Constitutions, Federalism, and National Integration

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Abstract

A constitution defines a vertical and horizontal division of power. The vertical division is the power that regions transfer to the national government; the horizontal division is the relative power of each region in the national legislature. We explore what combinations of vertical and horizontal division of power arise when forming a nation or a union, and which combinations reduce the risk of dissolution. We present a new model of political bargaining among heterogeneous regions that design a common constitution. We show that scale economies translate into higher centralized systems, whereas cultural and political heterogeneity translate into more decentralized federal systems. Interestingly, the constitutions that minimize the risk of secession compensate with proportionally more power in the national legislature those regions that have less to gain economically from national integration. Such division of power contrast with other widely used that assign equal power to each region or power in proportion to population size. Our results suggest that compensations in the constitutional process need not be accomplished through direct transfers; it can be accomplished through the legislative process.

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Keywords: Nation formation; Federalism; Decentralization; Secession; Power division

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1   Introduction

Most nations in the world today join together many local cultures and heterogeneous economies under a common government and constitution. The regions' disparate cultures and distinct interests are a source of strength for a nation, as there are economic gains to nation-formation. However, the regions are also an existential threat to the nation. Local areas have their own languages and customs; they have their own legal systems; they have their own distinctive economies deriving from their people, their natural resources, and their histories. The nation's laws must accommodate those differences, or risk regional political schisms, civil conflict, and possible dissolution. Examples around the world abound where cultural differences have produced political schisms, the Flemish-Walloon divide in Belgium, the Catalans and Basques in Spain, the Slovenians in the former Yugoslav federation, the Scots in UK or the Quebecois in Canada among many others.

This paper presents a positive theory of endogenous constitutional design in plural societies. Specifically, we contribute to the literature on federalism, nation formation and constitutional economics by clarifying the link between these two key features of constitutions: federalism and legislation. The degree of federalism, chosen at the constitution, defines the vertical relation of the national government with each of the regions, that is, the degree to which the nation can impose laws on each region. The legislative power defines the horizontal relation among regions, that is, which laws or customs will be used as the national law, or the influence of each region in forming the national law. A constitutional arrangement defines the relation of the regions with the nation. Such relation is translated into a vertical and horizontal division of power. Our approach provides positive predictions about what constitutional arrangements will be reached in federal systems, and relevant normative implications concerning the threat of secessionism grounded on the participation of the regions in the national policy.

We present a model of the politics and economics of centralization and decentralization. We consider a single nation with several regions. The economy of each region achieves a level of outcome and growth if it is a separate nation on its own, and, in expectation, it can achieve a higher level of outcome and growth if it becomes part of the nation and its economy. For example, it is commonly believed that the nations of Europe have enjoyed higher growth by virtue of being part of the European Union and that the states of the United States have enjoyed higher growth by virtue of being part of the United States instead of being 50 separate countries.[1]

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[1]The economies of scale in big countries are recognized by several authors (see, e.g.,

The costs of joining the nation are borne through the laws enacted and the degree to which those laws conflict with the regional laws and customs. Several examples clarify what is meant by such a cost. First, if a region speaks a language that is very different from the nation, then adopting the national language to conduct business, education, government transactions, and so forth would be very costly to the region. A second example concerns economic structures. If a region is heavily agricultural and has local laws that favor debtors but the other regions and the national laws are favorable toward financial markets, then a national law that favors creditors would be very costly to the people in the agricultural region. Third, financial integration can be a barrier. A region may have its own currency and banking institutions and want to manage monetary policy in a way that is attuned to the fluctuations in the local economy rather than subject itself to the economic fluctuations and monetary policy favored by the rest of the nation. Such is the case with monetary policy and the European Union, and was a problem in the early decades of the United States.

We view the integration of regions into a nation as a bargaining game among distinct regions with the possibility of exit. We formalize the process of nation formation in three phases, namely, the constitution stage, the legislation stage, and the integration stage. At the constitutional stage, the regions must balance the benefits and costs of being in the nation when it chooses what relation it is to have with the national government. At the legislation stage, the national law is enacted and it determines how the costs of integration are divided among the regions. Finally, after the political process is complete, regions decide whether to remain in the nation.

Our model reveals that scale economies translate into higher centralized systems, whereas cultural heterogeneity translates into more decentralized federal systems. We show that the constitution that best insures the union against dissolution gives those regions that benefit more from national integration proportionally less power in the national legislature. Notably, such distribution of political power in legislatures is a form of credible commitment of the majority culture to the minority culture in a heterogeneous society. This criterion sharply contrast with other widely used divisions of power, which either dictate equal division among regions, or a division in proportion to population size.

Our research speaks directly to the voluminous literature on federalism (Elazar 1968, 1987; Amoretti and Bermeo, 2004) and constitutional economics (Voigt, 1997). Extensive theoretical and empirical research examines fiscal federalism. Musgrave (1959) and Oates (1972) describe federalism as

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Alesina and Spolaore, 1997; Le Breton and Weber, 2003).

a fiscal arrangement for the joint production of public goods. According to Oates (1999), higher degrees of centralization and large federal nations achieve a greater degree of efficiency, and yet, over time in the United States, the central government has turned back significant portions of federal authority to the states.

Political scientists and economists have viewed federalism primarily as a common pool problem. There are diverse benefits associated to federalism and decentralization of power: i) It allows tailoring policies to the particular preferences of the regions (Musgrave, 1959; Oates, 1972, 1999); ii) It provides better information to policymakers (Hayek 1945; Dewatripont and Maskin, 1995; Qian and Weingast, 1997); iii) It increases political participation (Inman and Rubinfeld, 1997); iv) It provides capacity to sustain an efficient system of markets due to the limited access of local governments to credit (Weingast, 1995; Brennan and Buchanan, 1980); v) It allows for flexibility in dealing with multiple ethnicities within a nation (Bednar, Eskridge and Ferejohn, 1999); vi) It enhances electoral accountability (Bardhan, 2002; Hindriks and Lockwood, 2009). From an empirical perspective Arzaghi and Henderson (2005) show that economic growth and country size increase decentralization. By contrast, several authors justify that centralization, over decentralization, internalizes positive externalities among regional specific projects and it produces efficiency gains (Oates, 1972; Lockwood, 2002; Besley and Coate, 2003), or that centralization over decentralization reduces rent extraction (Boffa, Piolatto and Ponzetto, 2016). Also, empirical research has found that centralization enhances economic performance (Davoodi and Zou, 1998; Rodden 2004). Hence, there is a tension between the higher growth potential with centralization and unitary authority, and the greater flexibility associated with more decentralized systems. Our model incorporates the two forces: decentralization reduces the political cost of joining together heterogenous region, and centralized federalism allows higher economic growth. A third force at work is the threat of exit. A nation imposes costs on local areas that grow as the nation expands and becomes more heterogeneous (Bolton, Roland and Spolaore, 1996; Alesina and Spolaore 1997; Alesina, Spolaore and Wacziarg 2000, Bolton and Roland 1997; Chu 2010; Flamand 2019). If these costs are too large, the nation may either break apart, or not form to begin with. Bolton and Roland (1997) argue that economic inequality can lead to dissolution of a nation. De Figueiredo and Weingast (2005) propose the idea of self-enforcing federalism by which, the central government designs a coordination device with penalties to avoid shirking or exiting by the subunits. Le Breton and Weber (2003) argue that different preferences over the level of the common or public good can lead some regions to secede. Desmet et al. (2011) refer to the costs of greater cul-

tural heterogeneity as increasing the probability of secession, and Wittman (2000) models the political cost of diverse preferences.

Building on these models, our analysis assumes that there are increasing costs associated with higher heterogeneity across regions, and these costs create the potential for secession. Much of the past literature argues that the costs imposed on each region by a common national law must be compensated by money-transfers (Bolton and Roland, 1997; Le Breton and Weber 2003; Haimanko, Le Breton and Weber, 2005). In contrast to these contributions, we show that the degree of decentralized federalism and the division of national power (i.e., the horizontal and vertical division of power) shape the ability of the nation to accommodate cultural heterogeneity, and minimize the possibility of dissolution. This is an important point suggested by political scientist (Riker, 1964; Filippov, Ordeshook and Shvetsova, 2004) that, as far as we know, has been disregarded in the economic literature.

Other contributions study how the constitutional design affects legislation and the durability of a union. Aghion, Alesina and Trebbi (2004) propose that individuals in a country, behind the veil of ignorance, select the majority required to pass legislation. Barberá and Jackson (2004) analyze self-stable constitutions, i.e., constitutions that once approved, the society will not vote to change it. Rather than studying the voting rule (e.g., majority rule or super majority rule) that is adopted in the constitution, we study the division of legislative power. In this respect, our analysis connects the literature on constitutions, legislation and federalism to the study of power indexes and voting weights, that among others, examines the divisions of power in the European Union (Laruelle and Widgrén, 1998; Barberá and Jackson, 2006).

Based on the existing literature, our analysis provides new lens to analyze integration in terms of how the union contributes to each partner's economic growth, and how the division of power can arise from the need to compensate some regions in order to gain their entry into the union and to hold the nation together in the future.

The structure of the paper is as follows. Section 2 presents the model. Section 3 the main results. Section 4 provides an analysis of social welfare. Finally, the last section offers the conclusion. All the proofs are in the Appendix.

2   A model on nation formation

There are two regions k and l. One region may be thought of as a single region and the other as all other regions in a country; or, the regions may be thought of as separate countries that might join another to form a new

nation.

When forming a nation (or a union), regions need to agree on a constitution that determines the degree of centralization, denoted by $\beta \in [0, 1]$, and the power of each region within the union denoted by $a \in [0, 1]$ with $a_k = a$ indicating the legislative power of region k and $a_l = 1 - a$ that of region l. We can interpret a as congressional or parliamentary apportionment.[2] We refer to every combination (β, a) as a *constitutional arrangement*.

The degree of centralization has two effects. On the one hand, β influences the amount of growth or outcome that the regions can achieve, for example, more centralization can imply common national set of laws that creates a more integrated economy and translates into higher growth. On the other hand, β indicates the degree to which the regions follow a common national law instead of their own laws. Depending on β, three types of constitution can be written:

A *centralized* federal system, in which the national laws have primacy over the regional laws. In this case $\beta = 1$, which means that regions operate strictly under the nation's laws.

A *decentralized* federal system, in which national laws and regional laws are each in operation within each region, and the degree of decentralization is determined by the level of national control versus regional control. In this case, $1 > \beta > 0$.

An *autonomous* system, which grants the regional laws primacy. In this case $\beta = 0$, which means that regions operate under their own laws.

The legislative power of the regions, a_k and a_l, determine the national law. The higher the legislative power of a region, the stronger the influence of the region when deciding the national law, and therefore, the less costly the integration into a nation is for the region.

The economic implications of nation formation

The level of income that the people in a region achieve depends on their relationship to the rest of the nation. We distinguish two types of economic benefits derived from forming a nation; there are fixed benefits and variable benefits with the latter being increasing in the degree of centralization. For each region, $F_j > 0$ denotes the fixed benefits or immediate benefits of being in the nation, and g_j denotes the maximal variable benefits or growth level associated with the future benefits of being in the nation. The level of income achieved by each region j is denoted by Y_j and depends on the degree of

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[2]For example, in the United States, Congressional apportionment is addressed in the Constitution.

centralization:

$$Y_j(\beta) = \underbrace{Y_j^0}_{\text{Regional income}} + \underbrace{F_j}_{\text{Fixed union benefits}} + \underbrace{\beta g_j}_{\text{Variable union benefits}}. \tag{1}$$

When $\beta = 0$, the region operates in an autonomous manner within the nation and its level of income equals $Y_j^0 + F_j$, that is, the level of income achieved in an autonomous way Y_j^0, plus the fixed benefits associated to the nation alliance. In this case, the regions do not benefit from the higher degree of economic integration achieved in a centralized or decentralized system. When $\beta = 1$, the regions achieve the highest possible joint outcome because they eliminate the cost associated to different laws in different regions. The values of β between 0 and 1 represent different levels of integration of the region into the nation's economy. If $g_j < 0$, the union operating under a centralized or decentralized form, provides variable losses instead of benefits.

The political implications of nation formation

Each region, were it autonomous, prefers laws that best suit the local culture and customs. The laws can be treated (but not restricted to) as a one-dimensional policy space, such as left versus right or pro-debtor versus pro-creditor. Let Z be the space of laws, and let $z \in Z$ be a specific law. The values z_l and z_k denote the most preferred laws for each regional government. For example, if this is a matter of language policy, each region wants its language to be the national language.

When a nation is formed, all regions join together in the national legislature and enact a national law $z^L \in Z$. Given a division of legislative power a, the national law is a policy compromise. Specifically, the national law will be a point between the most preferred policies of the two regions, z_l and z_k. The exact value of the compromise depends on the relative power of the regions inside the legislature, that is:[3]

$$z^L(a) = a_k z_k + a_l z_l. \tag{2}$$

The resulting law used by the regions depends on the degree of centralization. In a centralized system, the regions implement the national policy. In a decentralized system, the regions implement a law that is the combination of the national law and the most preferred regional law for the region. Let z_j^L denote the enacted regional law that depends on the specific constitutional

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[3]Both in a majoritarian and in a proportional representation systems, elected members of the majority party (or coalition) represent different regional constituencies, and the national law emerges as a compromise of heterogeneous regional preferences.

arrangement (β, a) :

$$z_j^L(\beta, a) = \beta z^L(a) + (1 - \beta)z_j. \tag{3}$$

In words, regions adjust their regional laws to accommodate the national law depending on the degree of centralization. Note how $\beta = 1$ implies that the national law supersedes the regional laws, and the regions use the national law, whereas $\beta = 0$ implies that the regional law supersedes the national law, and the regions use their own regional laws.

Regional preferences

The preferences of each region over constitutional arrangements are increasing in outcome and decreasing in political cost. Regions know the fixed gains derived from the union F_j and are uncertain about the variable benefits g_j. The variable benefits for each region j are distributed according to a distribution function with average $\bar{g}_j$. The political cost is measured by the quadratic distance between the most preferred law for the region z_j and the enacted regional law z_j^L.[4] Both level of outcome and political cost are measured in monetary units, and the marginal cost per unit of political distance is normalized to one.

The preferences of the regions over constitutional arrangements (β, a) are represented by the following utility function

$$u_j(\beta, a) = \underbrace{Y_j(\beta)}_{\text{Economic gains}} - \underbrace{(z_j^L(\beta, a) - z_j)^2}_{\text{Political cost}} \text{ for all } j \in \{k, l\}. \tag{4}$$

Substituting expressions (1), (2) and (3), regions maximize the following expected utility[5]

$$E[u_j(\beta, a)] = Y_j^0 + F_j + \beta \bar{g}_j - \beta^2(1 - a_j)^2(z_k - z_l)^2. \tag{5}$$

By Expression (5) and provided that $\bar{g}_j > 0$, the degree of centralization β produces a positive effect through economic growth, but a negative political cost since it increases the distance between the most preferred regional policy and the national policy. The legislative power affects the political cost of the union: the higher a_j, the closer the national law is to the most preferred regional law.

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[4]Quadratic distances imply that each unit of political cost generates less disutility the closer it is to z_j. Quadratic distances, instead of city block distances, provide a continuous range of most preferred levels of centralization as a function of legislative power.

[5]First, substituting z_j^L yields $E[u_j(\beta, a)] = E[Y_j(\beta)] - \beta^2(z^L(a) - z_j)^2$. Second, note that $(z^L(a) - z_k)^2 = (1 - a)^2(z_l - z_k)^2$ and $(z^L(a) - z_l)^2 = a^2(z_k - z_l)^2$. By substituting $a = a_k = 1 - a_l$, we deduce Expression (5).
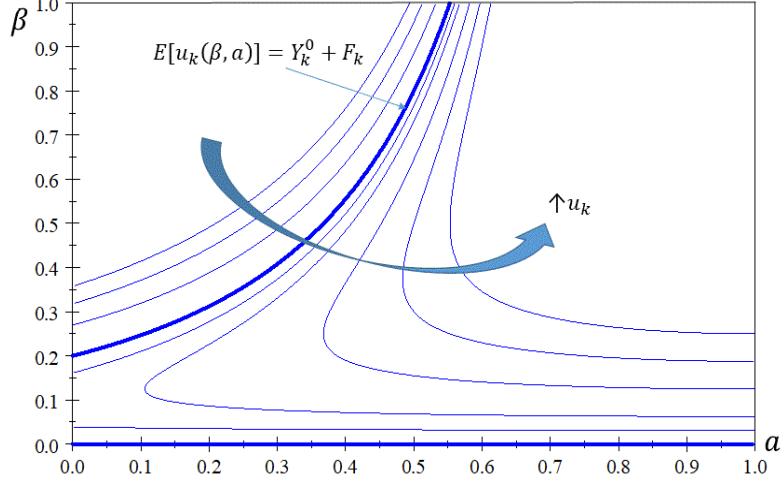
Figure 1: Indifference curves of region k over legislative power a and centralization β

We illustrate in Figure 1 the preferences of region k over levels of legislative power $a = a_j$ (horizontal axis), and degrees of centralization β (vertical axis).[6] The thick arrow that crosses the indifference curves indicates higher utility levels. Note how preferences are monotonic in the level of legislative power a_j, but only when the level of legislative power is sufficiently high, the region prefers a more centralized federal system. The bold indifference curve represented in Figure 1 provides the (expected) utility level $Y_k^0 + F_k$, it includes all the combinations (β, a) such that the variable union benefits are equal to the political cost, and where $\beta = 0$ is a particular case.

For each level of legislative power a_j, we calculate the unique most preferred degree of centralization for each region β_j^* solving:

$$\underset{\beta \in [0,1]}{Max} \quad E\left[u_j(\beta, a)\right].$$

The first order condition for an interior solution yields $\bar{g}_j - 2\beta(1 - a_j)^2(z_k - z_j)^2 = 0$. Thus, the most preferred level of centralization can be a corner solution in two cases: first, when $\bar{g}_j \leq 0$ or $\bar{g}_j < 2(1 - a_j)^2 [z_k - z_l]^2$, which implies $\beta_j^* = 0$ and, second, when $\bar{g}_j > 2(1 - a_j)^2(z_k - z_l)^2$, which implies $\beta_j^* = 1$. In the first situation, the variable benefits are negative or too low relative to the (marginal) political cost and the region's most preferred regime

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[6]This figure and the subsequent figures consider the exogenous parameter values $\bar{g}_j = .4$, $[z_k - z_l]^2 = 2$.

is an autonomous system. In the second situation, the variable benefits are very high in relation to the (marginal) political cost and the region's most preferred regime is centralization. For the remaining cases, the most preferred levels of centralization are interior solutions with $0 < \beta_j^* < 1$ and such that:[7]

$$\beta_j^*(a) = \frac{\bar{g}_j}{2(1-a_j)^2(z_k-z_l)^2} \text{ for all } j \in \{k,l\}. \qquad (6)$$

More legislative power for a region (e.g., $a_k = a$) implies less legislative power for the other (e.g., $a_l = 1 - a$) and, generally, the demand of centralization for the regions may not align: the region with more legislative power prefers more centralization, and the region with less legislative power prefers less centralization. We illustrate in Figure 2 the most preferred levels of centralization for the two regions. These functions show opposed slopes: $\beta_k^*(a)$ is increasing in a, whereas $\beta_l^*(a)$ is decreasing in a, and for the two regions, a sufficiently high legislative power implies that a centralize regime is preferred over every other federal system.
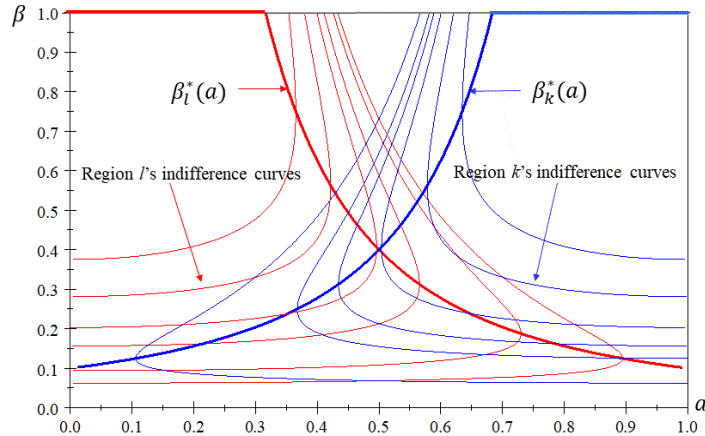

Figure 2: The most preferred levels of centralization
for each region: β_k^* and β_l^*

The mechanism of nation formation

The decision to join and integrate into a nation is based on the net benefits that the union provides to the regions. We divide the process of nation formation into three stages:

Constitutional Stage: at which regional governments select a constitutional arrangement (β, a).

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[7]The second order condition satisfies $\frac{\partial^2 u_j}{\partial \beta^2} = 2(1-a_j)^2(z_k-z_j)^2 < 0$ for $a_j \neq 1$.

Legislative Stage: at which the national law and the regional laws are enacted, and where regions have the option of vetoing the national law.

Integration Stage: at which regional governments observe the realization of the union benefits and decide whether to maintain or leave the union.

In line with De Figueiredo and Weingast (2005), at the legislative stage, we add the possibility of non-compliance with or veto of the national law by one of the regions. When a region vetoes the national law, the union persists but there is no national law and the regions enact their regional laws. Besides, if a region vetoes the national law, the nation does not benefit the higher degree of economic integration achieved in a centralized or decentralized system.[8]

The bargaining struck in the constitutional phase depends on many factors and different nations can follow different negotiation protocols. Instead of proposing a particular bargaining protocol, we stick to two criteria when figuring out what constitutional arrangements the regions accept: (ex-ante) *Pareto efficiency,* and *veto-proof.* The proposed criteria define a set of constitutional arrangements that the regions can select. The two principles state the following:

First, a constitutional arrangement (β, a) satisfies *ex-ante Pareto efficiency (PE)* if there is no other constitution (β', a') such that

$$E\left[u_j(\beta', a')\right] \geq E\left[u_j(\beta, a)\right] \text{ for every } j \in \{k, l\}, \tag{7}$$

with the above inequality being strict for at least a region. From a positive perspective, we interpret that no region accepts a constitutional arrangement when there is another arrangement with which it can improve without making its partner worse off.

Second, the *veto-proof* requirement holds that the regions will abide by the constitutional and legislative arrangement after the national law is passed. As already argued, when a region vetoes the national law, each region operates under its own most preferred regional law. We say that a constitutional arrangement (β, a) is *veto-proof (VP)* when no regional government improves vetoing the national law, i.e.,

$$E[u_j(\beta, a)] \geq Y_j^0 + F_j \text{ for every } j \in \{k, l\}. \tag{8}$$

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[8]A clear example of a failure to adopt a national law is the bankruptcy law in the United States during the 19th Century. Even though the Constitution explicitly gives Congress the power to pass bankruptcy laws, Congress did not create a sustainable bankruptcy law until the late 19th Century. The differences between the agricultural (debtor) regions and the industrial (creditor) regions made it difficult to find common ground on a law. It passed and then repealed in the 1840s and 1860s.

Note that $Y_j^0 + F_j > Y_j^0$, so that VP is a strong version of participation constraint by which, regions only accept forming a union when, in expectation, this union entails higher payoffs than remaining separated.[9] From a positive perspective, we interpret that regions anticipate the possibility of vetoing the national law and only those constitutions where the regions show ex-ante incentives to cooperate in the national legislature are accepted in the first place.

We say that (β, a) is an *acceptable constitutional arrangement* if it satisfies PE and VP.

At the integration stage, regions observe the realization of the variable union benefits and compare the utility of maintaining the union against the utility derived if forming its own separated nation, that is, regions check ex-post participation constraint. Given the accepted constitutional arrangement, denoted by (β_c, a_c), we say that the nation reaches integration when

$$u_j(\beta_c, a_c) \geq Y_j^0 \text{ for every } j \in \{k, l\}. \tag{9}$$

Otherwise, we say that there is secession.

Note how the proposed criterion to enter the union is more restrictive than the criterion to exit. While a utility below $Y_j^0 + F_j$ constraints the entry into a union, a utility below Y_j^0 provides enough incentives to dissolve the union. We can interpret that the right to veto a national law is an action that expresses a negative concern about the union. Specifically, a region can avoid a benefit below $Y_j^0 + F_j$ by vetoing the national law.[10]

3   Results

Examination of the PE and VP conditions allow us to characterize the constitutional arrangements and legislative behavior that will arise, and their relationship to the economic benefits and costs that each region will experience. In this section, we describe the set of constitutional arrangements that the regions can accept (Proposition 1). We then provide a comparative statics analysis that reveals how the set of acceptable constitutional arrangements

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[9]There is a large variety of non-cooperative approaches to bargaining theory that provide equilibrium agreements satisfying Pareto efficiency and participation constraint, among others, see Rubinstein (1982).

[10]Hirschman (1970) argues that when the members of a nation (or an organization) perceive that the nation is demonstrating a decrease in benefits, they can exit or they can voice (in an attempt to improve the relationship). We interpret that the interval $\left[Y_j^0, Y_j^0 + F_j\right]$ provides *voice* to the region that can exert its right of repealing the national law (e.g., Catalan political parties vetoed the Spanish national budget in February 2019 to express their discontent).

modify as a function of changes in the primitives of the model (Proposition 2). Finally, we examine the third phase of integration (Proposition 3).

The legislative and constitutional stages

Given some fixed utility level u, the set of PE arrangements solves the following problem:

$$\underset{\beta\in[0,1],a\in[0,1]}{Max}\quad E\left[u_l(\beta,a)\right]$$
$$\text{s.t.}\quad E\left[u_k(\beta,a)\right]\geq u. \tag{10}$$

We deduce the following result:

Lemma 1: *Every Pareto Efficient constitutional arrangement is defined by* $(\beta,a)=(\beta^*(a),a)$ *where*

$$\beta^*(a)=\left\{\begin{array}{rl} 0 & \text{if } a_k\bar{g}_k+a_l\bar{g}_l\leq 0 \\ \frac{a_k\bar{g}_k+a_l\bar{g}_l}{2(z_k-z_l)^2a_ka_l} & \text{if } 0<a_k\bar{g}_k+a_l\bar{g}_l<2(z_k-z_l)^2a_ka_l \\ 1 & \text{if } a_k\bar{g}_k+a_l\bar{g}_l\geq 2(z_k-z_l)^2a_ka_l \end{array}\right. . \tag{11}$$

We deduce that depending on the magnitude of the expected variable benefits $\bar{g}_k,\bar{g}_l$ and the political heterogeneity of the regions z_k-z_l, the three types of constitutions − autonomous system, decentralized and centralized − can be part of an efficient constitutional arrangement.

When $0<\beta^*(a)<1$, efficiency is characterized by the tangency between the slopes of the regions' indifference curves. The thick U-shaped curve in Figure 3 depicts the set of PE constitutional arrangements described in Lemma 1. We observe how Pareto efficiency implies lower levels of centralization when legislative power is more equally divided among the regions, and a highly centralized federal system when one of the region holds strong legislative power.

We derive three observations:

First, the set of efficient constitutional arrangements lies in between the most preferred levels of centralization for the two regions (the dash lines in Figure 3), defined by the expressions in (6): $\beta_l^*(a)$ and $\beta_k^*(a)$. In fact, when $0<\beta_l^*(a)<1$ and $0<\beta_k^*(a)<1$, then $\beta^*(a)=a_k\beta_l^*(a)+a_l\beta_k^*(a)$, that is, the PE condition defines a compensating mechanism by which the lower the legislative power assigned to a region (say $a_l<a_k$), the higher the impact of this region in the corresponding level of centralization ($\beta^*(a)$ is closer to $\beta_l^*(a)$ than $\beta_k^*(a)$). When the division of legislative power is very unequal, there is no possibility of activating such compensation mechanism and the PE arrangements unequally benefit one region over the other.

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Second, we can show that the Nash Bargaining solution with equal bargaining weights (Nash, 1953) is one of the PE arrangements characterized by the division of power $a_k = \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$, that is, a division inversely proportional to the the expected variable benefits of each region.[11] Notably, this solution has been widely used as a predictor to solve economic bargaining problems (Binmore et al., 1986).

Third, there is a minimal level of efficient centralization $a_k = \hat{a}$ satisfying $\frac{\partial \beta^*(\hat{a})}{\partial a} = 0$.[12] Solving for this value yields $\hat{a} = \frac{\bar{g}_l^{1/2}}{\bar{g}_l^{1/2} + \bar{g}_k^{1/2}}$ or, in relative terms $\hat{a} = \frac{1}{1 + \bar{g}^{1/2}}$, where $\bar{g} = \frac{\bar{g}_k}{\bar{g}_l}$. In $\hat{a}$, the fraction of power assigned to each region is inversely proportional to the square root of the region's expected variable gains. Besides, this is easy to show that in $\hat{a}$, the demands of centralization coincide, that is $\beta_l^*(\hat{a}) = \beta_k^*(\hat{a})$.[13]
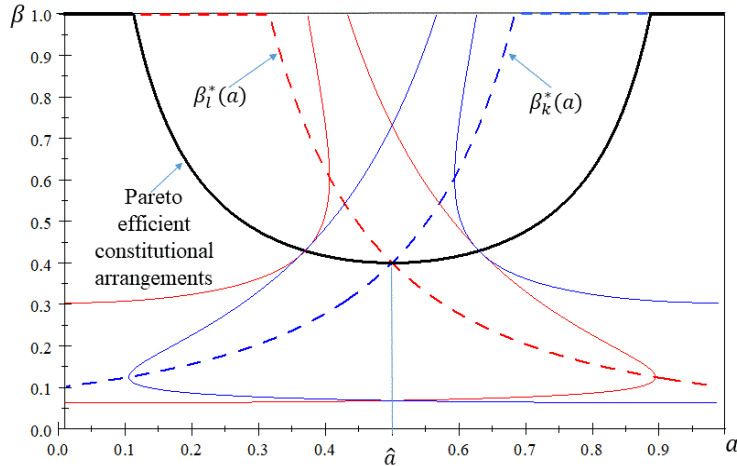

Figure 3: The U-shaped curve of the Pareto efficient constitutional arrangements

Next, we examine the VP condition by which, each region only accepts those constitutional arrangements satisfying $E[u_j(\beta, a)] \geq Y_j^0 + F_j$. Substituting (5) yields $\beta \bar{g}_l \geq \beta^2 a^2 (z_k - z_l)^2$ and $\beta \bar{g}_k \geq \beta^2 (1-a)^2 (z_k - z_l)^2$. Thus,

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[11] Let $Y_j^0 + F_j$ be each region's disagreement point. Then, solving for $E[u_k(\beta, a)] - (Y_k^0 + F_k) = E[u_l(\beta, a)] - (Y_l^0 + F_l)$ and substituting $\beta = \beta^*(a)$ yields the Nash Bargaining solution.

[12] $\frac{\partial \beta^*}{\partial a} = \frac{\bar{g}_k}{2(1-\hat{a})^2 [z_k - z_l]^2} - \frac{\bar{g}_l}{2\hat{a}^2 [z_k - z_l]^2} = 0$, and simplifying $\bar{g}_k a^2 = \bar{g}_l (1-a)^2$, from where we deduce $\hat{a}$. Since $\frac{\partial^2 \beta^*}{\partial a^2} > 0$, the second order condition is satisfied.

[13] $\beta_l^*(a) = \beta_k^*(a) \iff \frac{\bar{g}_l}{a^2} = \frac{\bar{g}_k}{(1-a)^2}$, from where $a = \hat{a}$.

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when either $\bar{g}_l \leq 0$ or $\bar{g}_k \leq 0$ or both, an autonomous system ($\beta = 0$) is the only federal system satisfying VP. For the remaining cases, we calculate which of the PE constitutional arrangements satisfy the VP condition. Region k imposes the minimal level of power that it can accept, denoted by $a_k = a_0$, and similarly, region l imposes the minimal level of power that it can accept, denoted by $a_l = 1 - a_1$. Thus, the interval $[a_0, a_1]$ represents the divisions of legislative power that the regions can accept according to the VP criterion.

Lemma 2: *The veto-proof criterion defines the lower bound (a_0) and the upper bound (a_1) of the division of power that the regions are willing to accept. In particular, if we consider that the regions are sufficiently heterogenous (specifically, if $(z_k - z_l)^2 \geq (\bar{g}_l^{1/2} + \bar{g}_k^{1/2})^2$) and $\bar{g}_l, \bar{g}_k > 0$, the regions only accept a **decentralized federal system** with a division of power defined by*

$$
\begin{aligned}
a_0 &= \tfrac{1}{3} \text{ and } a_1 = \tfrac{2}{3} \text{ when } \bar{g} = 1 \text{ and} &\text{(12)}\\
a_0 &= \tfrac{(\bar{g}(\bar{g}+8))^{1/2} - \bar{g} - 2}{2(\bar{g}-1)} \text{ and } a_1 = \tfrac{(8\bar{g}+1)^{1/2} - 3}{2(\bar{g}-1)} \text{ when } \bar{g} \neq 1.
\end{aligned}
$$

*If $\bar{g}_l \leq 0$ or $\bar{g}_k \leq 0$ or both, the veto-proof criterion implies that the regions only accept an **autonomous system**.*

The VP criterion restricts the division of legislative power that the regions can accept. For example, if the regions are symmetric, i.e., they obtain the same expected variable benefits $\bar{g} = 1$ (i.e., $\bar{g}_l = \bar{g}_k$), no region accepts a level of power below $\tfrac{1}{3}$.

Note that Lemma 2 describes the bounds a_0 and a_1 imposed by politically heterogeneous regions that do not select a centralized federal system. The more homogenous the regions are from a political perspective, the lower the political cost and therefore, the wider is the interval $[a_0', a_1']$ (where $a_0' < a_0$ and $a_1' > a_1$) that the regions are willing to accept. In the limit, when $z_k = z_l$, the VP criterion does not impose any bound on the division of power, and every division $a \in [0, 1]$ can be accepted by the regions. In the sequel, we restrict attention to those unions between *politically heterogeneous regions*, that is, unions that do not select a centralized federal system. Mathematically, Lemma 2 shows that the regions are politically heterogenous when $(z_k - z_l)^2 \geq (\bar{g}_l^{1/2} + \bar{g}_k^{1/2})^2$, i.e., when the maximal political cost is sufficiently high in relation to the variable union benefits.

The constitutional arrangements that satisfy both PE and VP are the following.

Proposition 1 *(The set of acceptable constitutional arrangements)*
Suppose that the regions are politically heterogenous and $\bar{g}_l, \bar{g}_k > 0$ then,

*the regions can accept those constitutional arrangements (β, a) that propose a **decentralized federal system** where $\beta = \beta^*(a) = \frac{a\bar{g}_k + (1-a)\bar{g}_l}{2(z_k - z_l)^2 a(1-a)}$ and a division of legislative power $a \in [a_0, a_1]$.*

*If $\bar{g}_l \leq 0$ or $\bar{g}_k \leq 0$ or both, the regions only accept an **autonomous system**.*
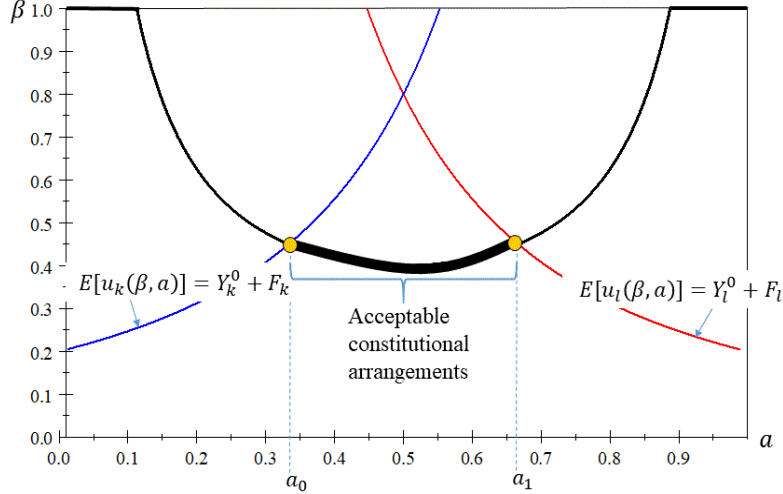

Figure 4: The set of acceptable constitutional arrangements

The proof of this proposition directly follows from Lemma 1 and 2. If the expected growth for one or both regions is zero or negative, the regions always opt for an autonomous system.[14] If the regions are politically hetero-geneous and its expected union gains are positive $\bar{g}_l, \bar{g}_k > 0$, there are several decentralized federal system that the regions can accept. We illustrate in Figure 4 the subset of the PE constitutional arrangements bounded above and below by the VP criterion. The thick black curve is the set of acceptable constitutional arrangements. Note how this set includes several degrees of centralization and not too unequal divisions of legislative power.

The divisions of legislative power a_0 and a_1, defined in Expression (12), are functions that only depend on the expected relative gains of the regions $\bar{g} = \frac{\bar{g}_k}{\bar{g}_l}$. We plot these expressions in Figure 5 and show how these functions, a_0 and a_1, are decreasing in $\bar{g}$. We plot an additional function, $\hat{a}$, that, as already argued, is the division of power that minimizes the efficient level of centralization.

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[14]Note that even when a decentralized federal system is unprofitable for a region, as-suming that the fixed benefits are positive $F_j > 0$ implies that an autonomous system benefits the region.

The graphical representation in Figure 5 reveals interesting facts:

First, the VP condition defines the minimum levels of legislative power that each region is willing to accept (a_0 and a_1) as a function of the variable union benefits.

Second, the higher $\bar{g}_j$ with respect to one's partner, the lower the minimal legislative power that each region j is willing to accept in order to pass the national law. For example, a region that enjoys twice the expected economic union gains as its partner region (e.g., $\bar{g} = 2$), is willing to pass the national law even when it holds about one fourth of the total legislative power. By contrast, a region that expects a half of the economic union gains of its partner region (e.g., $\bar{g} = .5$), needs more than two fifths of the total legislative power to pass the national law. Thus, the PE and VP requirements define a compensation mechanism by which, on average, *lower relative economic union benefits for a region need to be compensated with additional legislative power*.

Third, the dashed line in Figure 5 represents the division of power $\hat{a} = \frac{\bar{g}_l^{1/2}}{\bar{g}_l^{1/2} + \bar{g}_k^{1/2}}$ or, in relative terms $\hat{a} = \frac{1}{1 + \bar{g}^{1/2}}$, is the division of power associated to the efficient arrangement that minimizes the level of centralization. Function $\hat{a}$ is decreasing in $\bar{g}$ with $\hat{a} = \frac{1}{2}$ when $\bar{g} = 1$ and it is about equal to the midpoint between a_0 and a_1.[15] Note how the division of legislative power defined by $\hat{a}$ provides lower share of power to the region with higher expected gains.
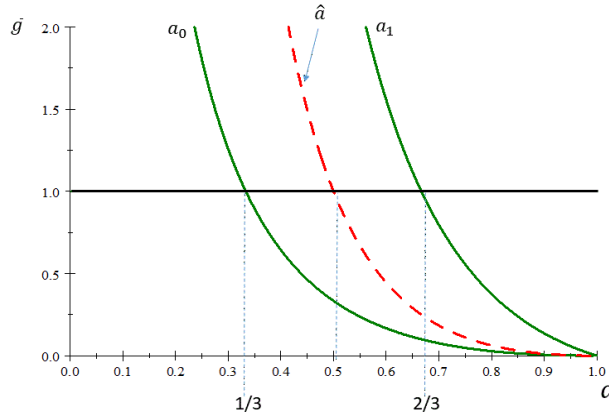

Figure 5: Divisions of legislative power that the regions can accept

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[15]Mathematically, the absolute distance between $\hat{a}$ and $\frac{a_0 + a_1}{2}$ satisfies $\left| \hat{a} - \frac{a_0 + a_1}{2} \right| <$ .035, that is, the distance between these functions is below the 3.5 percent of the total legislative power.

The following comparative statics analysis evaluates how the set of acceptable constitutional arrangements varies as a function of the expected variable benefits, the political distinctiveness between the regions, and the fixed union benefits.

Proposition 2 *(Comparative statics)*
i) If the expected variable union gains of a region increase (or decrease), the set of acceptable constitutional arrangements shows, on average, **lower legislative power** *for that region (with respect to higher legislative power) and* **higher centralization** *(with respect to lower centralization). [Left graph in Figure 6]*
ii) If the political distinctiveness between the regions decreases (or increases), the set of acceptable constitutional arrangements shows, on average, **higher centralization** *(with respect to lower centralization) and the acceptable levels of legislative power do not change. [Right graph in Figure 6]*
iii) The set of acceptable constitutional arrangements is independent of the fixed benefits.
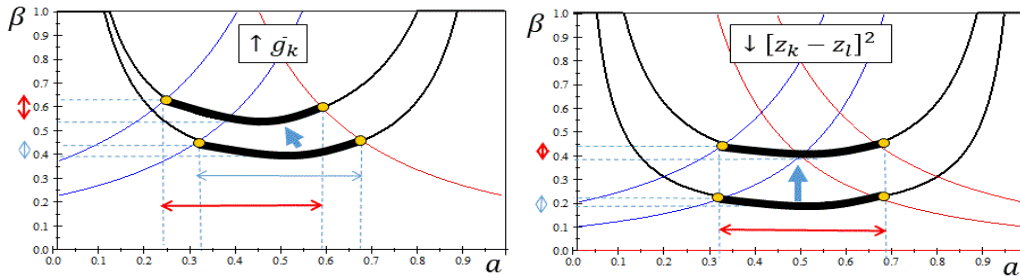

Figure 6: Variations in the set of acceptable constitutions when i) the expected variable union gains increase (left graph) and ii) the regions reduce their political distinctiveness (right graph)

When the variable union gains of a region increase, as we show in Figure 6, the indifference curve defining the VP condition moves up, and as well, the set of PE constitutional arrangements moves up. The new set of acceptable constitutional arrangements shows, on average, lower legislative power for region k, but some higher average level of centralization. When the political distance between the regions shrinks, we show in Figure 6, the set of acceptable constitutional arrangements moves up. In this last case, the indifference curves defining the VP condition move up. Thus, *higher expected variable gains and lower political distinctiveness between the regions increase the degrees of centralization that the regions can accept.* Intuitively,

higher expected variable gains make the union more profitable and, as a result, the region increases the demand for centralization. Lower political distinctiveness implies that the regions reduce the political cost associated to the union and, as a result, the regions are willing to accept higher levels of centralization. Besides, *higher expected variable gains for a region reduce the levels of legislative power that such region is willing to accept.* Intuitively, higher expected variable gains for a region imply that, on average, the region is willing to give away legislative power in exchange for a more centralized federal system.

The integration stage

At the integration stage, regions observe the benefits derived from the union. Let $\hat{g}_l$ and $\hat{g}_k$ denote the realized union gains and let (β_c, a_c) denote the accepted constitutional arrangement. Regions know with certainty if the constitution they agreed on, provides sufficient benefits as to maintain the union. Regions check ex-post participation constraint, defined as follows:

$$\text{if } F_j + \beta_c \hat{g}_j \geq (z_j^L(\beta_c, a_c) - z_j)^2, \quad \text{the region maintains the union and}$$
$$\text{otherwise,} \quad \text{the region secedes.}$$
$$(13)$$

If the regions opted for an autonomous system, we trivially deduce that regions maintain the union (note that $\beta = 0$ implies that $z_j^L = z_j$). However, if regions opted for a centralized or decentralized federal system, unexpected low realization of economic gains can motivate secession. Thus, when the realization of the union gains are below the expected union gains, that is, $\hat{g}_j < \bar{g}_j$, and the economic benefits derived from the union does not compensate its political cost, the union breaks up.

We analyze how to insure a region against the risk of secession. Our next result analyzes which acceptable constitutional arrangements reduce the possibility of dissolving the nation.

Let ε be the magnitude of an economic shock that equally affects both region, where ε is distributed (ex-ante) according to a continuous distribution function. We can express the realization of the union gains as a function of ε so that $\hat{g}_j = \bar{g}_j - \varepsilon$. The economic shock can be negative when reducing the expected variable benefits (in which case, $\varepsilon > 0$) or positive when increasing the expected variable benefits (in which case, $\varepsilon < 0$).

For every acceptable constitutional arrangement (β, a), we define a threshold shock $\tilde{\varepsilon}_j > 0$ below which region j maintains the union and above which, region j finds profitable to exit. This threshold shock is expressed as a function of (β, a), i.e., $\tilde{\varepsilon}_j(\beta, a)$, and it is implicitly defined by

$$F_j + \beta(\bar{g}_j - \tilde{\varepsilon}_j) - \beta^2(1 - a_j)^2(z_k - z_l)^2 = 0. \tag{14}$$

Note how the higher $\tilde{\varepsilon}_j$, the less likely it is that the region secedes. We deduce that a union is more resilient to secession when the constitutional arrangement accepted by the regions maximizes the value $\tilde{\varepsilon}_j$ for the two regions. Mathematically, let A be the set of acceptable constitutional arrangements then, the pair (β, a) in A that minimizes the risk of secession solves

$$(\beta, a) \in \underset{(\beta,a)\in A}{\arg\max}\min\left\{\tilde{\varepsilon}_k(\beta, a), \tilde{\varepsilon}_l(\beta, a)\right\}. \tag{15}$$

The acceptable constitutional arrangement (β, a) solving (15) defines the maximal threshold shock for the regions within those shocks that guarantee integration. Since both regions, we assume, are equally exposed to an economic shock, the above optimization problem aims at reducing the risk of secession of the most vulnerable region, the one with lower $\tilde{\varepsilon}_j$.[16] Our next result describes the solution to this problem for the case where regions derive equal fixed benefits. The next statement uses the division of power $\hat{a} = \frac{\bar{g}_l^{1/2}}{\bar{g}_l^{1/2}+\bar{g}_k^{1/2}}$ as a reference point.

Proposition 3 *Suppose that the regions derive equal fixed benefits ($F_k = F_l$). The Nash Bargaining solution, defined by $a_k = \frac{\bar{g}_l}{\bar{g}_l+\bar{g}_k}$ and the level of centralization $\beta = \frac{\bar{g}_l+\bar{g}_k}{(z_k-z_l)^2}$, minimizes the risk of secession among the efficient constitutional arrangement when the fixed benefits are null. Otherwise, there is an efficient constitutional arrangement $a_k \in \left[\frac{\bar{g}_l}{\bar{g}_l+\bar{g}_k}, \hat{a}\right]$ when $\bar{g}_k \geq \bar{g}_l$ and $a_k \in \left[\hat{a}, \frac{\bar{g}_l}{\bar{g}_l+\bar{g}_k}\right]$ when $\bar{g}_k < \bar{g}_l$, together with $\beta \in \left[\frac{\left(\bar{g}_l^{1/2}+\bar{g}_k^{1/2}\right)^2}{2(z_k-z_l)^2}, \frac{\bar{g}_l+\bar{g}_k}{(z_k-z_l)^2}\right]$ that minimize the risk of secession.[17]*

In the proof, we calculate the values $\tilde{\varepsilon}_j$ associated to each acceptable constitutional arrangement (i.e., the value of the economic shock above which one

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[16]We could generalize our result by assuming that the economic shocks do not equally affect the regions and, for instance, $\varepsilon_k = t\varepsilon_l$ where $t > 0$.

[17]If a region derives higher fixed and variable benefits ($\bar{g}_k > \bar{g}_l$ and $F_k > F_l$ or $\bar{g}_k < \bar{g}_l$ and $F_k < F_l$), we derive a similar result. We find that the constitutional arrangement that minimizes the risk of secession provides a division of power that compensates the economically disadvantaged region, that is, $a_k \in \left[\frac{\bar{g}_l^{1/2}}{\bar{g}_l^{1/2}+\bar{g}_k^{1/2}}, a_1\right]$ (when k is disadvantaged) and $a_k \in \left[a_0, \frac{\bar{g}_l^{1/2}}{\bar{g}_l^{1/2}+\bar{g}_k^{1/2}}\right]$ (when l is disadvantaged), and defines a level of centralization in between the minimal effcient level, either $\beta^*(a_1)$ (when k is disadvantaged) or $\beta^*(a_0)$ (when l is disadvantaged). The proof is in the Appendix.

region improves exiting). We then calculate the constitutional arrangements that minimize the risk of secession. Note how higher political distinctiveness between the regions and/or higher fixed benefits implies that the constitution that maximizes the resilience to secession provides higher legislative power to the region with lower variable benefits, and establishes a higher level of centralization.

In words, we find that integration implies compensating with power the economically disadvantaged region and selecting a low level of centralization. The constitutional arrangement that minimizes the risk of secession is located in between the Nash Bargaining solution and the constitutional arrangement that provides the minimal efficient level of centralization.[18] In this way, the region that benefits more from the union receives less legislative power, with a fraction of power inversely proportional to its variable union gains. Note that when the benefits of the union are equal for the regions, then $\hat{a} = \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k} = \frac{1}{2}$, and an equal division of power combined with the minimal level of centralization (within the efficient constitutional arrangements) is the constitutional arrangement that minimizes the risk of secession.
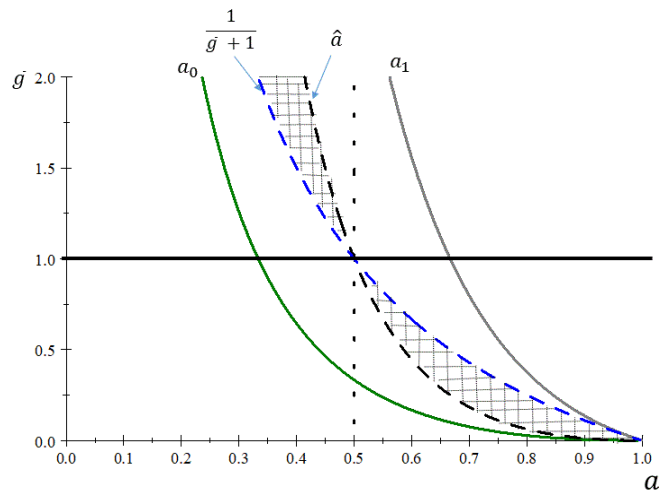
Figure 7: The divisions of legislative power
that minimize the risk of secession

For each value of the relative variable union gains $\bar{g} = \frac{\bar{g}_k}{\bar{g}_l}$, Figure 7 illustrates the interval that contains the division of power for which the regions minimize the risk of secession (when regions derive equal fixed benefits

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[18]The Nash-bargaining solution is characterized by maximizing the product of the regions' payoffs.

$F_k = F_l$). First, as explained in Figure 5, the bounds a_0 and a_1 define the minimal divisions of legislative power that the regions are willing to accept as a function of the relative variable union gains. The shadow area between $\frac{1}{1+\bar{g}}$ and $\hat{a}$ contains the division of legislative power that minimizes the risk of secession. Note how the shadow area provides more legislative power to the disadvantaged region, region l when $\bar{g} > 1$ (i.e., when $\bar{g}_k > \bar{g}_l$) and region k when $\bar{g} < 1$. Importantly, the levels of efficient centralization associated to the divisions of power in the shadow area discard the highest levels of efficient centralization, those above and close to $\beta(a_0)$ and those below and close to $\beta(a_1)$.

In the following table, we provide some numerical examples that illustrate our results in Proposition 3. Columns 1 through 4 provide different values for the levels of variable gains, the fixed benefits and political distinctiveness. All the examples in the table are such that $\bar{g}_k > \bar{g}_l$ and therefore, $\frac{\bar{g}_l}{\bar{g}_l+\bar{g}_k} < \hat{a}$. Following our result in Proposition 1, we describe in column 5 the interval $[a_0, a_1]$ defined by those divisions of legislative power included in the set of acceptable constitutional arrangements. According to Proposition 3, we describe in column 6 the subinterval defined by the division of power associated to the Nash Bargaining solution $\frac{g_l}{g_l+g_k}$ and the efficient division of legislative power that provides the minimum level of centralization $\hat{a}$. Finally, the last column describes the constitutional arrangement that minimizes the risk of secession, calculated as the solution to Expression (15). Note how the calculated division of legislative power is always included in the bounds of $[\frac{g_l}{g_l+g_k}, \hat{a}]$.

$\bar{g}_k$	$\bar{g}_l$	$F_k = F_l$	$\|z_k - z_l\|$	$[a_0, a_1]$	$[\frac{g_l}{g_l+g_k}, \hat{a}]$	Constitution
2	1	2	4	$[.24, .56]$	$[.33, .41]$	$a = .39$ $\beta = .18$
2	1	2	2	$[.24, .56]$	$[.33, .41]$	$a = .34$ $\beta = .75$
4	1	2	4	$[.15, .46]$	$[.2, .33]$	$a = .3$ $\beta = .28$
4	1	2	2	$[.15, .46]$	$[.2, .33]$	$a = .24$ $\beta = 1$
2	1	4	4	$[.24, .56]$	$[.33, .41]$	$a = .4$ $\beta = .18$
2	1	4	2	$[.24, .56]$	$[.33, .41]$	$a = .37$ $\beta = .73$
4	1	4	4	$[.15, .46]$	$[.2, .33]$	$a = .31$ $\beta = .28$
4	1	4	2	$[.15, .46]$	$[.2, .33]$	$a = .27$ $\beta = 1$

Table 1: Calculating the efficient constitutional arrangement that minimizes the risk of secession

In all the examples of the table, note that region l is the region that benefits less from the union. Several observations are in order:

First, note that the division of power that minimizes secession assigns to region l more than half of the legislative power, that is, $1 - a > .5$. This is due

to the fact that region l derives on average, lower variable benefits $\bar{g}_l < \bar{g}_k$.

Second, the higher the political distinctiveness between the regions $|z_k - z_l|$, ceteris paribus, the lower the legislative power for region l and the lower the degree of centralization. Note how starting from the first example, each pair of examples differ to each other in the political cost $|z_k - z_l| = 4$ and $|z_k - z_l| = 2$. The political cost negatively affects both regions and makes the union more vulnerable to negative economic shocks. Note how the level of centralization sharply reduces when we aim at preserving the integrity of the union.

Third, higher fixed benefits imply that region l has less incentives to withdraw from the union. Therefore, the level of legislative power that compensates region l reduces. For example, when comparing the examples in the first and fifth rows (where F_j increases in two points), we deduce that $1 - a$ diminishes in .01 points.

Finally, the larger the inequality between the regions in terms of expected variable benefits, $\bar{g}_k$ and $\bar{g}_l$, the higher the level of legislative power with which region l is compensated. For example, when comparing the examples in the first and third rows (where $\bar{g}_k$ increases in two points), we deduce that $1 - a$ increases in .09 points, and so, the level of centralization increases as well.

In sum, there is a variety of constitutional arrangements that the regions can accept. However, one of these arrangements provides the maximal insurance against dissolution. This arrangement is characterized by providing more than half of the legislative power to the economically disadvantaged region, and establishing a level of centralization around the minimal accepted by the regions. We find that the lower the union benefits of the disadvantaged region with respect to its partner, the lower the political distinctiveness between the regions, or the lower the fixed benefits derived from the union, the higher the legislative power of the disadvantaged region and the higher the level of centralization. On the contrary, the more similar the variable union benefits of the regions, the greater their political distinctiveness, and the higher the fixed benefits, the more equal the division of legislative power and the smaller the level of centralization.

In the Appendix we generalize and analyze the robustness of our results in two directions, first, by incorporating additional regions and second, by endowing regions with intertemporal preferences. In each of these models, we describe the set of acceptable constitutional arrangements. When there are more than two regions, we find the same compensation mechanism by which, regions with lower expected gains and *higher political distance to its partner regions* require additional legislative power for not vetoing the national law and not dissolving the union. When we consider intertemporal preferences, we find an additional result: the more the regions *care about the future*, the

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higher the degree of centralization that the regions can accept. Intuitively, the economic gains from the union accumulate over time and, as a result, the economic benefits, in comparison to the political cost, become more relevant.

4 Social Welfare Analysis

In this section, we examine the constitutional design from a normative perspective. Specifically, we calculate the constitutional arrangement that maximizes social welfare. We consider that n_j denotes the population size of region j. Then, according to the utilitarian criterion, the constitutional arrangement that provides maximal welfare is obtained when solving:

$$\underset{\beta \in [0,1], a \in [0,1]}{Max} \quad n_k E\left[u_k(\beta, a)\right] + n_l E\left[u_l(\beta, a)\right]. \tag{16}$$

Note that the solution to the above problem is by definition Pareto efficient. Solving for the first order condition with respect to a and simplifying yields:

$$\frac{\partial L}{\partial a} = n_k(1 - a) - n_l a = 0 \Leftrightarrow a = \frac{n_k}{n_k + n_l}.$$

Substituting the above condition in the equation that defines the set of Pareto efficient arrangements (Equation 11) we deduce that $\beta = \frac{(n_k \bar{g}_k + n_l \bar{g}_l)(n_k + n_l)}{2[z_k - z_l]^2 n_l n_k}$. We derive the following result.[19]

Proposition 4 *The constitutional arrangement (β, a) that maximizes social welfare provides to each region a legislative power in proportion to its population size, that is $a = \frac{n_k}{n_k + n_l}$, and assigns the following level of centralization*

$$\beta = \min\left\{1, \frac{(n_k \bar{g}_k + n_l \bar{g}_l)(n_k + n_l)}{2[z_k - z_l]^2 n_l n_k}\right\} \text{ if } n_k \bar{g}_k + n_l \bar{g}_l > 0,$$
$$\beta = 0 \text{ otherwise.}$$

The proof follows from the above arguments. We deduce that the constitution that provides maximal social welfare is the PE constitutional arrangement that divides legislative power in proportion to population size. Note that even when a region derives negative variable union benefits $\bar{g}_j < 0$, condition $n_k \bar{g}_k + n_l \bar{g}_l > 0$ can still hold and the utilitarian criterion selects certain decentralized federal system where $\beta > 0$. Clearly, there is no reason why the socially optimal constitution satisfies the VP condition and this could be the case that $\frac{n_k}{n_k + n_l} > a_1$ (or $\frac{n_k}{n_k + n_l} < a_0$), implying that region l (region k) has incentives to veto the national law. Likewise, there is no reason why the socially optimal constitution coincides with the constitutional arrangement that minimizes the risk of dissolution.

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[19]Equivalently, the level of centralization can be deduced from the f.o.c. so that $\frac{\partial L}{\partial \beta} = n_k \bar{g}_k + n_l \bar{g}_l - 2\beta a^2 [z_k - z_l]^2 \left[n_l a^2 + n_k(1 - a)^2\right] = 0$.

5 Conclusion

Prior theorizing has examined the justification for federalism as a means of efficient production of public goods or for joint economic gain. Other research has considered the effects of legislative arrangements as a way of holding constitutional bargains together. The model proposed here has sought to unite these strands of thinking to understand why federal systems might fall apart, why regions, such as Scotland, Quebec, Flanders or Catalonia threaten with leaving their parent nations, or why the UK decided to break from the European Union.

Two factors, we argue, drive the politics of nation formation: the economic gains to integration and the cultural or political costs borne by regions of adjusting to a common set of national laws and practices. The constitutional process defines a vertical and a horizontal division of power. The vertical division reflects the level of centralized federalism or power that regional governments transfer to the new national government; the horizontal division is manifest in the relative power of the regions in the new legislature. The vertical relation of the regions with the nation determines how much value the combined economy can create and the political cost of adjusting to a common national law; the horizontal division of power mitigates the political cost of transferring power to the union. We analyze those constitutional arrangements that are efficient and not vetoed as those that the regions approve in their political bargaining. From these two principles, we derive the following key insights:

First, we find an unexplored trade-off by which regions are willing to accept more centralization in exchange for additional power in the national legislature. By efficiency, unequal divisions of national power among regions are associated to high levels of centralization, whereas equitable divisions of power are linked to low levels of centralization. No vetoed restricts the set of acceptable constitutions to those that do not propose too unequal divisions of national power.

Second, we show that, on average, scale economies translate into higher centralized systems. Intuitively, scale economies can compensate the additional political cost associated to a highly centralized system.

Third, cultural, political or economic heterogeneity among regions translates, on average, into more decentralized federal systems. Note how the political cost of a common law is high when joining heterogeneous regions, and decentralized federalism can easily accommodate different cultures and customs.

Fourth, we explore what constitution, among the set of constitutions that the regions can accept, minimizes the possibility of dissolving the union. For

that, we assume that regions are equally exposed to a negative economic sock that affects their economic gains of integration. We show that the constitutional arrangement defined by the Nash Bargaining solution with equal bargaining weights, provides the maximal resilience to dissolution. This Nash Bargaining solution requires that those regions that have the most to economically gain from being in the union, will have to give proportionally more political power to its partner region. Besides, this division of power is associated to a low level of centralized federalism.

Our results suggest that compensations in the constitutional process need not be accomplished through direct transfers; it can be accomplished through the legislative process. Interestingly, the divisions of power that minimize the risk of secession sharply contrast with other widely used criteria on power division approved in many constitutions, which either dictate equal division of power among provinces, or a division of power in proportion to population size.[20]

Finally, we show that a constitution provides maximal social welfare when dividing legislative power in proportion to population size. We find however, that this criterion exposes the union to the risk of breaking up. For example, if the less populated region derives low economic union gains, then its marginal participation in the national legislative process can motivate its exit from the federal union.

Our contribution provides new lens through which to analyze the situation in countries (and unions such as the EU) struggling with independence or separation efforts. We have shown that the constitutional process determines the boundaries of what the regions can gain and lose. To minimize the risk of dissolving the union, regions need to compromise on their political and cultural distinctiveness, and regions that have the most to gain economically from being in a union, relative to not being in the nation, will have to give away more political power to the other regions in order to hold the nation together.

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APPENDIX A: Proofs

Proof of Lemma 1: Let $L = E[u_l(\beta, a)] - \mu[E[u_k(\beta, a)] - u]$ be the Lagrangian function of Problem (10). The first order condition defines the optimal interior values (a, β, μ) satisfying:

$$\frac{\partial L}{\partial a} = -a - \mu(1-a) = 0 \Leftrightarrow \mu = \frac{-a}{1-a}$$
$$\frac{\partial L}{\partial \beta} = \bar{g}_l - 2\beta a^2(z_k - z_l)^2 - \mu[\bar{g}_k - 2\beta(1-a)^2(z_k - z_l)^2] = 0.$$

Substituting the first condition into the second, we deduce Expression (11). The second order condition requires the determinant of the bordered Hessian being positive:

$$H = -(L_{aa}(h_\beta)^2 - 2L_{a\beta}g_a g_\beta + L_{\beta\beta}(h_a)^2)$$

where $h = E[u_k(\beta, a)] - u$ is the constraint function. The following derivatives prove that $H > 0$: $L_{aa} = 2\beta^2(z_k - z_l)^2(\mu - 1) < 0$, $L_{\beta\beta} = -2a^2(z_k - z_l)^2 + \mu 2(1-a)^2(z_k - z_l)^2 < 0$, $L_{a\beta} = -4\beta a(z_k - z_l)^2 - \mu 4\beta(1-a)(z_k - z_l)^2 = 0$.

Proof of Lemma 2: First, we show that when $(z_k - z_l)^2 \geq \left(\bar{g}_l^{1/2} + \bar{g}_k^{1/2}\right)^2$, the regions only accept a decentralized federal system. The VP condition implies that $\bar{g}_l \geq \beta_l a^2(z_k - z_l)^2$ and $\bar{g}_k \geq \beta_k(1-a)^2(z_k - z_l)^2$ from where, $\beta_l \leq \frac{\bar{g}_l}{a^2(z_k - z_l)^2}$ and $\beta_k \leq \frac{\bar{g}_k}{(1-a)^2(z_k - z_l)^2}$. These expressions intersect at $\hat{a} = \frac{\bar{g}_l^{1/2}}{\bar{g}_l^{1/2} + \bar{g}_k^{1/2}}$ and since β_l is strictly decreasing in a and β_k is strictly increasing in a, at $\hat{a}$ the regions are willing to accept the maximal level of centralization compatible with the VP condition. Thus, substituting $\hat{a}$ in any of the inequalities and imposing $\beta \leq 1$ we deduce that $(z_k - z_l)^2 \geq \left(\bar{g}_l^{1/2} + \bar{g}_k^{1/2}\right)^2$. Second, we calculate the intersection between the efficient set of constitutional arrangements $\beta^* = \frac{a\bar{g}_k + (1-a)\bar{g}_l}{2[z_k - z_l]^2 a(1-a)}$ and the VP conditions: $\beta_l \leq \frac{\bar{g}_l}{a^2(z_k - z_l)^2}$ and $\beta_k \leq \frac{\bar{g}_k}{(1-a)^2(z_k - z_l)^2}$. Solving for $\beta^* = \beta_l$ we deduce a_1, and solving for $\beta^* = \beta_k$ we deduce a_0. Substituting $\bar{g} = \frac{\bar{g}_k}{\bar{g}_l}$

$$\beta^* = \beta_k \iff \bar{g}_k a_0(1 + a_0) - \bar{g}_l(1 - a_0)^2 = 0$$
$$\beta^* = \beta_l \iff \bar{g}_l(1 - a_1)(2 - a_1) - \bar{g}_k a_1^2 = 0$$

From where, we deduce a_0 and a_1.

Proof of Proposition 2: *We show i). Let $\Psi_1 = \bar{g}_l(1-a_1)(2-a_1) - \bar{g}_k a_1^2 = 0$ and $\Psi_0 = \bar{g}_k(1 + a_0)a_0 - \bar{g}_l(1 - a_0)^2 = 0$. These functions Ψ_1 and Ψ_2 are continuous in its variables and have continuous first partial derivatives. In terms of relative expected benefits $\bar{g} = \frac{\bar{g}_k}{\bar{g}_l}$, $\Psi_1 = (1 - a_1)(2 - a_1) - \bar{g}a_1^2 = 0$*

and $\Psi_0 = \bar{g}(1 + a_0)a_0 - (1 - a_0)^2 = 0$. *By the Implicit Function Theorem, around $a = a_1$ and $a = a_0$:*

$$\left.\frac{\partial a}{\partial \bar{g}}\right|_{a=a_1} = -\frac{\frac{\partial \Psi_1}{\partial \bar{g}}}{\frac{\partial \Psi_1}{\partial a}} = -\frac{-a^2}{2a - 3 - 2\bar{g}a} < 0$$

$$\left.\frac{\partial a}{\partial \bar{g}}\right|_{a=a_0} = -\frac{\frac{\partial \Psi_0}{\partial \bar{g}}}{\frac{\partial \Psi_0}{\partial a}} = -\frac{(1+a)a}{\bar{g}(2a+1) + 2(1-a)} < 0$$

Thus, higher $\bar{g}_k$ implies higher $\bar{g}$ and lower a, and higher $\bar{g}_l$ implies lower $\bar{g}$ and higher a. *Besides, $\frac{\partial \beta^*}{\partial \bar{g}_k} > 0$ and $\frac{\partial \beta^*}{\partial \bar{g}_l} > 0$.*

We show ii). First, note that a_1 and a_0 do not vary with $[z_k - z_l]^2$. Second, $\frac{\partial \beta^*}{\partial [z_k - z_l]^2} < 0$.

We show iii). Since $\frac{\partial \Psi_1}{\partial F_j} = 0$ and $\frac{\partial \Psi_0}{\partial F_j} = 0$, then $\frac{\partial a}{\partial F_k} = 0$ and $\frac{\partial a}{\partial F_l} = 0$.

Proof of Proposition 3: Solving for $\tilde{\varepsilon}_j(\beta, a)$ in Expression (14) yields

$$\tilde{\varepsilon}_j(\beta, a) = \frac{F_j}{\beta} + \bar{g}_j - \beta(1 - a_j)^2(z_k - z_l)^2,$$

and evaluated at the efficient constitutional arrangements $\beta = \beta^*(a)$ defined by Expression (11)

$$\tilde{\varepsilon}_k(\beta^*(a), a) = \frac{F_k 2(z_k - z_l)^2 a(1-a)}{a\bar{g}_k + (1-a)\bar{g}_l} + \bar{g}_k - \frac{a(1-a)\bar{g}_k + (1-a)^2 \bar{g}_l}{2a}$$

$$\tilde{\varepsilon}_l(\beta^*(a), a) = \frac{F_l 2(z_k - z_l)^2 a(1-a)}{a\bar{g}_k + (1-a)\bar{g}_l} + \bar{g}_l - \frac{a^2 \bar{g}_k + (1-a)a\bar{g}_l}{2(1-a)}.$$

Since $F_k = F_l$, and taking $a_k = a$,

$$\tilde{\varepsilon}_k(\beta, a) = \tilde{\varepsilon}_l(\beta, a) \iff \bar{g}_k - \beta(1-a)^2(z_k - z_l)^2 = \bar{g}_l - \beta a^2(z_k - z_l)^2$$

$$\iff \bar{g}_k - \bar{g}_l = \beta(1 - 2a)(z_k - z_l)^2$$

substituting $\beta = \beta^*(a) = \frac{a\bar{g}_k + (1-a)\bar{g}_l}{2[z_k - z_l]^2 a(1-a)}$ and solving for a yields $a_k = \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$. This shows that the two functions only cut once. We next show that the value of a minimizing the risk of secession is located within the bounds $\left[\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}, \frac{\bar{g}_l^{1/2}}{\bar{g}_l^{1/2} + \bar{g}_k^{1/2}}\right]$ when $\bar{g}_k > \bar{g}_l$ and $F_k = F_l$. We calculate the slope of the above functions and simplify some of the terms:

$$\frac{\partial \tilde{\varepsilon}_k}{\partial a} = -\frac{F_k}{[\beta^*(a)]^2} \frac{\partial \beta^*}{\partial a} \underbrace{-\left[\frac{-(1-a^2)\bar{g}_l - a^2 \bar{g}_k}{2a^2}\right]}_{+} \tag{17}$$

$$\frac{\partial \tilde{\varepsilon}_l}{\partial a} = -\frac{F_l}{[\beta^*(a)]^2} \frac{\partial \beta^*}{\partial a} \underbrace{-\left[\frac{g_l(1-a)^2 + ag_k(2-a)}{2(1-a)^2}\right]}_{-} \tag{18}$$

If $F_k = 0$, $\frac{\partial \tilde{\varepsilon}_k}{\partial a} > 0$ for all a, and if $F_l = 0$, $\frac{\partial \tilde{\varepsilon}_l}{\partial a} < 0$ for all a. Then, for all $a \leq \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$, $\min \{\tilde{\varepsilon}_k(\beta, a), \tilde{\varepsilon}_l(\beta, a)\} = \tilde{\varepsilon}_k(\beta, a)$ and for all $a > \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$, $\min \{\tilde{\varepsilon}_k(\beta, a), \tilde{\varepsilon}_l(\beta, a)\} = \tilde{\varepsilon}_l(\beta, a)$ and the maximal argument of the function $\max \min \{\tilde{\varepsilon}_k(\beta, a), \tilde{\varepsilon}_l(\beta, a)\}$, is achieved when the two functions cut, that is, $a_k = \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$ and substituting $\beta^*(\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}) = \frac{\bar{g}_l + \bar{g}_k}{(z_k - z_l)^2}$. Consider now that $F_k = F_l \neq 0$. As already argued, function $\beta^*(\cdot)$ is decreasing up to $\hat{a}$, where $\hat{a} = \frac{\bar{g}_l^{1/2}}{\bar{g}_l^{1/2} + \bar{g}_k^{1/2}}$, and increasing afterward. Since $\frac{\partial \beta^*}{\partial a} < 0$ for all $a < \hat{a}$, and $\frac{\partial \beta^*}{\partial a} > 0$ for all $a > \hat{a}$, we deduce from (17) that for all $a \leq \hat{a}, \frac{\partial \tilde{\varepsilon}_k}{\partial a} > 0$ and, we deduce from (18), that for all $a \geq \hat{a}, \frac{\partial \tilde{\varepsilon}_l}{\partial a} < 0$. Besides, for every a, $\frac{\partial \tilde{\varepsilon}_k}{\partial a} > \frac{\partial \tilde{\varepsilon}_l}{\partial a}$ and in particular, this inequality holds in the unique cutting point $a = \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$. Thus, that for all $a \leq \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$, $\min \{\tilde{\varepsilon}_k(\beta, a), \tilde{\varepsilon}_l(\beta, a)\} = \tilde{\varepsilon}_k(\beta, a)$ and for all $a > \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$, $\min \{\tilde{\varepsilon}_k(\beta, a), \tilde{\varepsilon}_l(\beta, a)\} = \tilde{\varepsilon}_l(\beta, a)$. Consider that $\bar{g}_k > \bar{g}_l$ (the case $\bar{g}_k < \bar{g}_l$ is symmetric) then, $\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k} < \hat{a}$, and $\tilde{\varepsilon}_k(\beta, a)$ is increasing for all $a \in \left[a_0, \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}\right]$, $\tilde{\varepsilon}_l(\beta, a)$ can be increasing or decreasing in the interval $a \in \left[\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}, \hat{a}\right]$ and it is decreasing for every $a \in [\hat{a}, a_1]$. We deduce that $\max \min \{\tilde{\varepsilon}_k(\beta, a), \tilde{\varepsilon}_l(\beta, a)\} = \tilde{\varepsilon}_l(\beta, a)$ when $a \in \left[\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}, \hat{a}\right]$. When $\bar{g}_k > \bar{g}_l$, then $\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k} < \hat{a}$ (where $\hat{a} = \frac{\bar{g}_l^{1/2}}{\bar{g}_l^{1/2} + \bar{g}_k^{1/2}}$), and the minimal risk of secession is achieved at some value $a_k \in \left[\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}, \hat{a}\right]$ where note that $\hat{a} < \frac{1}{2}$. Note that when $\bar{g}_k < \bar{g}_l$, then $\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k} > \hat{a}$, and the minimal risk of secession is achieved at some value $a_k \in \left[\hat{a}, \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k},\right]$ where $\hat{a} > \frac{1}{2}$.

Next, we show that the associated level of centralization that minimizes the risk of secession diminishes when the fixed benefits are high and/or the political distinctiveness between the regions is high. Suppose that $\bar{g}_k > \bar{g}_l$, then condition $\frac{\partial \tilde{\varepsilon}_l}{\partial a} = 0$ defines the constitutional arrangement minimizing the risk of secession. Ceteris paribus, higher F_l implies that the positive component of expression (18) is greater and the value $a_k \in \left[\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}, \hat{a}\right]$ that maximizes $\tilde{\varepsilon}_l$ is closer to $\hat{a}$. Since $\beta^*(\hat{a})$ is the minimal level of efficient centralization, we deduce that the level of centralization diminishes. Following a similar argument, when $(z_k - z_l)^2$ increases, the first term of expression (18) increases $\left(-\frac{F_l}{[\beta^*(a)]^2} \frac{\partial \beta^*}{\partial a} = -\frac{2F_l(z_k - z_l)^2[\bar{g}_k a^2 - \bar{g}_l(1-a)^2]}{a g_k + (1-a)g_l}\right)$ and the value $a_k \in \left[\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}, \hat{a}\right]$ that maximizes $\tilde{\varepsilon}_l$ is closer to $\hat{a}$. Besides, by Proposition 2, higher $(z_k - z_l)^2$ implies that for all a, $\beta^*(a)$ decreases. Thus, we deduce that the level of centralization diminishes.

Finally, we show the statement in Footnote 18. Suppose that $\bar{g}_k > \bar{g}_l$ and

consider some F_k' such that $F_k' > F_k = F_l$. We have already shown that $a_k = \frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$ is the unique cutting point between $\tilde{\varepsilon}_k(\beta, a), \tilde{\varepsilon}_l(\beta, a)$ when $F_k = F_l$. According to (17), for all $a < \hat{a}$, $\frac{\partial \tilde{\varepsilon}_k}{\partial a}\big|_{F_k'} > \frac{\partial \tilde{\varepsilon}_k}{\partial a}\big|_{F_k} > \frac{\partial \tilde{\varepsilon}_l}{\partial a}\big|_{F_l}$, i.e., ceteris paribus, the slope $\frac{\partial \tilde{\varepsilon}_k}{\partial a}$ is steeper when evaluated at F_k' with respect to F_k and besides, the slope $\frac{\partial \tilde{\varepsilon}_k}{\partial a}$ is steeper than $\frac{\partial \tilde{\varepsilon}_l}{\partial a}$. Thus, the unique cutting point between $\tilde{\varepsilon}_k(\beta, a)$ and $\tilde{\varepsilon}_l(\beta, a)$ is below $\frac{\bar{g}_l}{\bar{g}_l + \bar{g}_k}$.

APPENDIX B: The case of n > 2 regions

Consider that there are n regions. When forming a nation, regions determine the degree of centralization, $\beta \in [0, 1]$, and the power of each region within the union $\vec{a} = (a_1, a_2, ..., a_n)$ where $\sum_{j=1}^{n} a_j = 1$. Given a division of legislative power $\vec{a}$, the national law is a compromise among the regions. For each region, let z_{-j} denote the perceived national law enacted by the remaining regions, were region j is excluded from the negotiation. From region j's perspective, the enacted national policy is a compromise between z_{-j} and region j's most preferred policy z_j, where each of these policies are weighted by its corresponding legislative power, $\sum_{i \neq j}^{n} a_i = 1 - a_j$ and a_j respectively. Thus, from regions j's perspective, the enacted national policy is $z^L(\vec{a}) = (1 - a_j)z_{-j} + a_j z_j$. Similarly to the case with two regions, the enacted policy in each region is defined by $z_j^L(\beta, \vec{a}) = \beta z^L(\vec{a}) + (1 - \beta)z_j$.

We then solve for the expected utility of each region and substitute $z_j^L(\beta, \vec{a})$ and $z^L(\vec{a})$:

$$
\begin{aligned}
E[u_j(\beta, \vec{a})] &= E[Y_j(\beta)] - (z_j^L(\beta, \vec{a}) - z_j)^2 = \\
E[Y_j(\beta)] - \beta^2 (z^L(\vec{a}) - z_j)^2 &= E[Y_j(\beta)] - \beta^2 (1 - a_j)^2 (z_{-j} - z_j)^2.
\end{aligned}
$$

Every efficient constitutional arrangements $(\beta, \vec{a})$ solves

$$
\begin{aligned}
&\underset{\beta, a_1, a_2, ..., a_n}{Max} \quad E[u_1(\beta, \vec{a})] \\
&\text{s.t.} \quad E[u_j(\beta, \vec{a})] \geq u_j, \ \forall j = 2, ...n \\
&\qquad \sum_{j=1}^{n} a_j = 1
\end{aligned}
$$

Substituting $a_1 = 1 - \sum_{j \neq 1}^{n} a_j$, the Lagrangian function is defined by

$$
\begin{aligned}
L &= E[Y_1(\beta)] - \beta^2 (\textstyle\sum_{j \neq 1}^{n} a_j)^2 (z_{-1} - z_1)^2 - \\
&\quad \textstyle\sum_{j=2}^{n} \mu_j \left[E[Y_j(\beta)] - \beta^2 (1 - a_j)^2 (z_{-j} - z_j)^2 - u_j \right]
\end{aligned}
$$

where μ_j is the Lagrangian multiplier. The first order condition for an interior solution defines the optimal values satisfying:

$$\frac{\partial L}{\partial a_j} = -2\beta^2(\sum_{i\neq 1}^n a_i)(z_{-1}-z_1)^2 + 2\mu_j\beta^2(1-a_j)(z_{-j}-z_j)^2 = 0, \quad (19)$$

$$\frac{\partial L}{\partial \beta} = \bar{g}_1 - 2\beta(\sum_{i\neq 1}^n a_i)^2(z_{-1}-z_1)^2 +$$
$$\sum_{j\neq 1}^n \mu_j[\bar{g}_j - 2\beta(1-a_j)^2(z_{-j}-z_j)^2] = 0. \quad (20)$$

From (19):

$$\mu_j = \frac{(\sum_{i\neq 1}^n a_i)(z_{-1}-z_1)^2}{(1-a_j)(z_{-j}-z_j)^2}, \quad \forall j = 2,...,n$$

Substituting in (20):

$$\bar{g}_1 - 2\beta(\sum_{i\neq 1}^n a_i)(z_{-1}-z_1)^2 + \sum_{j\neq 1}^n \frac{(\sum_{i\neq 1}^n a_i)(z_{-1}-z_1)^2}{(1-a_j)(z_{-j}-z_j)^2}[\bar{g}_j - 2\beta(1-a_j)^2(z_{-j}-z_j)^2] = 0,$$

and solving for β yields

$$\beta = \frac{\bar{g}_1 + (\sum_{i\neq 1}^n a_i)(z_{-1}-z_1)^2 \sum_{j\neq 1}^n \frac{\bar{g}_j}{(1-a_j)(z_{-j}-z_j)^2}}{2(\sum_{i\neq 1}^n a_i)(z_{-1}-z_1)^2(n-1)}.$$

In the above expression, multiplying and dividing $\bar{g}_1$ by $(\sum_{i\neq 1}^n a_i)(z_{-1}-z_1)^2$ we deduce:

$$\beta = \frac{\frac{\bar{g}_1}{(\sum_{i\neq 1}^n a_i)(z_{-1}-z_1)^2} + \sum_{j\neq 1}^n \frac{\bar{g}_j}{(1-a_j)(z_{-j}-z_j)^2}}{2(n-1)}$$

and since $\sum_{j\neq 1}^n a_j = 1 - a_1$ we obtain,

$$\beta^*(\vec{a}) = \frac{1}{2(n-1)}\sum_{j=1}^n \frac{\bar{g}_j}{(1-a_j)(z_{-j}-z_j)^2}.$$

Note how higher expected gains and smaller political distance among regions increase the degree of centralization. By VP, regions only accept those constitutional arrangements satisfying $E[u_j(\beta,a)] \geq Y_j^0 + F_j$, from where $\bar{g}_j \geq \beta(1-a_j)^2(z_{-j}-z_j)^2$ for all $j = 1,...,n$. Solving for a_j we deduce $a_j \geq 1 - \frac{\bar{g}_j^{\frac{1}{2}}}{\beta(z_{-j}-z_j)}$. Thus, given any β, those regions with lower expected gains and higher political distance require additional legislative power to not vetoing the national law in the first place, and also, for not dissolving the union.

APPENDIX C: The intertemporal utility model

Consider that the constitution is signed in period $t = 0$, and every later period $t > 0$ is a legislative period. For every period $t > 0$, the level of outcome for a region $j \in \{k, l\}$ is written $Y_j^t(\beta)$

$$Y_j^t(\beta) = \underbrace{Y_j^{t-1}}_{\text{Last period outcome}} + \underbrace{F_j^t}_{\text{Fixed union benefits}} + \underbrace{\beta g_j^t}_{\text{Variable union benefits}} . \tag{21}$$

That is, in a given period, the outcome of a region is the outcome of the previous period plus the benefits derived from the union. To simplify the analysis, we take $F_j^t = F_j$, i.e., the fixed union benefits are equal across periods. In each period, the preferences of the region are represented by $u_j^t(\beta, a) = Y_j^t(\beta) - \beta^2 \left[z^L(a) - z_j \right]^2$.

Regions are uncertain about their future growth path when forming a nation. The growth path for a region is described by a sequence $(g_j^1, g_j^2, ...)$. In every period $t > 0$, the believe of the region about its growth rate g_j^t is a random variable which realization depends on many unexpected factors such as economic global performance, quality of regional and national politicians, or international relations among others. Besides, we assume that no region has certainty about the dynamic of g_j^t, for example, there is no deterministic rule by which high economic growth in a period implies high growth in the next period.[21] We then consider that, when designing the constitution, each region accounts for some expected constant growth path $(g_j^1, g_j^2, ...) = (\bar{g}_j, \bar{g}_j, ...)$.[22]

Let $\delta \in (0, 1)$ be the discount factor applied by each region to each period or alternatively, δ can be interpreted as the probability in each period of both regions maintaining the union of the nation. The intertemporal preferences of a region when forming a nation are represented by the following discounted present utility function:

$$E\left[U_j(\beta, a)\right] = \sum_{t=0}^{\infty} \delta^t E\left[u_j^{t+1}(\beta, a)\right] = E\left[u_j^1(\beta, a)\right] + \delta E\left[u_j^2(\beta, a)\right] + ... =$$

$$Y_j^0 + F_j + \beta \bar{g}_j - \beta^2(z^L(a) - z_j)^2 + \delta Y_j^0 + \delta F_j + 2\delta\beta\bar{g}_j - \delta\beta^2(z^L(a) - z_j)^2 + ...$$

Substituting the value of $z^L(a)$, each region utility is described by[23]

$$E\left[U_j(\beta, a)\right] = \frac{Y_j^0 + F_j}{1 - \delta} + \frac{\beta \bar{g}_j}{(1 - \delta)^2} - \frac{\beta^2 a_j^2(z_k - z_l)^2}{1 - \delta}, \tag{22}$$

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[21] Even when policy makers are aware of economic cycles, there is no certainty on the magnitude and duration of each of these cycles.

[22] An alternative approach could consider a finite number of states and growth rates $(g_j^1, g_j^2, ...)$ following a Markov chain. Regions could enter in an expanding or contracting growth path, and decisions at the constitutional stage would be based on the probability of these two possibilities.

[23] Where $1 + \delta + \delta^2 + ... = \frac{1}{1 - \delta}$ and $1 + 2\delta + 3\delta^2 + ... = \frac{1}{(1 - \delta)^2}$

where $a_k = a$ and $a_l = 1 - a$. Regions estimate that a national law passes in every period if expected economic benefits compensate the political union cost, from where

$$\frac{\bar{g}_j}{1-\delta} \geq \beta a_j^2 (z_k - z_l)^2. \tag{23}$$

Solving for the ex-ante PE constitutional arrangements (interior solutions) yields

$$\beta^*(a) = \frac{\bar{g}_k a + (1-a)\bar{g}_l}{2(1-\delta)[z_k - z_l]^2 a(1-a)}. \tag{24}$$

Since $\frac{\partial \beta^*}{\partial \delta} > 0$, we deduce that *the more the regions care about the future, the more centralization they want.* Note that in every Pareto efficient arrangement, the level of centralization is above the one deduced in the non intertemporal model.

Substituting the above expression into expressions (23) we obtain

$$\bar{g}_l \geq \frac{\bar{g}_k a + (1-a)\bar{g}_l}{2(1-a)} a, \ \bar{g}_k \geq \frac{\bar{g}_k a + (1-a)\bar{g}_l}{2a}(1 - a).$$

These expressions do not depend on δ and coincide with those defining the upper and lower legislative power bounds $[a_0, a_1]$ in Proposition 1.