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Institutional flexibility, political alternation and middle-of-the-road policies*

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Abstract

Empirical observation shows that policies are usually gradually introduced in a society. This paper presents a model of repeated elections that captures this phenomenon, and that allows countries to differ in their institutional flexibility, thus in the speed of implementation of new policies. We show that with gradual implementation of policies there is an incentive for the voters to vote, each election, to a different party. Hence, our model produces equilibria with alternation. We further show that there is a tradeoff between efficiency and stability, with efficiency requiring moderate policies and stability pushing towards polarization. Last, we show that except for the partisan equilibria, the most stable ones convey policies that are bounded away from both the median and the extremes, with policies polarizing more when institutions are either too flexible or sufficiently rigid.

Keywords: Gradual implementation of policies; political alternation; polarization and moderation; efficiency; robustness

JEL: D02; D72

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1 Introduction

Although casual observation shows that left-wing and right-wing political parties take turns in office, there is relatively little work on the determinants of political alternation.[1] Empirical studies suggest that the electorate's natural disaffection with the incumbent party, particularly when the economy is not doing well, is the reason explaining party fluctuation. Examples are Bernhard and Leblang (2008), Walter (2009) and Chwieroth and Walter (2010). The idea of disaffection is also central to Roemer (1995)'s theory, who inspired by Hirschman (1982), considers an electorate whose preferences are continuously shifting away from the policy implemented by the elected party.

Though we believe disappointment is an important variable that explains part of the observed variation in political power, we also think that alternation is a way for the voters to assure that implemented policies do not move away too much from the median voter's ideal policy. In fact, by replacing the ruling party every certain number of elections, voters guarantee that the incumbent has not enough time to propose, pass and implement major policy changes. In this sense, alternation is a mechanism to keep policies at moderate positions.

Central to this argument is the idea that policy implementation takes time. That is, that a country's institutions, both formal and informal, preclude elected parties from setting rapid adjustments in policies and forces them to implement new policies through a gradual process.[2] Interestingly, casual observation supports this argument. Think for example on the ambitious Health Care Reform in the US, a process that started on March 23, 2010, when former President Obama signed the Affordable Care Act, and that before it was interrupted by current president Donald Trump, was expected to be fully implemented by 2022 (www.obamacarefacts.com). Other examples include the G-20 Initiative on Rationalizing and Phasing out Inefficient Fossil Fuel Subsidies, an agrement reached at the 2009 Pittsburgh Summit, that establishes a compromise to phase out subsidies for oil and other fossil fuels in the "medium term" (*Reuters*, 25 September 2009); the China's one-child policy that, according to the government, is to be gradually changed (*The Guardian*, 15 November 2013 and 29 October 2015); or the Reform of the Spanish Education System, with a calendar of implementation that goes from 2014 to 2017 (*La Vanguardia*, 3 December 2013).

This paper builds on this idea to explain political alternation. To the best of our knowledge, this is new in the literature. In fact, a standard assumption in the political economy literature is that once a new party wins the election, its pursued policy is immediately implemented. The literature on agenda control (Romer and Rosenthal (1978)), divided government (see Alesina and Rosenthal (1995, 1996, 2000)), legislative bargaining (see Austen-Smith and Banks (1988) and Baron and Diermeier (2001)) and within-party conflict (Roemer (2001)) make the more realistic assumption that the final policy is a compromise

[1] Since the II World War, alternation is a regularity in Western democracies. Countries like Denmark, Ireland and Norway count more than ten alternations in office, and Australia, Canada, France, UK and US are just below ten.

[2] There are various reasons for this. For example, division of powers, gridlock and regulation on filibusters, social resistance to major changes and such.

between two or more political actors, usually the executive and the legislature. In any case, all these papers consider that once the final policy is determined, it is fully operative. In contrast to this literature, in this paper we assume that the pursued policy of the elected party is gradually introduced in the society. Interestingly, we show that this new ingredient provides a logic to parties taking turns in power.

To model this idea, we base on Duggan (2000) and Van Weelden (2013) and propose a model of repeated elections between two political parties and a voter. The voter, who represents the median or decisive voter in the population, is assumed to have a moderate ideology. The political parties are policy-motivated and are assumed to have extreme ideologies, on each side of the ideology of the median voter. Elections run at discrete time. At every election, the elected party proposes the policy it seeks to implement in the term. We refer to this policy as the *pursued policy*. The key assumption is that pursued policies are gradually introduced in the society. It has a direct implication: The utility that political parties and voters receive from the policy implemented change on a daily basis, as policies are in continuous movement. Accordingly, in the paper we treat both time and utility as continuous variables.

We formalize the idea of the gradual implementation of policies by means of a parameter, namely the *country's institutional flexibility*, that measures the speed of implementation of a new policy in a society. Note that the value of this parameter, together with the status quo policy at a given term and the pursued policy for that term, determines whether the time between two elections is enough for the pursued policy to be fully operative at the end of the term or not. In this sense, moving the institutional flexibility parameter allows us to affect the range of policies that can be indeed implemented, which affects the equilibrium behavior of political parties. Note also that with gradual implementation of policies there is a linkage between terms, as the voter's optimal choice in a given term depends on the status quo policy, which further depends on the previous status quo, the previous proposed policy and the institutional flexibility parameter.

We consider the dynamic version of the game. Our results show that independently of the players' discount factor, there is an equilibrium in which political parties propose partisan policies, i.e., the policies that correspond to their party's lines, and the median voter votes, each election, to a different party. We refer to this equilibrium as the *partisan-alternating equilibrium*, as policies are continuously swinging back and forth from one party's partisan policy to the other.[3] In this equilibrium a different political party wins office each term; hence there is alternation of parties in power. We next analyze whether the partisan-alternating equilibrium is efficient. We obtain that any symmetric alternating outcome in which policies do not suffer from such sharp swings Pareto dominates the partisan-alternating equilibrium. Moreover, we show that the milder the swings in policy, the more efficient the alternation is. Accordingly, in the rest of the paper we study which more efficient alternating outcomes can be sustained as an equilibrium of the game. In particular, we focus our attention on symmetric and stationary alternating strategies.

We show that when a country has very rigid institutions, in which case the implementation of new

[3] Note that depending on the country's institutional flexibility, purposed policies could be finally implemented or not.

policies takes a long time, in equilibrium policies are as polarized as the speed of implementation allows. In contrast, when institutions are sufficiently flexible, the Pareto superior strategy in which parties propose the median voter's preferred ideology can be an equilibrium. These results suggest that despite rigid institutions limit the capacity of governments to introduce sharp changes, it induces politicians to push policies as far as legal, political or social restrictions allow.[4] This is not so when institutions are flexible, in which case more efficient outcomes can be sustained as an equilibrium. The reason is that the institutional flexibility of a country determines the effective cost of a given punishment. Roughly speaking, when institutions are rigid a punishment consisting in moving to the partisan-alternating equilibrium cannot be fully implemented, as rigidities limit the capacity of the opposing party to take the policy too close to its ideology when its turn in office comes. This is not so when institutions are flexible. Accordingly, in the former case the incentive for parties is to use all the loose that the institutions allow, whereas in the later case the fear of a costly punishment allows parties to sustain more efficient outcomes.

Last, we study whether the more efficient alternating outcomes are robust to the consideration of impatient players. We refer to this equilibria as the *robust-interior equilibria*. We obtain that a robust-interior equilibrium only exists when the institutions of a country are neither too flexible nor too rigid. We also find that the equilibrium policies in the robust-interior equilibria stand midway between the median voter's ideology and the parties' ideal policies, i.e., they are *middle-of-the-road policies*; and that the more rigid the institutions, the milder the swings in policy from one election to the other, hence the more efficient the robust-interior equilibrium is. Finally, we show that for values of the institutional flexibility for which the robust-interior equilibrium does not exist, the only equilibrium that is robust to variations of the discount factor is the partisan-alternating equilibrium, which recall is the most inefficient one.

Though it is difficult to move from theory to empirical work, we consider that our results find some support in the real world. The current debate on the increasing polarization of the US Congress invites us to consider it as a possible example. According to Poole and Rosenthal (1991), the average distance between parties in US Congress was remarkably high in the last decades of the XIX Century, then decreased and stabilized until the II World War, decreased again until the 1970s and increased afterwards. The question is whether this tendency is to some extend related to the existence of a powerful executive, namely either Democrats or Republicans controlling both the White House and the US Congress at the same time. Interestingly, casual observation shows that the US President political strength (roughly measured as controlling both the executive and the legislature or not) expanded in the last decades of the XIX Century and the beginning of the XX Century, slightly decreased and stabilized until the 1970s and significantly decreased afterwards. In this sense, our results that both sufficiently rigid institutions (higher control of the legislature) and too flexible institutions (powerful executives) can lead to polarization, accord with empirical observation.[5]

[4] The literature on divided government finds that the higher the power of the legislature (therefore the smaller the power of the executive), the higher the polarization of platforms. See Alesina and Rosenthal (1995, 1996, 2000) and Ghosh (2002).

[5] Another piece of evidence that gives some support to our results is to be found in the empirical work by MacIntyre

We consider that our result of a non-monotonic relationship between institutional flexibility and policy extremism makes a contribution to the understanding of how institutions affect policy outcomes. This is a debate that has received much attention in the last decades. See Cox (1990), Myerson (1993*a*,*b*), Alesina and Rosenthal (1995, 1996, 2000), Persson and Tabellini (2000, 2005), Besley (2006); Besley and Case (2003) and more recently Pettersson-Lidbom (2008) and Besley et al. (2010), among others. Much of this debate has focused on the effects of constitutions (electoral rules, term limits, size of districts, separation of powers, and such) on policies. We contribute to this literature by showing that a country's institutional flexibility may have effects on its political outcomes, more precisely, on how moderate or extreme policies are.

Our paper also speaks to the literature on alternation. This is a fundamental feature of democracies, however the logic of parties taking turns in office remains unclear. Empirical research has extensively documented the benefits of alternation (see Horowitz et al. (2009), Milanovic et al. (2010) and Besley et al. (2014)); an idea that is also central to some theoretical papers. For example, Lagunoff (2001) proposes a dynamic game in which greater political turnover leads to higher support of civil liberties, and Acemoglu et al. (2011) show that political cooperation increases with party alternation, it facilitating the sustainability of better policies. In contrast to this, it is often argued that alternation may also lead to time-inconsistent and inefficient policies that undermine economic growth. See Alesina (1987), Persson and Svensson (1989), Alesina and Tabellini (1990) and Battaglini and Coate (2008). In a different line of research, the work by Wittman (1977), Kramer (1977), Bendor et al. (2006) and Forand (2014) yield alternation as an equilibrium outcome, building on models in which the challenger can experiment with new platforms whereas the incumbent is constrained to propose the past implemented policy. We contribute to this discussion by considering a new ingredient, namely the gradual implementation of policies, that provides a new logic to explain alternation.

The gradual implementation of policies introduces a dynamic linkage in our model, as the status quo becomes endogenous. This is not a feature in the classical models of repeated elections (see Alesina (1987, 1988) and Alesina and Rosenthal (1989)). The relevance of the status quo in models of party competition was first highlighted by Grofman (1985), who nevertheless considers a static model with an exogenous status quo. In models of legislative bargaining the endogeneity of the status quo and its effects is receiving a lot of attention nowadays. Baron (1996) and Zápal (2018) show that an endogenous status quo has a moderating effect, whereas Dziuda and Loeper (2016, 2017) find a polarizing effect. The literature has also identified other channels through which the dynamic linkage may arise. Callander and Hummel (2014) focus on the logic to preemptive experimentation and rely on an information channel, and Callander and

(2003), who for a group of Asian economies posits the existence of a U-shaped relationship between a country's level of centralization and the scope for governance problems. The focus of this work is on how institutions, in particular political fragmentation, affect the capacity of a country to cope with crisis. For the Asian financial crisis in the late 1990s, he finds that neither Indonesia or Malaysia (with dictatorship regimes), nor Thailand (with a highly fractionalized parliament) were able to manage a solution to the crisis, whereas Philippines (situated in between) did it relatively better.

Raiha (2017) study how policy choices shape voter's future preferences. To this literature we contribute by identifying a new channel for the linkage between periods: the country's institutions, and in particular the institutional rigidities.

Last, our paper is related to the literature on repeated elections games. The seminal work by Alesina (1988) first showed that the classical result of convergence to the median voter (see Downs (1957) and Wittman (1977, 1983)) is time-inconsistent and therefore cannot be sustained as an equilibrium if elections repeat in time, platforms are non-binding and voters are rational and forward looking. We share with Alesina (1988) and all the subsequent work the result that equilibrium policies will usually diverge. Formally, our paper is closest related to Duggan (2000) and Van Weelden (2013), who consider models of political competition in which parties and voters do not communicate before the election and voters use a retrospective voting rule to decide their vote. We build on these papers to propose a model of political competition where to analyze the effects of institutional flexibility on policy outcomes.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we analyze the stage game and in Section 4 we move to the repeated game, where we first focus on the partisan-alternating equilibrium and its properties. In Section 5 we propose a class of alternating equilibria that Pareto dominates the partisan-alternating equilibrium and analyze their robustness properties for the different values of the institutional flexibility parameter. Section 6 discusses some extensions and variations of the game considered in the previous sections and finally, Section 7 concludes.

2 The model

There are two political parties, L and R, and a median voter, M. The ideological space is $[-1, 1]$. Political parties differ in their partisan lines or ideal policies, which we denote by $\bar{x}^L$ and $\bar{x}^R$, respectively. We assume $\bar{x}^L = -1$ and $\bar{x}^R = 1$. The median voter's ideal policy is $\bar{x}^M = 0$. There is complete information.

Each $t \in \mathcal{N} \equiv \{1, 2, ...\}$ represents a term, which is the (usually 4 year) period of time between two elections. At the beginning of each term t there is an election, in which the median voter elects one party. We denote by $v_t \in \{L, R\}$ the choice of the median voter at term t. Then, the elected party, v_t, announces a policy $p_t \in [-1, +1]$ to be implemented during the term.[6] We refer to p_t as the *pursued policy* at term t. For simplicity, we assume that $p_t \in [-1, 0]$ if $v_t = L$ and $p_t \in [0, 1]$ if $v_t = R$, i.e., parties cannot propose policies too different from their party's lines.[7]

We assume that the implementation of new policies takes time. The argument is that societies and institutions cannot change drastically from one day to the next. Hence, even if an incoming party would

[6] This structure is reminiscent of Duggan (2000) and Van Weelden (2013), who consider candidates and voters that do not communicate before the election. Note that this is analogous to considering a model with a first stage in which political parties make announcements and the platforms are non-binding.

[7] This is a reasonable assumption given we consider policy-motivated parties, as described in expression (2). See Alesina and Rosenthal (1989) and James M. Snyder and Ting (2002) for other papers with similar assumptions.

like its pursued policy to be fully operative right after the election, social and institutional variables impede it. According to this, we consider that pursued policies are gradually introduced in the economy along the term. We denote by $r > 0$ the speed of implementation of a new policy in the society during a term. Thus, given r, if we denote by $z \in [-1,1]$ the policy at work at the end of the previous term, by $y \in [-1,1]$ the pursued policy in the current term, and by $\tau = 0,1$ the beginning and end, respectively, of the current term, the policy at work at time $\tau \in [0,1]$ within a term is:

$$\varkappa_\tau (z,y) = \begin{cases} \min\{y, z + r\tau\} & \text{if } y \geq z, \\ \max\{y, z - r\tau\} & \text{if } y < z. \end{cases} \tag{1}$$

Two comments are worth mentioning here. First, note that the smaller r, the more rigid the society is and/or the weaker the power of the government, thus the slower the pace of implementation of a change. Reversely, the higher r, the more flexible the institutions of a country are, thus the more rapid pursued policies can be fully operative. For obvious reasons, we refer to $r > 0$ as the *institutional flexibility* of a country. This parameter is central to our analysis.[8]

Second, our formulation considers that within a term, the policy at work is continuously changing until either the term finishes or the pursued policy is met, in which case the policy at work coincides with the pursued policy for the rest of the term. That is, for any $t \in \mathcal{N}$, if we denote by x_t the policy at work at the end of term t, this policy is either $x_t = \min\{p_t, x_{t-1} + r\}$ if $p_t \geq x_{t-1}$, or $x_t = \max\{p_t, x_{t-1} - r\}$ if $p_t < x_{t-1}$. In the paper we refer to x_t as the *final implemented policy* at term t. Note that since $x_t \in [-1,+1]$, when $r \geq 2$ we are in a situation in which a term is enough to move the policy from one extreme of the ideological space to the other. In this case, the final implemented policy x_t always coincide with the pursued policy p_t. That is, for each $t \in \mathcal{N}$, $x_t = p_t$. However, if $r < 2$, we are in a situation in which the period of time between two elections may not be enough for a pursued policy to be reached. In this case the pursued policy may differ from the final implemented policy.

At each time τ in a term, the utility to player $i \in \{M, L, R\}$ depends on the distance between the policy at work at τ and the player's ideal policy $\bar{x}^i$. Since the policy at work is continuously changing, the utility to player $i \in \{M, L, R\}$ in the stage game that corresponds to a term is:

$$u_i (z,y) = \int_{\tau=0}^{\tau=1} -\left(\bar{x}^i - \varkappa_\tau (z,y)\right)^2 d\tau, \tag{2}$$

where $\varkappa_\tau(z,y)$ is given by (1), with $x_{t-1} = z$ and $p_t = y$. Note that we assume that all the players have utility functions that are concave in the distance between the policy at work and the player's ideal policy, and that the two parties have preferences over the policy implemented both when in and out of office. This is in line with the literature pioneered by Wittman (1977, 1983) that considers policy-motivated parties.

[8] The literature on political economy assumes that pursued policies are fully operative right after an election. This means $r \to \infty$. To the best of our knowledge, our model is the first one that relaxes this assumption and that studies the effects of a country's institutional flexibility on its policies.

We assume that the game is infinitely repeated, corresponding each period t to a term. Each term starts with an election, where the voter chooses one party to elect, v_t, and the elected party announces a pursued policy p_t to be gradually introduced during the term. For any $t \in \mathcal{N}$, a history at time t, denoted by h^t, consists of the list of previously elected parties and their pursued policies,

$$h^t = ((v_0, p_0), (v_1, p_1), ..., (v_{t-1}, p_{t-1})),$$

with $v_0 \in \{L, R\}$ being randomly drawn with uniform probability. In order to make the game fully symmetric, we parametrize the initial policy by $x \in [0, 1]$ such that, if $v_0 = L$ then $x_0 = -x$ and if $v_0 = R$ then $x_0 = x$. We denote by H^t the set of all possible histories at time t, with $\mathcal{H} = \bigcup_{t \geq 1} H^t$.

Restricting attention to pure strategies, a strategy for the median voter in the infinitely repeated game is $s_M : \mathcal{H} \longrightarrow \{L, R\}$. Regarding political parties, a strategy for party L and R is $s_L : \mathcal{H} \longrightarrow [-1, 0]$ and $s_R : \mathcal{H} \longrightarrow [0, 1]$, respectively.[9] Let $s = (s_M, s_L, s_R)$ denote a (pure) strategy profile and S be the set of all (pure) strategy profiles.

Now, for any $s \in S$, player i's payoff in the infinitely repeated game is the discounted flow of the stage game payoffs,

$$U_i(s) = \sum_{t=1}^{\infty} \delta^{t-1} u_i(x_{t-1}, p_t), \tag{3}$$

where $\delta \in (0, 1)$ is the discount factor. The equilibrium concept is subgame perfect equilibrium. We denote by $S^* \subset S$ the set of equilibrium strategy profiles.

3 The stage game

Prior to the analysis of the repeated game, let us first solve for the equilibria of the stage game. Recall that we consider r to be finite. It has one important implication: Our stage game is path-dependent.[10] That is, at each term t, the players' payoffs depend not only on the actions played at t but also on the status quo policy denoted by x_{t-1}, which is the final implemented policy at the previous term $t-1$. Note that x_{t-1} depends furthermore on the actions players chose in former rounds, and so on and so forth. Despite this apparent complexity, given a status quo x_{t-1}, the equilibrium of the stage game at period t is unique in terms of outcomes, as shown next.

Proposition 1. *Let x_{t-1} be the status quo at t, which is the final implemented policy at term $t-1$. The equilibria of the stage game corresponding to period t are those satisfying conditions (i)-(ii) below:*

(i) The median voter chooses

$$v_t = \begin{cases} L \text{ if } x_{t-1} > 0, \\ R \text{ if } x_{t-1} < 0. \end{cases}$$

[9] For the sake of simplifying the definition of the parties' strategies, we assume that at every term t both parties propose a policy to implement, even if a party is not elected. This has no effect on the results.

[10] When $r \to \infty$ the stage game is not path-dependent. This is the standard approach in games of repeated elections.

(ii) If elected, party R chooses $p_t \in [\min\{x_{t-1}+r, 1\}, 1]$*, and party L chooses* $p_t \in [-1, \max\{x_{t-1}-r, -1\}]$.

Proof. In the Appendix.

There are some important comments on the result of Proposition 1. First, for a given x_{t-1} and r, all the equilibria (if not unique) are outcome equivalent. Indeed, it is the final implemented policy x_t what determines the equilibrium outcome, hence the payoffs, and policy x_t is either $\min\{x_{t-1} + r, 1\}$ (if $v_t = R$) or $\max\{x_{t-1} - r, -1\}$ (if $v_t = L$), independently of p_t. Related to this, note that a particular and special equilibrium is the one in which parties propose their partisan policies and the voter votes for the party whose ideal policy is *farthest* away from the status quo. For obvious reasons, we will refer to this equilibrium of the stage game as the *partisan equilibrium.* The intuition for this result is provided below.

Second, note that the equilibria described in Proposition 1 and in particular the prediction of the partisan equilibrium is sharply in contrast to the idea of policy convergence and the median voter theorem, which states that in equilibrium political parties propose the median voter's ideal policy. The reason why our framework does not produce convergence to the median is because in our model the elected party is free to propose any policy. Recall that we do not model any electoral campaign, which is the same as modeling it but considering that any announcement made during this period is non-binding. This is in line with the work by Alesina (1988), who shows that when parties are policy-motivated, platforms are non-binding and voters are forward-looking, convergence to the median is never an equilibrium. The logic to the result in Alesina (1988) also explains the result in our case. What is different between Alesina's work and our work is that in Alesina there is no incentive for a voter to vote for the party whose ideal policy is farthest away from the previous policy. This incentive, which will be key to explain the alternation result that will appear next when analyzing the repeated game, crucially depends on r being finite, hence on the gradual implementation of policies. In fact, note that if r were rather infinite, as in Alesina (1988), the median voter would never have a strict incentive to change her previous vote (hence the policy implemented), but she would simply vote for the party with a party line closer to her own. If $-\bar{x}^L = \bar{x}^R$, as it is the case in our model, she would vote for any party indistinctively.

4 Partisan-alternating strategy profile: Stability and (in)efficiency

Next we move to the analysis of the repeated game. We start defining a strategy profile that consists on the repetition, from term to term, of the players' behavior in the partisan equilibrium of the stage game. We will refer to this as the *partisan-alternating strategy profile.*

Definition 1. ***(Partisan-alternating strategy profile)*** *A profile is a partisan-alternating strategy profile* $s^{PA} \in S$ *if at the election of each term* t*:*

1. *The median voter chooses*

$$v_t = \begin{cases} L \text{ if } x_{t-1} > 0, \\ R \text{ if } x_{t-1} < 0, \\ v \in \{L, R\} \backslash v_{t-1} \text{ otherwise.} \end{cases} \tag{4}$$

2. *Party $i \in \{L, R\}$ proposes its ideal policy $\bar{x}^i$. That is, $p_t = -1$ if $v_t = L$ and $p_t = 1$ if $v_t = R$.*

Note that if $r \geq 2$, s^{PA} implies $x_t \in \{-1, 1\}$ $\forall t$, with $x_t \neq x_{t-1}$. That is, the final implemented policy swings back and forth, from -1 to 1 and viceversa, from election to election. In the case $r < 2$, the voter's strategy at t prescribes to start voting for the party whose party line is farther away from the status quo at $t-1$ and to continue doing so until the policy at work trespass the cutoff value of 0. From that term onwards, s^{PA} describes an alternation of political parties in office, with policies swinging back and forth with distance $r = |x_t - x_{t-1}|$.

We are now in position to study whether the partisan-alternating strategy profile constitutes an equilibrium and whether it is efficient. With respect to the first idea, note that as in any induced multistage game, the repetition of the equilibrium of the stage game every round t defines a subgame perfect equilibrium of the repeated game. In this sense, we can assert that the partisan-alternating strategy profile constitutes an equilibrium of the repeated game, for any discount factor. This idea constitutes point (i) of Proposition 2 below. Points (ii) and (iii) of this proposition announce results for the case of very impatient players. To this situation, Proposition 2 states that the partisan-alternating strategy profile is either the only equilibrium (it is so if $r \geq 2$) or, if it is not unique, then all the equilibrium strategy profiles are outcome equivalent to s^{PA} (it is so if $r < 2$).

Proposition 2. *(i) $s^{PA} \in S^*$ for all $\delta \in (0, 1)$. (ii) If $r \geq 2$, there exists $\bar{\delta} \in (0, 1)$ such that for all $\delta < \bar{\delta}$, $S^* = \{s^{PA}\}$. (iii) If $r < 2$, there exists $\bar{\delta} \in (0, 1)$ such that for all $\delta < \bar{\delta}$, all the equilibria are outcome-equivalent to s^{PA}.*

Proof. In the Appendix.

The partisan-alternating equilibrium illustrates very well the logic behind the alternation result. The idea is simple: Given the behavior of the parties and the fact that r is finite, to alternate and so, to vote each election to a different party is a way for the voter to keep policies at moderate positions, at least for some period of time every term. If the voter were rather not to alternate, the policy implemented would never be moderate but a extreme one; hence she would clearly be worse off. Note that as previously pointed out, the alternation result crucially depends on r being finite, for if it were infinite the utility to the voter from both an alternation and a no-alternation strategy would be the same, as the policy implemented at any time at any term would always be an extreme one, in both cases.

Next, we analyze whether the partisan-alternating equilibrium is efficient, i.e., it maximizes individual players' payoffs. Recall that by our symmetry assumption, initial conditions are either $(v_0, x_0) = (R, x)$

or $(v_0, x_0) = (L, -x)$, each with probability $1/2$. Now, for each t, let us define $|x_t^{PA}|$ as the absolute value of the final implemented policy at term t in the path induced by the strategy s^{PA}, and $\hat{t} = \min\{t \in \mathcal{N} : -x + t \cdot r \geq 0\}$ as the first term in which alternation occurs. The result is the following.

Proposition 3. *s^{PA} is Pareto dominated by any strategy profile in which, for all $t \geq 1$, v_t is given by (4) and $|p_t| = a \in [0, \min\{|x_{\hat{t}}^{PA}|, |x_{\hat{t}+1}^{PA}|\})$. Additionally, the lower a, the more efficient the strategy is.*

Proof. In the Appendix.

Proposition 3 shows that the partisan-alternating equilibrium is Pareto dominated, hence it is inefficient. To see the reason for this, let us start with the median voter, for which the argument is straightforward: Since the voter's ideology is 0, the more extreme the a-profile, the lower the voter's utility. Regarding the political parties, recall that they have extreme ideologies. Despite it, the result above says that their individual payoffs are smaller in the partisan-alternating equilibrium than in any other more moderate alternating profile. Two ideas explain this result. First, the symmetry in the initial conditions and second, the concavity of the players' utility function. The first idea makes that any initial gain to a party from being in office in the first period is compensated by the reversed situation in which it is out of office (recall that both situations have probability $1/2$ to occur). The second idea makes that a party loses a lot when, while being out of office, the policy implemented is very far away from the party's ideal policy. The combination of both ideas yields the result.

The same kind of reasoning can also be used to explain why the profile $a = 0$ Pareto dominates any other symmetric a-profile, with $a = 0$ corresponding to the median voter result. Despite $a = 0$ being the most efficient outcome, it is interesting to note that the strategy profile described in Proposition 3 is not an equilibrium, neither for $a = 0$ nor for $a \in (0, \min\{\left|x_{\hat{t}}^{PA}\right|, \left|x_{\hat{t}+1}^{PA}\right|\}]$. The reason is that such a strategy profile does not punish deviations. To account for this problem, in the next section we introduce a family of strategy profiles that can be Pareto ranked and that punishes deviations with the reversion to the inefficient but stable strategy profile s^{PA}. We will show that such Pareto improving strategies can constitute an equilibrium.[11]

5 Stability of Pareto improving alternating strategy profiles

In this section we aim to identify the strategy profiles that Pareto dominate s^{PA} and still constitute an equilibrium of the game for the widest range of the discount factor δ. Following the standard approach in the literature of repeated elections, we will concentrate on equilibria in which political parties use stationary strategies (see e.g. Duggan (2000), Banks and Duggan (2008), Bernhardt et al. (2009, 2011) and Van Weelden (2013)). We will further assume that strategies are symmetric, of the class $|p_t| = a$. Without loss of generality we consider $a \in [0, \hat{x}]$, with $\hat{x} = \min\{\frac{r}{2}, 1\}$ being the most extreme policy that

[11] This family includes the partisan-alternating strategy profile as an extreme case.

can be implemented at any term, given r. As for the median voter, we will follow Van Weelden (2013) and consider that the voter's strategy may depend both on her payoff in the last term and in the identity of the governing party. In this way, a party's reelection can be contingent on its policy choice.[12] We will refer to an element of this class of (symmetric and stationary) strategy profiles as an *alternating a-profile*, with $a \in [0, \hat{x}]$, and will denote it by $\widetilde{s}^a$.[13] We will further denote by $\widetilde{S} \equiv \{\widetilde{s}^a\}_{a\in[0,\hat{x}]} \subset S$ the set of (symmetric and stationary) alternating profiles, parametrized by a.

Definition 2. ***(Alternating*** a***-profile)*** *For each* $a \in [0, \hat{x}]$*, a strategy profile is an alternating* a*-profile* $\widetilde{s}^a \in S$ *if:*

(1) At the election of each term t*, the median voter chooses* v_t *according to (4).*

(2) Political parties propose:

$$\text{(i) At } t = 1,\ |p_1| = \begin{cases} a \text{ if } v_1 \text{ is given by (4)}, \\ 1 \text{ otherwise.} \end{cases}$$

$$\text{(ii) For any } t > 1,\ |p_t| = \begin{cases} a \text{ if, for all } t' \leq t,\ v_{t'} \text{ is given by (4) and } |p_{t-1}| = a, \\ 1 \text{ otherwise.} \end{cases}$$

Additionally, with some abuse of terminology, and in order to make the stationary path induced by the strategy profile $\widetilde{s}^a$ *already present in the initial condition, we will consider that* $x = a$*, i.e., initial conditions are either* $(v_0, x_0) = (R, a)$ *or* $(v_0, x_0) = (L, -a)$*, each with probability* $1/2$*.*

Two comments are worth mentioning here. First, the alternating a-profile prescribes the voter to vote, each election, to a different party; and the political parties to propose policy $|p_t| = a$ at t if and only if no player has previously deviated. In case of a deviation, it prescribes the parties to propose their party lines and the voter to keep on alternating.[14] Second, the alternating a-profile imposes that the initial conditions are in accordance with the strategy profile. This formulation allows us to abstract from the particularities that different initial conditions may introduce and to focus on the interesting part of the analysis, which is to understand when more efficient outcomes can be achieved in equilibrium.

Note that all the elements of $\widetilde{S}$ can be Pareto ranked, according to Proposition 3. The ranking establishes that any a-profile, with $a < 1$, Pareto dominates the partisan-alternating profile s^{PA}, and that

[12] As Van Weelden (2013) indicates: "*This still incorporates elements of stationarity, however, as she (the voter) gives each candidate a clean slate and evaluates him based only on her utility in the last period*".

[13] Note that in the case $r \geq 2$, $\hat{x} = 1$. In the case $r < 2$, we could extend the definition of $\widetilde{s}^a$ to all $a \in [0, 1]$, but it is straightforward to see that for each $a \in (\hat{x}, 1]$, $\widetilde{s}^a$ is equivalent to $\widetilde{s}^{\hat{x}}$.

[14] The punishment pattern that we consider consists on the natural idea of playing the partisan-alternating equilibrium, which is the repetition of the partisan equilibrium of the stage game, every term t. Back since Abreu (1983), it is well known that in order to keep cooperation, players can use worse punishments than the reversion to the one-shot Nash equilibrium. In this paper we abstract from these more general but artificial punishment patterns, which typically include punishment phases in which players (including the deviator himself) need to cooperate for a pre-specified number of periods in order to punish the deviator. Here, we follow Alesina (1988), who also uses the equilibrium of the stage game as the punishment in the repeated game.

the smaller the a, i.e., the milder the swings in policy from one election to the other, the more efficient the outcome is.

Corollary 1. *For all $a', a'' \in [0, \hat{x}]$ such that $a' < a''$, $\tilde{s}^{a'}$ Pareto dominates $\tilde{s}^{a''}$.*

Proof. The result follows from Proposition 3.

With the efficiency result in mind, next we move to the equilibrium analysis. Let $\tilde{S}^* = S^* \cap \tilde{S}$ be the set of (symmetric and stationary) *alternating equilibria*. The first result is the following.

Proposition 4. *(i) $\tilde{s}^{\hat{x}} \in \tilde{S}^*$.*

(ii) If $r \leq \sqrt{3}$, then $\tilde{S}^* = \{\tilde{s}^{\hat{x}}\}$.

(iii) If $r > \sqrt{3}$ there exists a function $\bar{a}(r)$ such that, for all $a \in [0, \bar{a}(r))$, there is a threshold $\bar{\delta}(a, r) \in (0, 1)$ such that $\tilde{s}^a \in \tilde{S}^*$ if and only if $\delta \geq \bar{\delta}(a, r)$. If $a \in [\bar{a}(r), \hat{x})$, then for each $\delta \in (0, 1)$, $\tilde{s}^a \notin \tilde{S}^*$.

(iv) The function $\bar{a}(r)$ is continuous; if $r < 2$, $\bar{a}(r)$ is strictly increasing, with $\bar{a}(r) \in (0, r-1]$ and $lim_{a \to \sqrt{3}^+} \bar{a}(r) = 0$; if $r \geq 2$, $\bar{a}(r) = 1$.

Proof. In the Appendix.

The results of this Proposition are illustrated in Figure 1, which depicts the amplitude of the alternating equilibria as a function of the country's institutional flexibility. In the vertical axis we represent parameter a, which is a measure of policy polarization. Note that the higher a, the higher the extremism of the policies implemented in the alternating a-equilibria. In the horizontal axis we represent parameter r, which captures the country's institutional flexibility. Recall that the higher r the higher the country's institutional flexibility.

[Fig. 1 about here]

We observe that the alternating $\hat{x}$-profile, with $\hat{x} = \min\{\frac{r}{2}, 1\}$, is always an equilibrium. This is point i) of Proposition 4. It is important to note that since the $\hat{x}$-profile is equivalent to the partisan-alternating profile s^{PA} (see footnote 13), and the latter is an equilibrium for any $\delta \in (0, 1)$, then $\tilde{S}^* \neq \emptyset$. Points ii) and iii) characterize the alternating a-equilibria as a function of the institutional flexibilities of a country. Point ii) refers to the case $r \leq \sqrt{3}$. Here, the result is that the only alternating a-equilibrium conveys political parties that propose, every term t, the most extreme policy compatible with r, i.e., $|p_t| = \hat{x}$.[15] Point iii) refers to the case $r > \sqrt{3}$. Here we obtain that new alternating a-profiles can be sustained as equilibria. In particular, we obtain that the set of alternating a-equilibria is $\tilde{S}^* = \{\tilde{s}^a\}_{a \in [0, \bar{a}(r))} \bigcup \{\tilde{s}^{\hat{x}}\}$,

[15] Note that $\sqrt{3} < 2$. Hence, $r < \sqrt{3}$ illustrates a situation in which a term is not enough to move the policy from one extreme of the ideological space to the other.

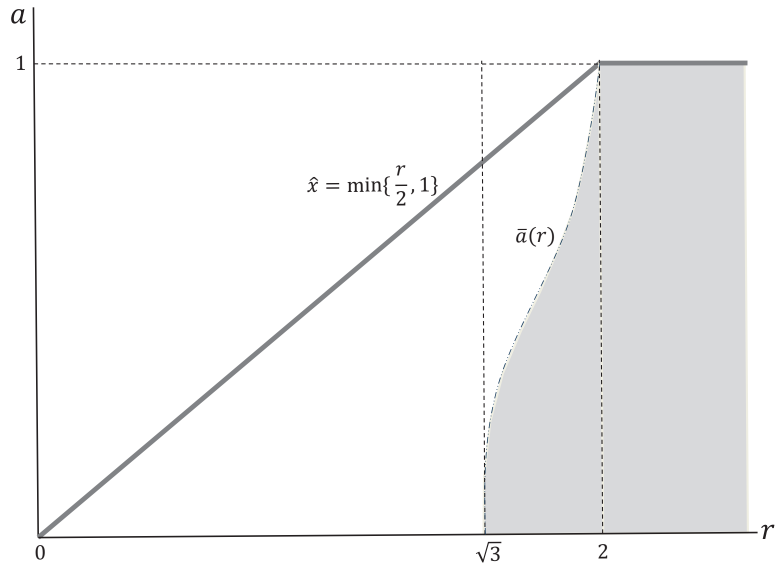

Figure 1: We represent $\hat{x}$ and function $\bar{a}$, which defines the amplitude of the alternating equilibria, as a function of a country's institutional flexibility r, for $\delta > \bar{\delta}(a, r)$. The equilibria are given by the thick grey lines and the grey area.

provided that players are sufficiently patient, i.e., $\delta \geq \bar{\delta}(a, r)$, with threshold $\bar{a}(r)$ having the properties described in point iv) of the proposition.

In words, the results in Proposition 4 suggest that rigid institutions limit the capacity of politicians to introduce rapid changes and to implement their partisan policies, but induce them to push policies as far as legal, political or social restrictions allow. As a consequence, in equilibrium, alternation features political polarization. Interestingly, our results also suggest that countries with flexible institutions can feature alternation with lower levels of political polarization (including convergence to the median voter's ideal policy), hence more efficient outcomes. How can we explain that even though flexible institutions give freedom to the parties to implement extremist policies, it does not necessarily yield greater polarization? To have an intuition for this result, note that a political party that thinks on deviating has to take into account two effects: The first one is that by deviating and taking the policy closer to its ideal policy, the party will enjoy higher utility when in office. The second effect is that after a deviation the opponent party will always propose it's ideal policy, something that hurts the deviating party. The key point is that for the second effect to really matter (in the sense of a costly punishment), r must be sufficiently high so that the opponent party can take the policy close to its party line when its turn in office comes. Otherwise, the deviating party can use all the loose that r allows to take the policy all the way to its ideal one, thus reducing the punishment that it will experience when the opponent party gets into office. It explains why small values of r induce parties to exploit all the loose that r allows, hence impeding any moderate alternation (unless r imposes so). It also explains why higher values of r allows for an alternation of parties with more efficient policies (including the median voter's ideology).

A corollary of Proposition 4 is presented next. It states the conditions under which the Pareto superior profile $a = 0$ is an equilibrium. The result says that, in our model, the median voter's result is an equilibrium provided that the country has sufficiently flexible institutions and that political players are sufficiently patient.

Corollary 2. *The efficient strategy profile* $\widetilde{s}^0$ *is an equilibrium if and only if* $r > \sqrt{3}$ *and* $\delta \geq \bar{\delta}(0, r) \in (0, 1)$.

Proof. The result follows from Proposition 4.

Last, we study which of the previous alternating equilibria are robust to variations in the discount factor. We know that the inefficient partisan-alternating profile s^{PA} constitutes an equilibrium for any discount factor $\delta \in [0, 1]$. So the question now is which other more efficient profiles remain equilibria when players are sufficiently impatient. We will refer to this class of alternating a-profiles as a *robust-interior equilibrium strategy*, and will denote it by $\widetilde{s}^{\hat{a}}$. Note that the $\hat{a}$-profile will constitute a second best in terms of stability.

Definition 3. ***(Robust-interior equilibrium strategy)*** *A strategy* $\widetilde{s}^{\hat{a}} \in \widetilde{S}$, *with* $\hat{a} \in [0, 1)$, *is a robust-interior equilibrium if there exist* $\hat{\delta} \in (0, 1)$ *such that:*

(i) For all $\delta \geq \hat{\delta}$, $\widetilde{s}^{\hat{a}} \in \widetilde{S}^*$.

(ii) For all $\delta < \hat{\delta}$, $\widetilde{S}^* = \{\widetilde{s}^{\hat{x}}\}$.

We are now in position to characterize the robust-interior equilibria of the game. This is done next.

Proposition 5. *(i) If* $\sqrt{3} < r < 2\sqrt{2}$, *there exists a unique robust-interior equilibrium.*

(ii) If $r \leq \sqrt{3}$ *or* $r \geq 2\sqrt{2}$, *there is no robust-interior equilibrium.*

(iii) Let $\hat{a}(r) \in [0, 1)$ *be the policy such that, given* $r \in (\sqrt{3}, 2\sqrt{2})$, $\widetilde{s}^{\hat{a}(r)} \in \widetilde{S}$ *is a robust-interior equilibrium. Then,* $\hat{a}(r)$ *is increasing in* r, $\lim_{r \to \sqrt{3}} \hat{a}(r) = 0$ *and* $\lim_{r \to 2\sqrt{2}} \hat{a}(r) = 1$.

(iv) Let $\hat{\delta}(r) = \bar{\delta}(\hat{a}(r), r)$, *with* $\bar{\delta}(\cdot)$ *as defined in Proposition 4. Then,* $\lim_{r \to \sqrt{3}^+} \hat{\delta}(r) = 1$ *and, for all* $\sqrt{3} < r < 2\sqrt{2}$, $\hat{\delta}(r)$ *is strictly decreasing in* r.

Proof. In the Appendix.

The results of this Proposition are illustrated in Figure 2, that consists of two panels. The upper panel represents the amplitude of both the robust-interior equilibrium (thin black curve) and the partisan-alternating equilibrium (thick grey line), as a function of the country's institutional flexibility. The bottom panel represents the pairs (r, δ) for which the robust-interior equilibrium (dark grey) and the partisan-alternating equilibrium (dark and light grey) exist.

[Fig. 2 about here]

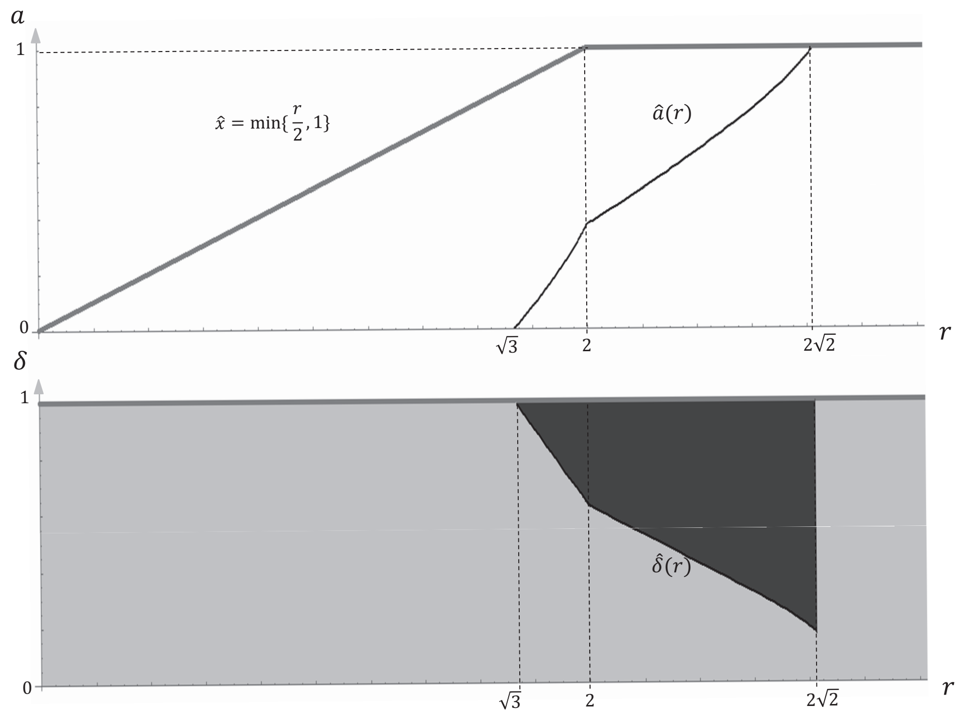

Figure 2: In the upper panel we represent function $\hat{a}$ (thin black curve), which defines the amplitude of the robust-interior equilibrium; and $\hat{x}$ (thick grey line), which defines the amplitude of the alternating $\hat{x}$-equilibrium. In the bottom panel we represent in grey (both dark and light) the pairs (r, δ) for which the alternating $\hat{x}$-equilibrium exists; and in dark grey the pairs (r, δ) for which the corresponding robust-interior equilibria (represented above) exist. In both graphs, the horizontal axis represents the country's institutional flexibility r.

We observe that a robust-interior equilibrium only exists for intermediate values of r, more precisely, for $\sqrt{3} < r < 2\sqrt{2}$. These are points i) and ii) of Proposition 5. Point iii) of the proposition states that within the range $\sqrt{3} < r < 2\sqrt{2}$, the higher the institutional flexibility of a country r, the more extreme and thus inefficient the robust-interior equilibrium is. Last, point iv) states that the higher the institutional flexibility of a country, the smaller the requirement on δ that allows us to sustain the corresponding robust-interior equilibrium.

In words, the results in Proposition 5 suggest that only when a country's institutions are neither very rigid nor very flexible, we can have a system of political alternation with stable and moderate policies, even if the players are impatient (up to a certain level). Interestingly, in this case, the implemented policies at every term stand midway between the median voter's ideology and the party's ideal policies, i.e., they are middle-of-the-road policies. We also obtain that when the conditions on r do not hold, in particular when either $r \leq \sqrt{3}$ or $r \geq 2\sqrt{2}$, the only equilibrium that can be sustained for sufficiently impatient players is the inefficient partisan-alternating equilibrium, in which policies swing back and forth from one election to the next as much as the institutions of the country allow. Coming back to the interesting case of $2\sqrt{2} < r < \sqrt{3}$, note also that the higher the players' impatience, the higher the value of r that sustains the robust-interior equilibrium. Thus, for example, when the players' patience moves from $\delta \sim 0.8$ to $\delta \sim 0.5$ (so that impatience in the society increases), the requirement on r to have a robust-interior equilibrium increases from $r > 1.9$ to $r > 2.3$. In that sense, our results suggest that more impatient societies require more flexible institutions. Last, note that within the range $2\sqrt{2} < r < \sqrt{3}$, the higher the institutional flexibility of a country the more extreme (and thus inefficient) the robust-interior equilibrium is. Reversely, the more rigid the institutions the more efficient such an equilibrium is. In the limit, i.e., when $r \longrightarrow \sqrt{3}$, we obtain that the Pareto efficient outcome, the $\widetilde{s}^0$ profile, can be sustained as a robust-interior equilibrium of the game, provided that the condition on δ is met.

6 Discussion

This section discusses some relevant variations of the model: i) Rent seeking political parties, ii) asymmetric distribution of voters and/or political parties, iii) inefficiencies associated to a change in government and iv) alternation every two terms. All the extensions below consider the punishment pattern that we use in the main body of the text (after a deviation, parties propose their partisan policies forever), and all the extensions, but for the last one, focus on (stationary and symmetric) alternating a-profiles.

6.1 Office-motivated political parties

A common assumption in the political economy literature is to consider that politicians receive rents from holding office. Let P denote this rent. In this section we incorporate this payoff into the utility function of the political parties that, for other aspects, is given by expressions (2) and (3). Of course, the rent P is exclusive to the party in office, hence it only adds utility when the party is in power.

Note that as long as political parties are policy-motivated, as given by expression (2), the introduction of office-concerns, i.e., a rent P for holding office, will not affect the outcomes of the game. To see it, take any r and focus on the party in power at a particular t, i.e., the one that may consider the possibility of deviating. Note that both in the equilibrium and out of the equilibrium path, the strategy of the median voter is given by expression (4), which roughly speaking prescribes alternation. Because the voting strategy is invariant to deviations, the number of terms that a particular party will be in office and the order of these terms will be the same both in the equilibrium and out of the equilibrium path. Accordingly, introducing P will affect a party's payoff both in the equilibrium and out of the equilibrium path in the same way; hence, it will not affect the outcome of the game. Accordingly, we can conclude that the results of the paper are robust to the consideration of a rent P for holding office.

6.2 Asymmetric distribution of voters and/or political parties

The model considered so far assumes that political parties have bliss points which are equidistant to the median voter's ideology. In this section we break this symmetry. In particular, without loss of generality, we consider players with the following ideal policies: $|\bar{x}^L| \in (0,1)$, $\bar{x}^R = 1$ and $\bar{x}^M = 0$. Note that in this case $|\bar{x}^L - \bar{x}^M| < |\bar{x}^R - \bar{x}^M|$, i.e., the median voter is closer to the left-wing party than to the right-wing party.[16]

Suppose first that r is small. Moreover, suppose $\frac{r}{2} \leq |\bar{x}^L|$. Here, the results in the main body of the paper hold and so we can state that $\widetilde{S}^* = \{\widetilde{s}^{\hat{x}}\}$. The reason is simple: Because r is very small, any a-profile that does not exploit all the loose allowed by r cannot be an equilibrium, as party R always gains by deviating (it could also be the case to party L). Additionally, the strategy $\widetilde{s}^{\hat{x}}$ is an equilibrium because players maximize their payoffs, given the strategies of the other players. Suppose now $r > 2|\bar{x}^L|$. An observation here is that the profile $\widetilde{s}^a$ with $a = |\bar{x}^L|$, in which party L proposes its ideal policy (and party R the symmetric one), is never an equilibrium. To see why, note that party R gains again by deviating, as by doing so it can take the policy closer to its policy and suffer no punishment (when R is out of office, the policy proposed by L will always be $a = |\bar{x}^L|$).

Finally, consider r to be high enough. Recall that in the main body of the paper there are multiple equilibria in this case, with the Pareto efficient outcome $a = 0$ being one of the equilibria. It is interesting to note that to break the symmetry has one important consequence: The efficient profile $a = 0$ may no longer be an equilibrium (nor the profiles $a \sim 0$). To see it, suppose $r = 2$. It is clear that if $|\bar{x}^L|$ is small, then party R would win by deviating, as the punishment it would incur if deviating is not costly enough to prevent the deviation. This idea suggests that only if $|\bar{x}^L|$ is sufficiently high we could have an equilibrium in which the policy implemented is the median voter's ideal policy. The reason for this simple: How effective the punishment is in this case depends on r (as in the main body of the text) and on the parties' ideologies $|\bar{x}^L|$ and $|\bar{x}^R|$. When $|\bar{x}^L||$ is small, the punishment to party R is small as well; hence equilibria are difficult to sustain.

Summarizing, the results on this variation of the model suggest that breaking the symmetry (while using the punishment pattern of the text) would push equilibria towards the partisan lines, making it more difficult for the moderate (and more efficient) alternating profiles to survive as equilibria of the game.

6.3 Inefficiencies associated to a change in government

Suppose now that there are costs to the voters from the alternation. These costs illustrate any sort of switching costs and/or inefficiencies that accompany any change in government. Let c denote this cost and let us take it as a fixed cost. Of course, the effect of this cost is to reduce the voter's gain from the

[16] We can break the symmetry of the main body of the paper in at least two ways: i) Changing the location of one party (and keeping constant the location of the median voter); ii) changing the location of the median voter (and maintaining the location of the parties). Both alternatives are similar in terms of the resulting setup and outcomes.

alternation.

Suppose c is small enough so as not to offset the voter's gain from alternation. In this case, even with cost c, the voter will find it optimal to alternate; hence the equilibrium/a that we predict are the same than those in the case without c. Suppose now c is high enough so as to affect the voter's optimal choice, but not as high as to make alternation unprofitable to the voter. Here, consider first an alternating a-profile where policies are moderate, i.e., a is small. In this case, it is reasonable to consider that the a-profile constitutes an equilibrium even if the voter has to pay cost c for alternating, since the payoff to the voter from the alternation is high when a is small and if deviating (and voting for the same party forever, which we believe is the best deviation) her payoff would be smaller. Consider now an alternating a-profile with more extremist policies, i.e., a is high. Because the payoff to the voter from the alternation decreases in a, it is easy to see that it will exist a cost c for which this alternating a-profile is not an equilibrium. It suggests the existence of a threshold for the cost such that for values of the cost higher than the threshold, only the alternating a-profiles with moderate policies can be an equilibrium.

Putting all together, the results in this section suggest that introducing a cost for the alternation would probably eliminate the equilibria in which alternation conveys drastic policy changes and would not affect the equilibria with moderate alternation. In this sense, we predict that in the presence of switching costs and/or inefficiencies that imply a cost to consumers, if there are equilibria with alternation, then alternation would probably convey moderate (then more efficient) policies.

6.4 Alternation every two terms

Last, we discuss whether our model can produce equilibrium in which alternation occurs with a frequency different to one. Casual observation shows that in many situations alternation occurs every two terms. For example, all the last three ex-presidents of the US: Barack Obama, George W. Bush and Bill Clinton, were each given two mandates. In this section we take this case to the study and discuss whether our model can shed light on any rationality behind this empirical observation.

First, note that for alternation every two terms to be an equilibrium, a necessary condition is that the median voter finds it optimal to vote in such a way (to vote for a different party only after the second term). It requires the parties not to propose their partisan policies the two terms in a row that they each are in office. For if they did so, the median voter would find it optimal to alternate after the first term, as it would guarantee a period of time with moderate policies that otherwise she will not enjoy. Hence, alternation every two terms requires the elected party to propose different policies in the first and the second mandate.

In this line, consider the following profile of strategies, with r being high enough (for simplicity): The median voter alternates every two periods and the elected party proposes the voter's ideal policy in the first term and the party's partisan policy in the second term, i.e., $p_t = 0$ at a party's first term in office and $|p_{t+1}| = 1$ when $v_t = v_{t+1}$ at a party's second term in office.[17] Note that given such strategies, for δ sufficiently high, no player would benefit from a unilateral deviation: Neither the voter would strictly gain by deviating, nor the parties gain by proposing a different policy. Accordingly, the strategies above would constitute an equilibrium of the game.

In a similar vein, we can consider a profile of strategies in which everything is as before except for the policies proposed in the first and the second mandate that a party is given. Suppose here that the order is reserved, i.e., the elected party now proposes its partisan policy in the first term and the voter's ideal policy in the second term. We believe that the same type of reasoning that explains the equilibrium above serves us to say that, for sufficiently patient players, the profile may also constitute an equilibrium of the game.

Interestingly, the last profile describes a situation that seems to fit very well with real world observation. In fact, casual evidence suggests that in some occasions parties in governments start their mandates with partisan policies and it is during second mandates that more moderate policies come into the agenda. Professor Daniel W. Drezner, in reference to former US Presidents Ronald Reagan, Bill Clinton and George W. Bush and their second term policies, argues: "Recent second-termers have not reverted to their ideological bliss point -if anything, it's been the reverse; they've tacked away from their starting point".[18]

7 Conclusions

Despite political alternation is a fundamental feature of established democracies, there is not yet a clear answer as to why parties take turns in power. Political disaffection and disillusion with the policies implemented by ruling parties is surely something to take into account. Beyond this argument, our work contributes to this discussion by proposing a new ingredient to explain political turnover: Institutional rigidities.

Our work speaks to (at least) two literatures: The aforementioned one on the logic to alternation, and the research on institutions and their effects on policymaking. To the former, our contribution is to propose a new ingredient, namely institutional rigidities, that helps explain why rational voters may find it optimal to vote, each election, to a different political party. To the later, we posit a non-monotonic relationship between institutional flexibility and policy extremism.

The results in this paper have a clear policy implication: Since moderate alternating profiles are preferred (Pareto superior) to extreme alternating profiles, a country's optimal design of its institutions must account for both rigidity (that constrains the power of the government to implement capricious policies) and flexibility (that gives the executive certain freedom to allow rapid action in response to new circumstances), in a very precise combination.

Our results allow us to understand how moderate or extreme policies will be as a function of the

[17] Recall that we use the same punishment pattern as in the text.

[18] See "The dirty little secret about second-term presidents", published in the *Foreign Policy* magazine on March 29, 2012.

institutional rigidities of a country. They also tell us which equilibria are expected to survive as a function of the players' discount factor. However, how much flexibility is optimal and whether this is something we can measure in the real world are important and challenging questions that remain unexplored. As usual, moving from theory to empirical work is not easy, as some of the essential features of the model are unobservable (such as the speed of implementation of new policies). Beyond the anecdotal evidence discussed in the Introduction that supports our results, we consider that testing our predictions with real world data is a very interesting exercise. Another important question is to better understand the incumbent's advantage in setting the status quo, a privilege that is here due to the gradual implementation of policies. How important this advantage may be in shaping future policies and outcomes is another interesting question that we leave for future research.

8 Appendix

Proof of Proposition 1

Given (1)-(2), each party maximizes its utility by proposing a pursued policy p_t such that, in case of being elected, the implemented policy at the end of the term, x_t, is as close as possible to the party's partisan lines. Since $\bar{x}^R = 1$, all $p_t \in [\min\{x_{t-1} + r, 1\}, 1]$ are outcome equivalent for party R, yielding $x_t = \min\{x_{t-1} + r, 1\}$. Likewise, since $\bar{x}^L = -1$, all $p_t \in [-1, \max\{x_{t-1} - r, -1\}]$ are outcome equivalent for party L, yielding $x_t = \max\{x_{t-1} - r, -1\}]$. This proves (i).

Since the median voter's ideology is $\bar{x}^M = 0$, given (i), $u_M(x_{t-1}, x_t)$ defined by (1)-(2) is maximized by choosing $v_t = L$ ($v_t = R$) if $x_{t-1} > 0$ ($x_{t-1} < 0$). In case $x_{t-1} < 0$, both choices yield the same utility. This proves (ii). Q.E.D.

Proof of Proposition 2

Point (i) follows from the fact that the strategy profile s^{PA} prescribes to play the repetition of an equilibrium of the stage game (cf. Proposition 1) at every period.

When $r \geq 2$, by Proposition 1, there is a unique equilibrium of the stage game in which at each t, if elected, party R chooses $p_t = 1$, party L chooses $p_t = -1$, and the median voter chooses the party whose ideal policy is farther away from x_{t-1}. Since s^{PA} consists on the repetition of the unique equilibrium of the stage game, for δ low enough s^{PA} is the unique equilibrium of the repeated game. This proves point (ii).

When $r < 2$, by Proposition 1, there are multiple equilibria in the stage game, but all of them are outcome equivalent. Hence, for δ low enough any equilibrium of the repeated game must produce a path that replicates the (unique) equilibrium outcome of the stage game at every period t. Thus, any equilibrium of the repeated game is outcome equivalent to s^{PA}. This proves point (iii). QED.

Proof of Proposition 3

The utility to player $i \in \{M, L, R\}$ from strategy s^{PA} is

$$\begin{aligned}
U_i(s^{PA}) &= \frac{1}{2}\Bigg(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(-x+kr, -x+(k+1)r) + \delta^{\hat{t}-1} u_i(-x+(\hat{t}-1)r, -x+\hat{t}r) + \delta^{\hat{t}} u_i(-x+\hat{t}r, -x+(\hat{t}-1)r) \\
&\quad + \delta^{\hat{t}+1} u_i(-x+(\hat{t}-1)r, -x+\hat{t}r) + \delta^{\hat{t}+2} u_i(-x+\hat{t}r, -x+(\hat{t}-1)r) + ...\Bigg) \\
&\quad + \frac{1}{2}\Bigg(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(x-kr, x-(k+1)r) + \delta^{\hat{t}-1} u_i(x-(\hat{t}-1)r, x-\hat{t}r) + \delta^{\hat{t}} u_i(x-\hat{t}r, x-(\hat{t}-1)r) \\
&\quad + \delta^{\hat{t}+1} u_i(x-(\hat{t}-1)r, x-\hat{t}r) + \delta^{\hat{t}+2} u_i(x-\hat{t}r, x-(\hat{t}-1)r) + ...\Bigg) \\
&= \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(-x+kr, -x+(k+1)r) + \frac{\delta^{\hat{t}-1} u_i(-x+(\hat{t}-1)r, -x+\hat{t}r) + \delta^{\hat{t}} u_i(-x+\hat{t}r, -x+(\hat{t}-1)r)}{1-\delta^2}\right) \\
&\quad + \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(x-kr, x-(k+1)\,r) + \frac{\delta^{\hat{t}-1} u_i(x-\left(\hat{t}-1\right)r, x-\hat{t}r) + \delta^{\hat{t}} u_i(x-\hat{t}r, x-\left(\hat{t}-1\right)r)}{1-\delta^2}\right),
\end{aligned}$$

which, since $\left|-x+\hat{t}r\right| = \left|x_{\hat{t}}^{PA}\right|$ and $\left|-x+\left(\hat{t}-1\right)r\right| = \left|x_{\hat{t}-1}^{PA}\right|$, can be rewritten as

$$\begin{aligned}
U_i(s^{PA}) &= \sum_{k=0}^{k=\hat{t}-2} \delta^k \frac{u_i(-x+kr, -x+(k+1)\,r) + u_i(x-kr, x-(k+1)\,r)}{2} + \\
&\quad \delta^{\hat{t}-1} \frac{u_i(-\left|x_{\hat{t}-1}^{PA}\right|, \left|x_{\hat{t}}^{PA}\right|) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -\left|x_{\hat{t}}^{PA}\right|)}{2} + \\
&\quad \frac{\delta^{\hat{t}} + \delta^{\hat{t}+1}}{1-\delta^2} \frac{u_i(-\left|x_{\hat{t}-1}^{PA}\right|, \left|x_{\hat{t}}^{PA}\right|) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -\left|x_{\hat{t}}^{PA}\right|)}{2}.
\end{aligned}$$

Now consider a strategy, say s' in which, for all $t \geq 1$, v_t is given by (4) and $|p_t| = a \in [0, \min\{|x_{\hat{t}}^{PA}|, |x_{\hat{t}+1}^{PA}|\})$. It follows that the utility to player $i \in \{M, L, R\}$ from strategy s' is

$$\begin{aligned}
U_i(s') &= \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(-x+kr, -x+(k+1)\,r) + \delta^{\hat{t}-1} u_i(-x+\left(\hat{t}-1\right)r, a) + \frac{\delta^{\hat{t}} u_i(a, -a) + \delta^{\hat{t}+1} u_i(-a, a)}{1-\delta^2}\right) \\
&\quad + \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(x-kr, x-(k+1)\,r) + \delta^{\hat{t}-1} u_i(x-\left(\hat{t}-1\right)r, -a) + \frac{\delta^{\hat{t}} u_i(-a, a) + \delta^{\hat{t}+1} u_i(a, -a)}{1-\delta^2}\right),
\end{aligned}$$

which, since $\left|-x+\hat{t}r\right| = \left|x_{\hat{t}}^{PA}\right|$ and $\left|-x+\left(\hat{t}-1\right)r\right| = \left|x_{\hat{t}-1}^{PA}\right|$, can be rewritten as

$$\begin{aligned}
U_i(s') &= \sum_{k=0}^{k=\hat{t}-2} \delta^k \frac{u_i(-x+kr, -x+(k+1)\,r) + u_i(x-kr, x-(k+1)\,r)}{2} + \\
&\quad \delta^{\hat{t}-1} \frac{u_i(-\left|x_{\hat{t}-1}^{PA}\right|, a) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -a)}{2} + \\
&\quad \frac{\delta^{\hat{t}} + \delta^{\hat{t}+1}}{1-\delta^2} \frac{u_i(a, -a) + u_i(-a, a)}{2}.
\end{aligned}$$

Hence,

$$U_i(s') - U_i(s^{PA}) = \delta^{\hat{t}-1} \frac{u_i(-\left|x^{PA}_{\hat{t}-1}\right|, a) + u_i(\left|x^{PA}_{\hat{t}-1}\right|, -a) - \left(u_i(-\left|x^{PA}_{\hat{t}-1}\right|, |x^{PA}_{\hat{t}}|) + u_i(\left|x^{PA}_{\hat{t}-1}\right|, -\left|x^{PA}_{\hat{t}-1}\right|)\right)}{2} + \frac{\delta^{\hat{t}} + \delta^{\hat{t}+1}}{1-\delta^2} \frac{u_i(a,-a) + u_i(-a,a) - \left(u_i(-\left|x^{PA}_{\hat{t}-1}\right|, |x^{PA}_{\hat{t}}|) + u_i(\left|x^{PA}_{\hat{t}-1}\right|, -\left|x^{PA}_{\hat{t}-1}\right|)\right)}{2}.$$

Since $a \in [0, \min\{|x^{PA}_{\hat{t}}|, |x^{PA}_{\hat{t}+1}|\})$ and, by (4), the three players' utility functions are concave with respect to the distance between the policy at work and their ideologies (with $\bar{x}^L = -1$, $\bar{x}^R = 1$ and $\bar{x}^M = 0$), we get that, for all $i \in \{M, L, R\}$,

$$\begin{aligned} u_i(-\left|x^{PA}_{\hat{t}-1}\right|, a) + u_i(\left|x^{PA}_{\hat{t}-1}\right|, -a) &> \left(u_i(-\left|x^{PA}_{\hat{t}-1}\right|, |x^{PA}_{\hat{t}}|) + u_i(\left|x^{PA}_{\hat{t}-1}\right|, -\left|x^{PA}_{\hat{t}-1}\right|)\right), \\ u_i(a,-a) + u_i(-a,a) &> \left(u_i(-\left|x^{PA}_{\hat{t}-1}\right|, |x^{PA}_{\hat{t}}|) + u_i(\left|x^{PA}_{\hat{t}-1}\right|, -\left|x^{PA}_{\hat{t}-1}\right|)\right), \end{aligned}$$

which in turn implies that $U_i(s') - U_i(s^{PA}) > 0$.

An analogous reasoning shows that if we consider a strategy, say s'' in which, for all $t \geq 1$, v_t is given by (4) and $|p_t| = a' \in [0, a)$, then, for all $i \in \{M, L, R\}$,

$$U_i(s'') - U_i(s') = \delta^{\hat{t}-1} \frac{u_i(-\left|x^{PA}_{\hat{t}-1}\right|, a') + u_i(\left|x^{PA}_{\hat{t}-1}\right|, -a') - \left(u_i(-\left|x^{PA}_{\hat{t}-1}\right|, a) + u_i(\left|x^{PA}_{\hat{t}-1}\right|, -a)\right)}{2} + \frac{\delta^{\hat{t}} + \delta^{\hat{t}+1}}{1-\delta^2} \frac{u_i(a',-a') + u_i(-a',a') - (u_i(a,-a) + u_i(-a,a))}{2},$$

and, by the same argument, $U_i(s'') - U_i(s') > 0$. This completes the proof. QED.

Proof of Proposition 4

Point (i) follows directly from Proposition 2 (recall that $\hat{x} = \min\{\frac{r}{2}, 1\}$).

We then turn to analyze the additional equilibria in $\widetilde{S}^*$. To check if a strategy $\widetilde{s}^a$ with $a \in [0, \hat{x})$ is an equilibrium, given the symmetry of the path induced by $\widetilde{s}^a$, without loss of generality we can focus on any period t at which $x_{t-1} = -a$ and $v_{t-1} = L$ and check that party R does not have any incentive to deviate in the continuation of the game. To see this, note that M does not have any incentive to deviate from $\widetilde{s}^a$ at any t, since a deviation induces both parties (and the median voter himself) to turn to the repetition, forever, of the (inefficient) equilibrium of the stage game, which cannot yield a higher utitity to M than the continuation utility of $\widetilde{s}^a$. On the other hand, we only need to check the incentives to deviate from $\widetilde{s}^a$ at period t by party R, to its ideal policy $p_t = 1$ and, from $t+1$ onwards (i.e., after the deviation), we can safely assume that all players will keep playing according to $\widetilde{s}^a$, since it prescribes to turn to play according to the extreme alternating strategy profile, which as shown in Proposition 2, is an equilibrium for all $\delta \in (0,1)$.

Hence consider some period t such that $x_{t-1} = -a$ and $v_{t-1} = L$. The utility induced by following the strategy $\widetilde{s}^a$ to R in the continuation of the game is:

$$\begin{aligned} U^F_R(a,r,\delta) &= u_R(-a,a) + \delta u_R(a,-a) + \delta^2 u_R(-a,a) + \delta^3 u_R(a,-a) + ... \\ &= \frac{u_R(-a,a)}{1-\delta^2} + \frac{\delta u_R(a,-a)}{1-\delta^2}. \end{aligned} \quad (5)$$

In contrast, if R deviates to its ideal policy $p_t = 1$ (and all players follow the prescription of $\widetilde{s}^a$ hereafter), then the utility to R in the continuation of the game is:

$$\begin{aligned} U_R^D(a,r,\delta) &= u_R(-a,x^D) + \delta u_R(x^D,x^C) + \delta^2 u_R(x^C,x^D) + \delta^3 u_R(x^D,x^C) + ... \\ &= u_R(-a,x^D) + \frac{\delta u_R(x^D,x^C)}{1-\delta^2} + \frac{\delta^2 u_R(x^C,x^D)}{1-\delta^2}, \end{aligned} \tag{6}$$

where, the"deviation policy", x^D, and the subsequent "continuation policy in the next period", x^C, are defined as:

$$\begin{aligned} x^D &= \min\{r-a,1\}, \\ x^C &= \max\{x^D - r, -1\} = \max\{\min\{-a, 1-r\}, -1\}. \end{aligned}$$

Therefore

$$\widetilde{s}^a \in S^* \iff U_R^F(a,r,\delta) \geq U_R^D(a,r,\delta). \tag{7}$$

The next lemma restricts the set $\widetilde{S}^*$.

Lemma 1: If $a < \frac{r}{2}$ and $r - a \leq 1$, $\widetilde{s}^a \notin \widetilde{S}^*$.

Proof of Lemma 1. Assume, for the sake of contradiction, that $a < \frac{r}{2}$, $r - a < 1$ and $\widetilde{s}^a \in \widetilde{S}^*$. Consider some period t such that $x_{t-1} = -a$ and $v_{t-1} = L$. Then, $U_R^F(a,r,\delta)$ and $U_R^D(a,r,\delta)$ are given by (5) and (6). Since $r - a \leq 1$, $x^D = r - a$ and $x^C = -a$, we get

$$U_R^F(a,r,\delta) - U_R^D(a,r,\delta) = \frac{u_R(-a,a) - u_R(-a,r-a)}{1-\delta^2} + \frac{\delta(u_R(a,-a) - u_R(r-a,-a))}{1-\delta^2}. \tag{8}$$

Given (2), since $\bar{x}^R = 1$, we know that the function $u_R(z,y)$ is strictly increasing in both arguments. Since $a < \frac{r}{2}$, it follows that $r - a > a$ which, by (8), yields $U_R^F(a,r,\delta) - U_R^D(a,r,\delta) < 0$, a contradiction with $\widetilde{s}^a \in \widetilde{S}^*$. This proves Lemma 1.

Given the condition $r - a \leq 1$, since $a \leq 1$, Lemma 1 is only relevant for the case $r \in (0,2]$. In particular, Lemma 1 implies that:

(I) If $r \leq 1$, for all $a \in [0, \frac{r}{2})$, $\widetilde{s}^a \notin \widetilde{S}^*$.

(II) If $r \in (1,2]$, for all $a \in [r-1, \frac{r}{2})$, $\widetilde{s}^a \notin \widetilde{S}^*$.

Point (I) already shows that, if $r \leq 1$, then $\widetilde{S}^* = \{\widetilde{s}^{\frac{r}{2}}\}$. Hence, in the following we consider the case $r > 1$. Moreover, by Point (II) we can focus on strategies with

$$a \in [0, \min\{r-1, 1\}). \tag{9}$$

Assuming $r > 1$ and $a \in [0, \min\{r-1,1\})$, we have that, in (6),

$$\begin{aligned} x^D &= 1, \\ x^C &= \max\{1-r, -1\}. \end{aligned}$$

Hence

$$
\begin{aligned}
u_R(-a,a) &= -\int_0^{\frac{2a}{r}} (1+a-r\tau)^2 d\tau - \int_{\frac{2a}{r}}^1 (1-a)^2 d\tau, && (10)\\
u_R(a,-a) &= -\int_0^{\frac{2a}{r}} (1-a+r\tau)^2 d\tau - \int_{\frac{2a}{r}}^1 (1+a)^2 d\tau, && (11)\\
u_R(-a,1) &= -\int_0^{\frac{1+a}{r}} (1+a-r\tau)^2 d\tau, && (12)\\
u_R(1,\max\{1-r,-1\}) &= -\int_0^{\min\{\frac{2}{r},1\}} (r\tau)^2 d\tau - \int_{\min\{\frac{2}{r},1\}}^1 2^2 d\tau, && (13)\\
u_R(\max\{1-r,-1\},1) &= -\int_0^{\min\{\frac{2}{r},1\}} \left(\min\left\{\frac{2}{r},1\right\}\cdot r - r\tau\right)^2 d\tau. && (14)
\end{aligned}
$$

By replacing (10)-(14) in expressions (5) and (6), we obtain the function:

$$f(a,r,\delta) = (1-\delta^2)\left(U_R^F(a,r,\delta) - U_R^D(a,r,\delta)\right),$$

where $f(a,r,\delta)$ is a degree-3 polynomial in a:

$$f(a,r,\delta) = \sum_{j=0}^{j=3} \beta_j(r,\delta)\cdot a^j,$$

with coefficients

$$
\begin{aligned}
\beta_0(r,\delta) &= \frac{1-\delta^2+r(9\delta-3)-12r\delta\min\{1,\frac{2}{r}\}+r^3\delta(1+\delta)\min\{1,\frac{2}{r}\}^3}{3r},\\
\beta_1(r,\delta) &= -\frac{(\delta-1)(1+2r+\delta)}{r},\\
\beta_2(r,\delta) &= -\frac{3+r+(r-4)\delta+\delta^2}{r},\\
\beta_3(r,\delta) &= -\frac{(\delta-5)(1+\delta)}{3r}.
\end{aligned}
$$

Hence, for any $a \in [0,\min\{r-1,1\})$, given (7), to verify if $\widetilde{s}^a$ is an equilibrium we need to check if $f(a,r,\delta) \geq 0$.

We first consider the case $r \in (1,2]$. In such a case condition (9) becomes $a \in [0,r-1)$ and the conditions such that $f(a,r,\delta) \geq 0$ are:

$$
\begin{aligned}
\sqrt{3} &< r \leq 2, && (15)\\
0 &\leq a < \bar{a}_L(r), && (16)\\
\bar{\delta}_L(a,r) &\leq \delta < 1, && (17)
\end{aligned}
$$

where $\bar{a}_L(r)$ is the second highest real root of the following cubic polynomial in α,

$$h(\alpha;r) = 4\alpha^3 - 3r\alpha^2 + r(r^2-3),$$

and

$$\bar{\delta}_L(a,r) = \frac{12a^2+4a^3-3r-6ar-3a^2r+r^3}{2(1+3a+3a^2+a^3-r^3)} + \frac{1}{2}\sqrt{\frac{\sigma(a,r)}{(1+3a+3a^2+a^3-r^3)^2}}, \quad (18)$$

with

$$\begin{aligned}\sigma(a,r) &= 4+24a+12a^2-48a^3+108a^4+120a^5+36a^6-12r \\ &\quad -12ar-48a^2r-144a^3r-132a^4r-36a^5r+9r^2 \\ &\quad +36ar^2+54a^2r^2+36a^3r^2+9a^4r^2-4r^3-12ar^3 \\ &\quad +60a^2r^3-12a^3r^3+6r^4-36ar^4+6a^2r^4+r^6, \end{aligned} \tag{19}$$

being $\bar{\delta}_L(a,r) \in (0,1)$ for all (a,r) satisfying (15)-(16). This proves point (ii).

Since all the coefficients of $h(\alpha;r)$ are themselves polynomials in r (hence, continuous functions) and, for the whole interval $\sqrt{3} \leq r \leq 2$, the three roots (in α) of the polynomial $h(\alpha;r)$ are real,[19] it follows that $\bar{a}_L(r)$ is continuous in r in such an interval (see Uherka and Sergott (1977)). Moreover, algebraic calculations show that the derivative of $\bar{a}_L(r)$ is strictly positive in the interval $\sqrt{3} < r < 2$, and that

$$\begin{aligned}\lim{}_{a\to\sqrt{3}^+}\bar{a}_L(r) &= 0, \\ \text{and } \bar{a}_L(2) &= 1.\end{aligned}$$

We now study the equilibrium conditions for the case $r > 2$. Note that, in such a case, condition (9) becomes $a \in [0,1)$ and the conditions such that $f(a,r,\delta) \geq 0$ are:

$$r > 2, \tag{20}$$

$$0 \leq a < 1, \tag{21}$$

$$\bar{\delta}_H(a,r) \leq \delta < 1, \tag{22}$$

where

$$\bar{\delta}_H(a,r) = \frac{16+16a+4a^2-9r-3ar}{2(7+4a+a^2)} + \frac{\sqrt{3}}{2}\sqrt{\frac{\rho(a,r)}{(7+4a+a^2)^2}}, \tag{23}$$

with

$$\begin{aligned}\rho(a,r) &= 76+128a+152a^2+64a^3+12a^4-68r-140ar \\ &\quad -68a^2r-12a^3r+27r^2+18ar^2+3a^2r^2, \end{aligned} \tag{24}$$

being $\bar{\delta}_H(a,r) \in (0,1)$ for all (a,r) satisfying (20)-(21).

To prove points (iii) and (iv) we just need to define

$$\bar{a}(r) = \begin{cases} \bar{a}_L(r) \text{ if } \sqrt{3} < r \leq 2 \\ 1 \text{ if } r > 2 \end{cases} \tag{25}$$

$$\bar{\delta}(a,r) = \begin{cases} \bar{\delta}_L(a,r) \text{ if } \sqrt{3} < r \leq 2 \\ \bar{\delta}_H(a,r) \text{ if } r > 2. \end{cases} \tag{26}$$

[19] Three distinct real roots in the interior of the interval and two distinct real roots (one of them with multiplicity two) at the extremes.

QED.

Proof of Proposition 5

First consider the case $\sqrt{3} < r \leq 2$ and $a < \bar{a}(r)$, i.e., conditions (15) and (16) defined in the proof of Proposition 4 hold. Then, given (25) and (26), we get that:

$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \leq 0 \Longleftrightarrow a \leq \hat{a}_L(r), \tag{27}$$
$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \geq 0 \Longleftrightarrow \hat{a}_L(r) \leq a < \bar{a}(r), \tag{28}$$

where $\hat{a}_L(r)$ is the third highest real root of the following degree-7 polynomial in α:

$$k_L(\alpha; r) = \sum_{j=0}^{j=7} \lambda_j(r) \cdot \alpha^j,$$

with all the coefficients of $k_L(\alpha; r)$ being themselves polynomials in r (hence, continuous functions), given by the following expressions:

$$\begin{aligned}
\lambda_0(r) &= -6r^2 + 3r^3 + 2r^4 + 2r^5 + 6r^6 - r^7 - 2r^8, \\
\lambda_1(r) &= 24r - 2r^2 + 7r^3 - 10r^4 - 52r^5 + 16r^6 + 13r^7, \\
\lambda_2(r) &= -12r + 16r^2 + 26r^3 + 106r^4 - 94r^5 - 14r^6, \\
\lambda_3(r) &= 32 - 172r - 12r^2 - 6r^3 + 198r^4 - 40r^5, \\
\lambda_4(r) &= 144 - 208r - 62r^2 - 101r^3 + 88r^4, \\
\lambda_5(r) &= 208 - 56r - 50r^2 - 57r^3, \\
\lambda_6(r) &= 112 + 28r - 12r^2, \\
\lambda_7(r) &= 16 + 12r.
\end{aligned}$$

Hence, given (27)-(28), if $\sqrt{3} < r \leq 2$, we conclude that within the interval $0 \leq a < \bar{a}(r)$, the threshold $\bar{\delta}(a,r)$ reaches its minimum at $a = \hat{a}_L(r)$.

Now consider the case $r > 2$. In such a case, by (25), $\bar{a}(r) = 1$. Then, assuming $r > 2$ and $a < 1$, given (26), we get that:

$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \leq 0 \Longleftrightarrow [r < 2\sqrt{2} \wedge a \leq \hat{a}_H(r)] \vee [r = 2\sqrt{2} \wedge a < \hat{a}_H(r)] \vee r > 2\sqrt{2}, \tag{29}$$
$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \geq 0 \Longleftrightarrow r \leq 2\sqrt{2} \wedge a \geq \hat{a}_H(r), \tag{30}$$

where $\hat{a}_H(r)$ is the third highest real root of the following degree-4 polynomial in α:

$$k_H(\alpha; r) = \sum_{j=0}^{j=4} \eta_j(r) \cdot \alpha^j,$$

with all the coefficients of $k_H(\alpha;r)$ being themselves polynomials in r (hence, continuous functions), given by the following expressions:

$$\begin{aligned}
\eta_0(r) &= -48-60r+52r^2+15r^3,\\
\eta_1(r) &= 96-272r-44r^2+18r^3,\\
\eta_2(r) &= 352-32r-68r^2+3r^3,\\
\eta_3(r) &= 160+64r-12r^2,\\
\eta_4(r) &= 16+12r.
\end{aligned}$$

Since $\hat{a}_H(2\sqrt{2})=1$ and we are considering the case $r>2$ and $a<1$, the expressions (29)-(30) can be rewritten as:

$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \leq 0 \Longleftrightarrow [r<2\sqrt{2}\wedge a\leq \hat{a}_H(r)]\vee r\geq 2\sqrt{2}, \tag{31}$$

$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \geq 0 \Longleftrightarrow r<2\sqrt{2}\wedge a\geq \hat{a}_H(r), \tag{32}$$

which, in turn, imply that:

a) If $r<2\sqrt{2}$, the threshold $\bar{\delta}(a,r)$ reaches its minimum at $a=\hat{a}_H(r)$.

b) If $r\geq 2\sqrt{2}$, then $\frac{\partial\bar{\delta}(a,r)}{\partial a}<0$ and, therefore, there is not a minimum of $\bar{\delta}(a,r)$ in the interval $a\in[0,1)$, so there is no robust-interior equilibrium. This proves (ii).

To prove (i) and (iii), we define

$$\hat{a}(r)=\begin{cases}\hat{a}_L(r) \text{ if } \sqrt{3}<r\leq 2\\ \hat{a}_H(r) \text{ if } 2<r<2\sqrt{2}\end{cases} \tag{33}$$

and note that algebraic calculations show that:

I) If $\sqrt{3}<r\leq 2$, $\frac{\partial\hat{a}_L(r)}{\partial r}>0$.

II) If $2<r<2\sqrt{2}$, $\frac{\partial\hat{a}_R(r)}{\partial r}>0$.

III) $\hat{a}_L(\sqrt{3})=0$, $\hat{a}_L(2)=\hat{a}_L(2)$ and $\hat{a}_H(2\sqrt{2})=1$.

Finally, by substituting (33) in (26), we obtain $\hat{\delta}(r)=\bar{\delta}(\hat{a}(r),r)$. Algebraic calculations show that:

IV) $\lim_{r\to\sqrt{3}^+}\hat{\delta}(r)=1$.

V) If $\sqrt{3}<r\leq 2$, $\frac{\partial\hat{\delta}(r)}{\partial r}<0$.

VI) If $2<r<2\sqrt{2}$, $\frac{\partial\hat{\delta}(r)}{\partial r}<0$.

VII) $\bar{\delta}_L(\hat{a}_L(2),2)=\bar{\delta}_H(\hat{a}_H(2),2)$.

This proves point (iv). QED.

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