, that changes the relative price of both energy sources, reducing the price of the fossil fuel energy relative to the price of the renewable energy source. This provokes a substitution of renewable energy by fossil fuels energy. The rise in the demand of fossil fuels is larger than the down of renewable energy, resulting in a total rise in the demand of energy.

In sum, an emission efficiency technological shock is equivalent to a reduction in the user cost of the "dirty" energy, and hence, increasing the relative price of the "clean" energy relative to the "dirty" energy. Indeed, the literature suggests that the relative low prices of fossil fuels had driven technological progress to fossil fuels intensive industries. Emission efficiency technological change provokes similar results, having a negative impact on energy transition to renewable energy sources. Overall, the social cost of energy declines following this shock, increasing the quantity of energy used in the production. Nevertheless, the effect of the rise in the quantity of energy used for production is compensated by the reduction in emissions by energy unit, resulting in an environmental quality improvement.

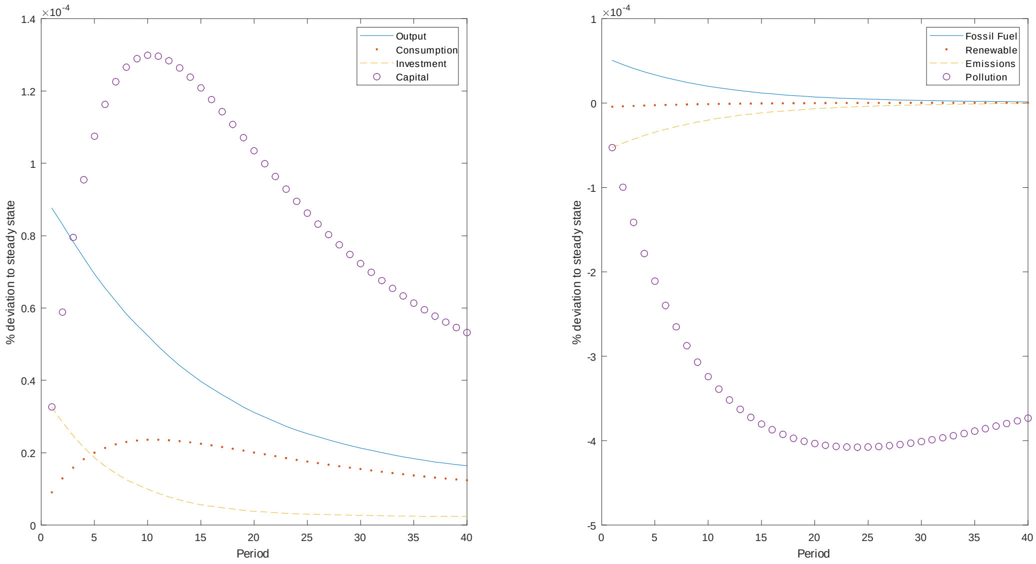

Figure 5: Impulse-response functions to an emissions effiency technological shock

5. Fossil fuels price shock

Finally, we analyze the case of a shock to the fossil fuels prices, representing a change in the relative price of "dirty" to "clean" energies. The effects of fossil fuel prices over the business cycle have been widely studied in the literature, resulting that a rise in oil prices turns out into an economic slowdown. Kim and Loungani (1992) show that energy price shocks have a significant impact in predicting output volatility. Rotemberg and Woodford (1996) study the relationship between oil price and macroeconomics variables and found strong implications of crude oil shocks for the design and implementation of economic policies. De Miguel *et al.* (2003) analyze the impact of oil price shocks on the business cycle and welfare. Atalla *et al.* (2017) estimates the output impulse-response to a price shock for oil, gas and coal. They show that relative prices are the main determinant of the primary fossil fuel mix for the U.S., but not for Germany and the U.K. Punzi (2019) studies the effects of energy price shocks and find that whereas a positive energy price shock causes an economic slowdown, energy price volatility shocks generates an increase in GDP in the short-run and a reversal in the long-run.

We extend this literature by studying the implications of fossil fuels price shocks and changes in the relative prices of non-renewable to renewable energy sources on energy transition. The relative price of fossil fuels to renewable energies is a key variable for determining the relative quantity of each energy source in the final energy input used for production in the economy. The weight of each energy source in the energy mix of an economy, and hence emissions, are directly related not only to technological factors and energy policies, but also to the relative price between the two energy sources. Therefore, it is of interest to study how the energy mix and emissions respond to a shock affecting the price of one of the energy sources. In particular, here we study the implications of a positive shock to the price of fossil fuels. As indicated above, the social relative price of the two energy sources depends, on one side, on the relative quantity used (demand) of each energy source, but on the other side, also depends on the costs of pollution relative to the renewable energy productivity. This second term becomes fundamental in the determination of the optimal relative quantity of each energy source used in the production activity once the cost of emissions has been internalized. As the cost of damages from pollution increases, the change in the relative price produces a substitution of fossil fuel energy by renewable energy sources, as the second becomes cheaper compared to the first. This is equivalent to an investment-specific technological change in the renewable energy sector which induces a change in the relative price of the two energy sources, increasing the use of renewable energy and reducing the use of fossil fuels energy. This energy transition path leads to a lower level of emissions.

Figure 5 plots the impulse-response function of the main variables of the model to this shock. The increase in the price of fossil fuels produces two important effects. First, output reduces as a consequence of the rise in the energy input cost. The rise in the price of fossil fuels reduces the demand of energy, as the substitution of the more expensive energy source by the less expensive is not complete. As a consequence, final output reduces. This negative effect is reinforced by the contraction in investment, which reduces physical capital stock. During this process, it is also observed a drop in consumption. The increase in the cost of the energy input also affects to the other two inputs, reinforcing the negative impact on output. All these effects are consistent with the previous literature studying the effects of an energy price shock on the economy. Second, the change in the relative prices of the two energy sources will change the energy mix. The rise in the price of fossil fuels induces a substitution effect of the two types of energy. As expected, the economy reduces the use of fossil fuels and increases the use of renewable energy. However, as noted above, the reduction of the first is larger than the rise in the second, resulting in a reduction in the amount of energy used in the production. Finally, the reduction in the use of dirty energy decreases carbon emissions, and hence, carbon concentration declines. Therefore, this price shock has a positive effect on environmental quality and accelerates energy transition to renewable energy sources, but at the cost of reducing output. This result clearly illustrates the dilemma between protecting the environment and economic growth, and explains why energy transition path to renewable energy sources, to avoid or minimize environmental damages, is too slow.

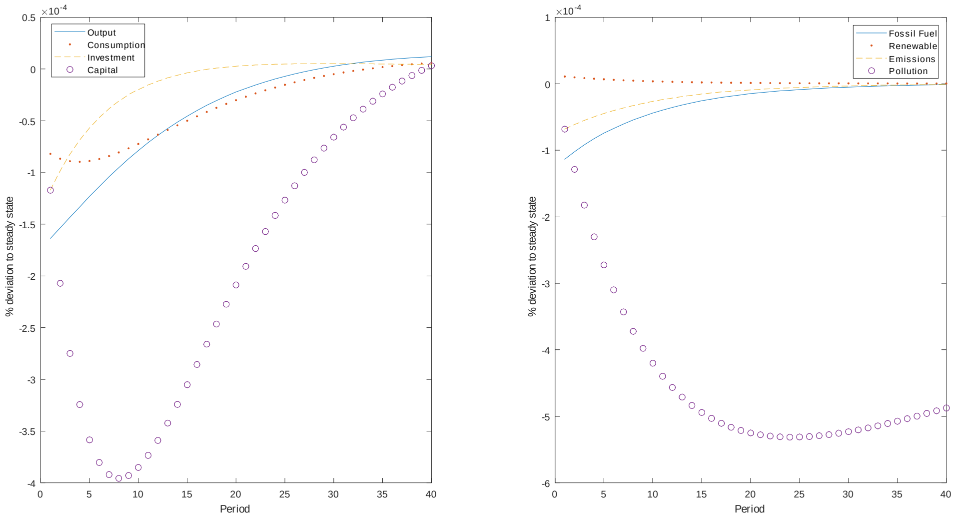

Figure 6: Impulse-response functions to a fossil fuel price shock

6. Concluding remarks

This paper studies how the energy mix of renewable versus non-renewable energy sources is affected by technological and energy price shocks, and their implications for energy transition, carbon emissions, and the environment. Our starting evidence is that pollutant emissions varies widely depending on the energy source, and hence, alternative technological and price shocks have different effects on the environment depending on how they change the energy mix. The paper investigates those links using an E-DSGE model where final good sector productivity is negatively affected by pollutant emissions. The model uses a three factor production function: capital, labor, and energy, where two energy sources, fossil fuel and renewable energy, are considered.

The summary of the main results of the paper are the following. First, energy consumption and emissions are procyclical, but emissions per unit of output are countercyclical, consistent with empirical evidence. This is a direct consequence of the decline in energy intensity as economic activity expands. We find that a neutral technological shock provokes an expansion of the economic activity, a higher energy consumption, increasing both fossil fuels and renewable energy consumption, and generates more emissions, resulting in a harmful impact on the environment. Second, an energy efficiency shock provokes a rebound effect above 100% (the so-called backfire effect), resulting in an increase in emissions, indicating that energy efficiency environmental policies must include additional instruments to avoid non-anticipated negative effects on the environment. By contrast, emissions are reduced in the case of an emission efficiency technological improvement, and in the case of an increase in the price of the "dirty" energy, although the transmission mechanisms are different. The emission efficiency technological shock reduces carbon emissions but also increases energy consumption and output. However, a fossil fuels price shock also reduces emissions but at the cost of reducing output. As pointed out by Acemoglu *et al.* (2012), if "dirty" and "clean" energy are sufficiently substitutable, then there is room for implementing directed technical change policies under alternative environmental policies in order to redirect technical change to renewable energy sources and reduce environmental damage.

In our theoretical framework, energy prices are assumed to follow an exogenous stochastic process but in which the social relative price of renewable versus nonrenewable energy is endogenously determined by the cost of pollutants measured as foregone output. The underlying mechanism works as follows. Emissions are generated by the use of fossil fuel in the energy production activity. As fossil fuel consumption increases, the pollution externality cost also increases, reducing productivity. This makes the use of "dirty" energy more expensive. The rise in the relative price, measured in units of final output, provokes a substitution of fossil fuels energy by renewable energy, resulting in a decline in the level of emissions. Therefore, it is worth noting to study the implications for the design and implementation of environmental policies driving energy prices, increasing "dirty" energy prices relative to clean energies in order to reduce the demand of fossil fuel and the transition to renewable energies.

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Appendix A. Technical Appendix: First Order Conditions

The central planner maximization problem can be defined using the following Lagrangian auxiliary function:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \omega \frac{L_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} \right] \qquad \text{(A.1)} \\
& -\lambda_{1,t} \left[\begin{array}{c} C_t + K_{t+1} - (1-\delta_k)K_t + P_{o,t}O_t + P_{s,t}S_t \\ -A_t \exp(-\phi Z_t) K_t^{\alpha_1} \left(D_t \left[\mu O_t^{\frac{\sigma-1}{\sigma}} + (1-\mu) S_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right)^{\alpha_2} L_t^{1-\alpha_1-\alpha_2} \end{array} \right] \\
& -\lambda_{2,t} \left[Z_{t+1} - (1-\delta_z) Z_t - \eta B_t O_t \right]
\end{aligned}
$$

The first order conditions for the consumer maximization problem are:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &: \; \beta^t C_t^{-\gamma} - \lambda_{1,t} = 0 && \text{(A.2)} \\
\frac{\partial \mathcal{L}}{\partial L_t} &: \; -\beta^t \omega L_t^{\frac{1}{v}} + \lambda_{1,t}(1-\alpha_1-\alpha_2)\frac{Y_t}{L_t} = 0 && \text{(A.3)} \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &: \; -\lambda_{1,t} + \lambda_{1,t+1}\left[(1-\delta_k) + \alpha_1 \frac{Y_{t+1}}{K_{t+1}} \right] = 0 && \text{(A.4)} \\
\frac{\partial \mathcal{L}}{\partial O_t} &: \; -\lambda_{1,t}\left[P_{0,t} - \alpha_2 \mu \frac{Y_t O_t^{\frac{-1}{\sigma}}}{\mu O_t^{\frac{\sigma-1}{\sigma}} + (1-\mu) S_t^{\frac{\sigma-1}{\sigma}}} \right] + \lambda_{2,t}\eta B_t = 0 && \text{(A.5)} \\
\frac{\partial \mathcal{L}}{\partial S_t} &: \; -\lambda_{1,t}\left[P_{s,t} - \alpha_2 (1-\mu) \frac{Y_t S_t^{\frac{-1}{\sigma}}}{\mu O_t^{\frac{\sigma-1}{\sigma}} + (1-\mu) S_t^{\frac{\sigma-1}{\sigma}}} \right] = 0 && \text{(A.6)} \\
\frac{\partial \mathcal{L}}{\partial Z_{t+1}} &: \; -\lambda_{1,t+1}\phi Y_{t+1} - \lambda_{2,t} + \lambda_{2,t+1}(1-\delta_z) = 0 && \text{(A.7)}
\end{aligned}
$$

From first order conditions we obtain the following values for the Lagrangian multipliers:

$$
\lambda_{1,t} = \beta^t C_t^{-\gamma} \tag{A.8}
$$

$$
\lambda_{2,t} = \frac{\beta^t C_t^{-\gamma} \left[P_{0,t} - \alpha_2 \mu \frac{Y_t O_t^{\frac{-1}{\sigma}}}{\mu O_t^{\frac{\sigma-1}{\sigma}} + (1-\mu) S_t^{\frac{\sigma-1}{\sigma}}} \right]}{\eta B_t} \tag{A.9}
$$

Whereas $\lambda_{1,t}$ is the shadow price of consumption, $\lambda_{2,t}$ is the shadow price of fossil fuel consumption. Substituting in first order conditions for the central planner problem we obtain the equilibrium condition for the working hours:

$$
L_t^{\frac{1}{v}+1} = \frac{(1-\alpha_1-\alpha_2)Y_t}{C_t^{\gamma}\omega} \tag{A.10}
$$

The optimal consumption path is given by:

$$\frac{C_{t+1}^{\gamma}}{C_t^{\gamma}} = \beta\left[(1-\delta_k) + \alpha_1\frac{Y_{t+1}}{K_{t+1}}\right] \tag{A.11}$$

The optimal stock of pollution is defined by the following first order condition:

$$\lambda_{1,t+1}\phi Y_{t+1} = \lambda_{2,t+1}(1-\delta_z) - \lambda_{2,t} \tag{A.12}$$

By substituting the Langrange's multipliers, we obtain optimal quantities for the use of fossil fuel and renerwable energy:

$$\begin{aligned} Y_{t+1} \ &= \ \frac{\left[P_{0,t+1} - \alpha_2\mu\frac{Y_{t+1}O_{t+1}^{\frac{-1}{\sigma}}}{\mu O_{t+1}^{\frac{\sigma-1}{\sigma}}+(1-\mu)S_{t+1}^{\frac{\sigma-1}{\sigma}}}\right](1-\delta_z)}{\phi\eta B_{t+1}} \\ &\quad - \frac{C_t^{-\gamma}\left[P_{0,t} - \alpha_2\mu\frac{Y_tO_t^{\frac{-1}{\sigma}}}{\mu O_t^{\frac{\sigma-1}{\sigma}}+(1-\mu)S_t^{\frac{\sigma-1}{\sigma}}}\right]}{\beta C_{t+1}^{-\gamma}\phi\eta B_t} \end{aligned} \tag{A.13}$$

$$P_{s,t} = \alpha_2(1-\mu)\frac{Y_tS_t^{\frac{-1}{\sigma}}}{\mu O_t^{\frac{\sigma-1}{\sigma}} + (1-\mu)S_t^{\frac{\sigma-1}{\sigma}}} \tag{A.14}$$