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Guadalupe Correa-Lopera

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Departamento de Teoría e Historia Económica
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Why Delegate? Comparing Direct And Representative Democracy

Guadalupe Correa-Lopera [*] [**]

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Abstract

The growing demand for referendums challenges the traditional model of representative democracy. Why may voters prefer to decide by themselves (direct democracy) rather than delegate on their representatives (representative democracy)? We propose a model in which voters select either a policy or a representative under uncertainty over the socially correct policy. Under direct democracy, the policy selected by voters is implemented, while under representative democracy the elected representative decides the policy. We assume that representatives have informational advantage. Our main result shows that a society in which the majority of voters are selfish may prefer a system of political representation when they are strongly ideologically polarized. If, instead of ideological confrontation, there is consensus among these selfish voters, referendum is the preferred instrument for making decisions. Non-selfish societies, however, always prefer to delegate on better informed representatives.

Keywords: Direct democracy; Representative democracy; Ideological electorate; Pragmatic electorate; Polarization; Information.

JEL classification numbers: D72, D82.

[*]Departamento de Teoría e Historia Económica, Facultad de Ciencias Económicas y Empresariales, Campus de El Ejido, Universidad de Málaga, 29071 Málaga, Spain. E-mail: guacorlop@uma.es

[**]PhD Program in Economics and Business, Universidad de Málaga, Málaga, Spain.

1. Introduction

Why might voters prefer to make their own decisions (direct democracy) rather than delegate those on representatives (representative democracy)? Our aim in this paper is to determine which of these two democratic systems is the regime preferred by the electorate.

Some part of the society openly expresses its opposition to the implementation of direct democracy practices such as referendums. The most recurring argument is the lack of information that the ordinary citizen has when she has to make a decision. Instead, this part of the society argues that decisions should be made by politicians, since they are more likely to have appropriate information to make a wise decision. In essence, what these people defend is the establishment of a representative democracy in which voters delegate decisions on representatives.

It is often claimed that politicians may have private information that would contribute to increase the overall welfare of society. However, arguing that representative democracy has advantages over direct democracy because of this fact might not be as clear as it seems. We suggest two main reasons. First, not all voters necessarily have to be interested in the improvement of the general welfare. This idea is in line with the distinction between the theories of *public-regardingness* and *selfishness* in voting behaviour. While the first theory highlights the importance of seeking the good of the community as a whole, the second one emphasizes the search for personal benefits. The second reason is that there is no guarantee that every representative will use such superior information to increase the common welfare, that is, politicians may pursue their personal interests to the detriment of the general interests of the population.

In order to determine whether direct democracy or representative democracy is the regime preferred by the electorate we propose a model in which there are two possible policies, l and r, to choose from. In direct democracy, voters directly cast their ballots for one of the two alternatives, whereas in representative democracy, they choose a representative, who will then decide which policy to implement. One of the two policies is *socially correct*, that is, the policy that would be appropriate to implement from a neutral perspective. For example, l and r may represent left-wing and right-wing policies respectively and the socially correct policy be the

one that, given the economic situation of the country, would allow a long-term GDP growth. Agents may be biased towards one or the other policy, so they may differ by the policy they like the most. Furthermore, they receive additional non-negative utility in case the socially correct policy becomes implemented.

The socially correct policy is determined by a random shock. Voters are uncertain about the optimal policy whereas representatives are experts who observe the shock. The realization of the shock takes place after a policy has been chosen under direct democracy or a politician has been selected under representative democracy. The underlying idea is that certain events that could change the existing conditions in society might happen unexpectedly in the future, modifying so what is considered to be socially correct. Consider the case of an economic crisis. Voters may have to choose a policy *today* without knowing if *tomorrow* such a policy will either be socially correct or not after a possible economic crisis has arisen. Voters may have to choose a representative instead, who would know which is the optimal policy once the economic crisis has broken out. There are two candidates, L and R, who are biased towards policies l and r respectively. However, the winner is not committed to her private bias. We distinguish two different types of candidates: *pragmatic candidates*, who are willing to implement the socially correct policy even if such policy is not the one towards which they are biased, and *ideological candidates*, who are not. Although voters know each candidate's private bias, they are uncertain about the type of each of them.

We identify two different types of electorate: *pragmatic electorate* and *ideological electorate*. The former is an electorate characterized by having a majority of voters who are interested in the implementation of the socially correct policy, regardless of their private biases. We denote them by *pragmatic voters*. On the contrary, the latter is an electorate in which there is a majority of voters concerned about the implementation of the policy towards which each of them is biased, regardless of the socially correct policy. We denote them by *ideological voters*. We determine the preference for direct democracy or representative democracy as measured by voters' expected utility, that is, we compare the expected utility of voters when a certain policy is chosen under direct democracy to the expected utility of voters when a certain candidate is selected under representative democracy. Our first result shows that the policy chosen under direct democracy and the candidate selected under representative democracy depend on the median voter of the distribution of voters' biases intensity (Theorem 1). Regarding

the preference for one voting system or another, we find that representative democracy is preferred to direct democracy by a pragmatic electorate (Theorem 2). Intuitively, given that citizens are interested in the implementation of the socially correct policy and candidates have better information about it, they are willing to delegate the decision making process on representatives. In this sense, it might be thought that direct democracy would be preferred to representative democracy by an ideological electorate: given that citizens are interested in the implementation of the policy towards which each of them is biased, they would not be willing to allow representatives to make decisions because there would exist the risk that they end up choosing a totally different policy. However, this is not always necessarily the case. In fact, this only happens when such majority of ideological voters are biased towards the same policy (Theorem 2). Otherwise, we find that representative democracy is the system preferred by an ideological electorate (Theorem 2). That is, if instead of being an ideological society with homogeneity among voters' biases it is an ideologically divided society between two different policies, a system of political representation is preferred. This event happens as a result of the ideological polarization of the electorate. Given that the outcome of a referendum held in a society like this might be extremely uncertain, some ideological voters fear that their preferred policy will not be chosen under direct democracy. In contrast, they believe that its implementation might be more likely under representative democracy: with certain probability, such policy is socially correct and the selected candidate is pragmatic, which would guarantee the implementation of their preferred policy.

Related Literature

The advantages and disadvantages of each of these regimes in the face of the other have been widely studied in the literature.

Several papers express the benefits of using the instruments of direct democracy, understood as a form of democracy in which people decide policy directly. According to Besley and Coate (2008), policy outcomes on specific issues may differ substantially from what the majority desires when citizens have only one vote to cast for candidates who have to decide on a bundle of issues. They show that citizens' initiatives and referendums prevent such problems from occurring. Matsusaka (2005) states that allowing citizens to participate in lawmaking leads

to the prevalence of the median voter's preferences along different dimensions and therefore reduces the discretionary performance of the government. Empirical evidence on how direct democracy prevents politicians from increasing spending to favor special interest groups is offered by Sanz (2015).

For its part, another strand in the literature highlights the inability of voters to make decisions due to lack of information. The seminal works of Madison (1787) and Siéyès (1789) stand up for the establishment of a representative democracy in which politicians with an informational advantage decide. Having politicians better informed than voters is a generally used assumption in the literature. The superior information available for the politicians may be of diverse nature. Roemer (1994) and Cukierman and Tommasi (1998) consider models where candidates are better informed than the electorate about how different policies map into outcomes. The former focuses exclusively on economic outcomes while the latter adopts a broader definition of outcome. Schultz (1996, 2002), Martinelli (2001), and Jensen (2009) assume that politicians are better informed than voters about conditions relevant for policy choice (*i.e.*, about the state of the world).

We suggest two main reasons why such informational advantage might not be enough to ensure the better performance of representative democracy over the direct democracy. Regarding the theories of *public-regardingness* and *selfishness* in voting behaviour, Wilson and Banfield (1964) support the hypothesis that only certain classes of voters (distinguished on the basis of ethnic and income factors) act as if they were concerned about some concept of welfare of community. Martínez-Vázquez (1981) identifies a selfish behaviour of taxpayers when voting to maximize their respective individual net benefits. Even when he finds that high income groups may opt for redistributive policies of income to the poor, he argues that this behaviour may be due to a self-interest in real estate speculation, so it might not have to do with the public-regardingness hypothesis. With respect to the possibility that politicians do not use this additional information for the benefit of all citizens, Kartik and Preston McAfee (2007) propose a model in which only a fraction of candidates are committed to implement the policy that they consider to be the most appropriate to maximize the overall welfare of society, even when such policy may not be the most popular among voters. They denote this type of representatives as candidates "with character". In contrast, candidates "without character" might end up implementing a policy different from the one that increases the social welfare.

Papers cited above dealing with either direct or representative democracy focus exclusively on the analysis of one of these two systems, but without establishing a comparison between them as we will do. To the best of my knowledge, it is only one paper in the literature that develops a theoretical comparison between both regimes. Maskin and Tirole (2004) studies whether decisions should be made by the public directly, politicians subject to reelection, or independent judges when the aim is to maximize the social welfare. In their model, these authors assume that all voters have the same preference order over the two available policies and all of them prefer the implementation of the policy considered as optimal. In terms of our model, this is equivalent to say that all voters are biased towards the same policy and are pragmatic. We widely relax these assumptions in our work by assuming that *(i)* voters may be biased towards one or the other policy and *(ii)* not necessarily all voters are interested in the implementation of the optimal policy.

The remainder of the paper is organized as follows. In Section 2 we set out the model. In Section 3 we derive equilibria under direct and representative democracy determining so the outcome under each regime. In Section 4 we carry out an analysis of voters' expected utility in order to determine which of these regimes is preferred by the majority of the electorate. Finally, we discuss and conclude in Section 5. The Appendix A offers explanatory notes and examples. The Appendix B provides the proofs of the results.

2. The Model

We study two voting systems: direct democracy and representative democracy. In direct democracy, voters directly cast their ballots for an alternative. In representative democracy, voters choose the representative (from now on denoted as *candidate* to avoid confusion with the system notation), who will then choose the implemented policy.

Let N be a continuous set of voters and $X = \{l, r\}$ be the set of policies. Each voter $j \in N$ may have a private bias for one or the other policy. We assume that one of the two policies is **socially correct**. That is, one policy is better than the other from a neutral perspective: the

socially correct policy is the one which is preferred by unbiased voters.[1]

After voters have voted, a random shock $\theta \in X$ determines one of the two policies as socially correct. Candidates, but not voters, observe the shock. All voters have the same beliefs about the policy which is socially correct, that is, l with probability p and r with probability $1-p$, where $p \in [0,1]$.[2]

Given the realization of the shock $\theta \in X$, if policy $x \in X$ is implemented, then a voter $j \in N$ receives the utility:

$$u_j(x,\theta) = z_j(x) + \mathbb{1}\{\theta = x\}v \tag{1}$$

where $z_j : X \to \mathbb{R}$ is a function that gives a real number for each policy $x \in X$, and $\mathbb{1}\{\theta = x\}$ is an indicator function that takes unit value if the implemented policy is socially correct and zero otherwise, *i.e.*,

$$\mathbb{1}\{\theta = x\} = \begin{cases} 1 & \text{if } \theta = x \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

so $v \geq 0$ can be defined as the extra level of utility that any voter j receives from the implementation of the socially correct policy. We impose no restrictions on $z_j(\cdot)$, thereby accommodating many situations.

Note that a voter's utility consists of two components: the *private* component $z_j(x)$ and the *public* component $\mathbb{1}\{\theta = x\}v$. The distinction between the two components is motivated by the theories of *selfishness* and *public-regardingness* in voting behaviour. The private component reflects the individual preferences of each voter for one or another policy: voters obtain utility from the two possible policies, l and r, although they may differ by the policy they like the most. These individual preferences are independent of their beliefs about the socially correct policy. For its part, that unbiased voters prefer the socially correct policy suggests that its implementation leads to an increase in common welfare: all voters would receive an identical additional utility. The public component symbolizes this idea.

Let $\triangle z_j = z_j(r) - z_j(l)$ for every $j \in N$, so that $\triangle z_j < 0$ and $\triangle z_j > 0$ reflect private bias for l and r respectively. A voter j has no private bias if $\triangle z_j = 0$. The intensity of voters' biases

[1] See Jackson and Tan (2013).

[2] These probabilities are exogenous and common knowledge.

are distributed according to $F(\cdot)$ with a positive density function $f(\cdot)$ and median m.

Voters are either *ideological* or *pragmatic*. A voter $j \in N$ is **ideological** if $|\triangle\ z_j| > v$, while she is **pragmatic** if $|\triangle\ z_j| \leq v$. Voter j is ideological if the intensity of her bias is higher than the utility derived from the socially correct policy and pragmatic otherwise. An ideological voter is concerned about the implementation of the policy towards which she is biased, regardless of the socially correct policy. The reason is that the difference of utility between l and r is so large that it cannot be compensated by v. In contrast, a pragmatic voter is interested in the implementation of the socially correct policy, regardless of her private bias. This happens when the difference of utility between l and r is compensated by v.[3]

Let $\alpha > 0$ be the proportion of voters such that $\triangle z_j < -v$, *i.e.*, ideological voters biased for l. Let $\gamma > 0$ be the proportion of voters such that $\triangle z_j > v$, *i.e.*, ideological voters biased for r. Let $\beta > 0$ be the proportion of voters such that $-v \leq \triangle z_j \leq v$, *i.e.*, pragmatic voters regardless of their biases.[4] Let α, β, γ be such that $\alpha + \beta + \gamma = 1$.

By abuse of language, we distinguish two types of electorate: *ideological electorate* and *pragmatic electorate*. The electorate is **ideological** if the majority of voters are ideological, while the electorate is **pragmatic** if the majority of voters are pragmatic.

Definition 1. *The electorate is* ***ideological*** *if* $\alpha + \gamma > 0.5$.

Definition 2. *The electorate is* ***pragmatic*** *if* $\beta \geq 0.5$.

Note that a society is either ideological or pragmatic as long as a majority of voters are either ideological or pragmatic respectively, regardless of the configuration of the biases of the voters that compose such majority. We now define what we mean by *polarized electorate*. An electorate is **polarized** when a majority of voters are ideological but neither the group of ideological voters biased for l nor the group of ideological voters biased for r constitute a majority by themselves.

[3] See *Appendix A.1* for a graphical explanation of both private biases and types of voters.

[4] Note that $\alpha, \beta, \gamma > 0$ given that we assume a positive density function $f(\cdot)$.

Definition 3. *The electorate is* ***polarized*** *if* $\alpha + \gamma > 0.5$*,* $\alpha < 0.5$*, and* $\gamma < 0.5$*.*

If the electorate is polarized, then it is also ideological but not necessarily the opposite.

Under representative democracy there are two candidates, $C = \{L, R\}$. Both candidates are experts who observe the shock.[5] The candidates have the same form of utility functions as the voters.[6] Given the realization of the shock $\theta \in X$, if policy $x \in X$ is implemented, then a candidate $c \in C$ receives the utility:

$$u_c(x, \theta) = z_c(x) + \mathbb{1}\{\theta = x\}v \tag{3}$$

where $z_c : X \to \mathbb{R}$ is a function that gives a real number for each policy $x \in X$. Let $\triangle z_c = z_c(r) - z_c(l)$ for every $c \in C$, which is interpreted as in the case of voters. We assume that $\triangle\ z_L < 0$ and $\triangle\ z_R > 0$, so that candidate L is biased for l and candidate R is biased for r. This is known by all voters (and candidates).

Like voters, candidates can be *ideological* or *pragmatic*. A candidate $c \in C$ is **ideological** if $|\triangle\ z_c| > v$, while she is **pragmatic** if $|\triangle\ z_c| \leq v$. An ideological candidate always prefers to implement the policy towards which she is biased regardless of the socially correct policy, while a pragmatic candidate always prefers to implement the socially correct policy. For each candidate $c \in C$, all voters have the same beliefs about her type: they believe that c is pragmatic with probability μ_c, where $\mu_c \in [0, 1]$.[7]

Direct Democracy

Under direct democracy voters select a policy before θ is realized. Let $x^{DD} \in X$ be the policy selected under this voting system. Thus, the *outcome* under direct democracy is a *policy* (l or r).

[5] This is common knowledge.

[6] This assumption might be relaxed without affecting our results.

[7] These probabilities are exogenous and common knowledge. The probability of being pragmatic is not necessarily equal for both candidates.

Let $\mathbb{E}u_j(x^{DD}, \theta)$ denote the expected utility of a voter $j \in N$ under **direct democracy** when policy $x^{DD} \in X$ is implemented. Equations (4) and (5) represent the cases $x^{DD} = l$ and $x^{DD} = r$ respectively (see *Appendix A.2* for computations).

$$\mathbb{E}u_j(l, \theta) = z_j(l) + pv \tag{4}$$

$$\mathbb{E}u_j(r, \theta) = z_j(r) + (1 - p)v \tag{5}$$

Under direct democracy, each voter $j \in N$ casts her ballot for the policy (l or r) which maximizes her expected utility, and the policy selected by the majority rule is implemented.

Representative Democracy

Under representative democracy voters select a candidate before θ is realized. Let $c^{RD} \in C$ be the candidate selected under this voting system. Thus, the *outcome* under representative democracy is a *candidate* (L or R).

Let $\mathbb{E}u_j(c^{RD}, \theta)$ denote the expected utility of a voter $j \in N$ under **representative democracy** when candidate $c^{RD} \in C$ is elected and such candidate implements the policy that maximizes her utility. Equations (6) and (7) represent the cases $c^{RD} = L$ and $c^{RD} = R$ respectively (see *Appendix A.3* for computations).

$$\mathbb{E}u_j(L, \theta) = (1 - p)\mu_L\Big(v + z_j(r) - z_j(l)\Big) + pv + z_j(l) \tag{6}$$

$$\mathbb{E}u_j(R, \theta) = p\mu_R\Big(v - z_j(r) + z_j(l)\Big) + (1 - p)v + z_j(r) \tag{7}$$

Under representative democracy, each voter $j \in N$ votes for the candidate (L or R) which maximizes her expected utility, and the candidate selected by the majority rule chooses which policy to implement.

3. Equilibria

In this section, we derive equilibria under direct and representative democracy. We consider the dominant strategy equilibrium concept: under each voting system, each voter votes for the alternative (a *policy* under direct democracy and a *candidate* under representative democracy) that maximizes her expected utility. In order to determine the outcome under each regime, we identify an indifferent voter and the median voter whose preference determines the outcome.[8] The indifferent voter under direct democracy, i^{DD}, is the voter who is indifferent between policies l and r, *i.e.*, $\mathbb{E}u_{i^{DD}}(l,\theta) = \mathbb{E}u_{i^{DD}}(r,\theta)$. The indifferent voter under representative democracy, i^{RD}, is the voter who is indifferent between candidates L and R, *i.e.*, $\mathbb{E}u_{i^{RD}}(L,\theta) = \mathbb{E}u_{i^{RD}}(R,\theta)$.

Lemmas 1 and 2 characterize the indifferent voter under both systems. Proofs of these Lemmas are in *Appendix B*.

Lemma 1. *The indifferent voter under direct democracy is such that* $\triangle z_{i^{DD}} = v(2p-1)$. *Every voter* $j \in N$ *such that* $\triangle z_j < \triangle z_{i^{DD}}$ *prefers the policy* l*, whereas every voter* $j \in N$ *such that* $\triangle z_j > \triangle z_{i^{DD}}$ *prefers the policy* r.

If $v = 0$ or $p = \frac{1}{2}$, then the indifferent voter under direct democracy has no private bias (*i.e.*, $\triangle z_{i^{DD}} = 0$). The first condition implies that voters do not care at all about the socially correct policy, whereas the second condition implies that both realizations of the shock are equally expected by voters.

Lemma 2. *The indifferent voter under representative democracy is such that* $\triangle z_{i^{RD}} = \frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L}$. *Every voter* $j \in N$ *such that* $\triangle z_j < \triangle z_{i^{RD}}$ *prefers the candidate* L*, whereas every voter* $j \in N$ *such that* $\triangle z_j > \triangle z_{i^{RD}}$ *prefers the candidate* R.

If $v = 0$ or $p = \frac{1}{2}$ and $\mu_L = \mu_R$, then the indifferent voter under representative democracy has no private bias (*i.e.*, $\triangle z_{i^{RD}} = 0$). The condition on v is interpreted as before. The condition on beliefs has now two parts: both realizations of the shock are equally expected and both

[8] In case the median voter is indifferent we assume that the status quo alternative remains (consider, for instance, that status quo alternatives under direct and representative democracy are *policy* l and *candidate* L respectively).

candidates are equally likely to be pragmatic.

However, it is generally not the case that the indifferent voter has no private bias. A voter who has a private bias for one of the two policies may be indifferent between policies under direct democracy when she derives additional utility from the socially correct policy. For the same reason, a voter who has a private bias for one of the two policies may end up being indifferent between candidates. Example 1 in *Appendix A.4* shows a situation in which the indifferent voters under both regimes have private biases. Still, that a voter is indifferent between candidates does not imply that the voter is indifferent between policies. As we have shown in Lemma 1, a comparison between $\triangle z_j$ and $\triangle z_{i^{DD}}$ reveals voter j's policy preference under direct democracy. By studying the relationship between $\triangle z_{i^{RD}}$ and $\triangle z_{i^{DD}}$ we can know the policy preferred by i^{RD}. Lemma 3 shows that the outcomes of this analysis depend on the voters' beliefs about both the socially correct policy and candidates' types. The proof of this Lemma is in *Appendix B*.

Lemma 3. *For each distribution function* $F(\cdot)$*:*

i. if either $p \in \{0,1\}$ *or* $p \in (0,1)$ *and* $\mu_L = \mu_R$*, then* i^{RD} *is indifferent between* l *and* r*,*

ii. if $p \in (0,1)$ *and* $\mu_L < \mu_R$*, then* i^{RD} *prefers* l*, and*

iii. if $p \in (0,1)$ *and* $\mu_L > \mu_R$*, then* i^{RD} *prefers* r*.*

If $p \in \{0,1\}$ or $p \in (0,1)$ and $\mu_L = \mu_R$, then i^{DD} and i^{RD} coincide. The first condition implies that voters believe that a certain policy is socially correct for sure. Voters then do not longer perceive candidates as agents with superior information. The second condition implies that, although voters recognize that candidates have an informational advantage, they cannot distinguish candidates by their types. In both cases, their private biases are the only differentiating factor between candidates, so saying that a voter is indifferent between L and R would be equivalent to say that such voter is indifferent between l and r. However, if $p \in (0,1)$ and $\mu_L \neq \mu_R$, then i^{DD} and i^{RD} are different: if $\mu_L < \mu_R$, then i^{RD} prefers the policy l, whereas if $\mu_L > \mu_R$, then i^{RD} prefers the policy r. In the first case, since voters believe that R is more likely to be pragmatic, the voter who is indifferent between L and R is in fact a voter who prefers policy l. Analogous interpretation applies to the second case. Table 1 summarizes results in Lemma 3, in terms of beliefs.

	$p = 0$	$p \in (0,1)$	$p = 1$
$\mu_L < \mu_R$	$\triangle z_{iRD} = \triangle z_{iDD}$	$\triangle z_{iRD} < \triangle z_{iDD}$	$\triangle z_{iRD} = \triangle z_{iDD}$
$\mu_L = \mu_R$	$\triangle z_{iRD} = \triangle z_{iDD}$	$\triangle z_{iRD} = \triangle z_{iDD}$	$\triangle z_{iRD} = \triangle z_{iDD}$
$\mu_L > \mu_R$	$\triangle z_{iRD} = \triangle z_{iDD}$	$\triangle z_{iRD} > \triangle z_{iDD}$	$\triangle z_{iRD} = \triangle z_{iDD}$

Table 1 – Relationship between $\triangle z_{iRD}$ and $\triangle z_{iDD}$ depending on the values of p, μ_L, and μ_R

Although the indifferent voters under both direct and representative democracy may have private biases, they always prefer that the socially correct policy be implemented, even when contrary to their private biases. Lemma 4 states this result. Its proof is in *Appendix B*.

Lemma 4. *For every $p \in [0,1]$, $v \geq 0$, and $\mu_c \in [0,1]$ where $c \in C$, both the indifferent voters under direct and representative democracy are pragmatic.*

The selected policy under direct democracy and the selected candidate under representative democracy both depend on the relationship between indifferent and median voters in each case. Lemma 5 specifies the selected alternative under each regime. The proof of this Lemma is in *Appendix B*.

Lemma 5. *We have that:*

i. the selected policy under direct democracy is:
$x^{DD} = l$ *if* $\triangle z_m \leq \triangle z_{iDD}$ *and* $x^{DD} = r$ *if* $\triangle z_m > \triangle z_{iDD}$, *and*

ii. the selected candidate under representative democracy is:
$c^{RD} = L$ *if* $\triangle z_m \leq \triangle z_{iRD}$ *and* $c^{RD} = R$ *if* $\triangle z_m > \triangle z_{iRD}$.

Lemma 5 shows that the outcomes under both direct and representative democracy depend on the value of $\triangle z_m$ with respect to $\triangle z_{iDD}$ and $\triangle z_{iRD}$. Theorem 1 states, for each possible configuration of parameters $\triangle z_m$, $\triangle z_{iDD}$, and $\triangle z_{iRD}$, the policy and the candidate that would be selected under one and another regime respectively. The proof for this result directly follows from Lemmas 3 and 5 so we omit it.

Theorem 1. *The outcomes under direct and representative democracy are:*

i. if $\triangle z_m \in (-\infty, \min\{\triangle z_{iDD}, \triangle z_{iRD}\})$, *then* $x^{DD} = l$ *and* $c^{RD} = L$,

ii. if $\triangle z_m \in (\max\{\triangle z_{iDD}, \triangle z_{iRD}\}, \infty)$, *then* $x^{DD} = r$ *and* $c^{RD} = R$,

iii. if $\triangle z_m \in [\triangle z_{iRD}, \triangle z_{iDD}]$, *then* $x^{DD} = l$ *and* $c^{RD} = R$, *and*

iv. if $\triangle z_m \in [\triangle z_{iDD}, \triangle z_{iRD}]$, *then* $x^{DD} = r$ *and* $c^{RD} = L$.

From Theorem 1 we have that if $\triangle z_{iDD} = \triangle z_{iRD}$, then the median voter delegates the decision making process on the candidate whose private bias coincides with her preferred policy. From Lemma 3 we know that this happens when voters either believe that a certain policy is socially correct for sure or are not able to differenciate candidates by their types. Figure 1 illustrates this situation.

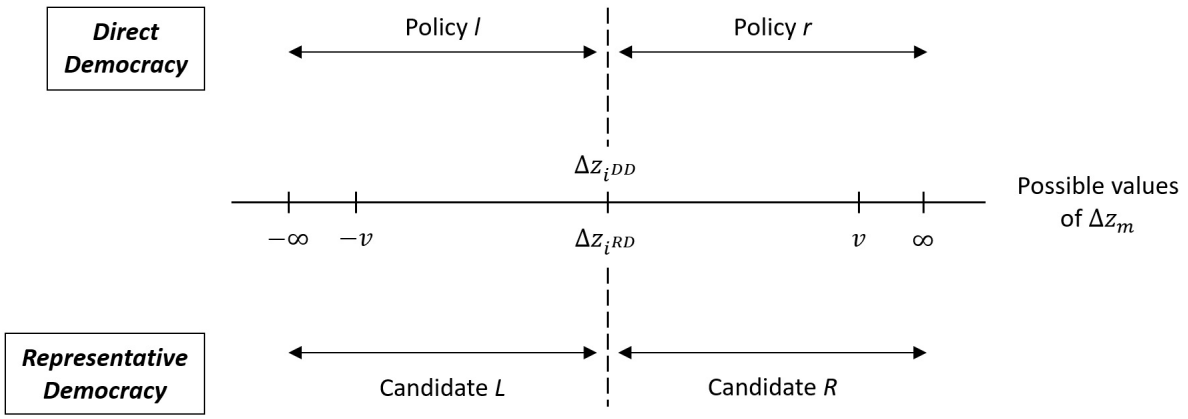

Figure 1 – Comparison of DD and RD when either $p \in \{0, 1\}$ or $p \in (0, 1)$ and $\mu_L = \mu_R$

However, this is not necessarily true when candidates can be distinguished by their types. In that case, the median voter might delegate the decision on a candidate with a private bias different from her preferred policy. Assume, without loss of generality, that $\triangle z_{iRD} < \triangle z_{iDD}$. From Lemma 3 we know that this happens when voters are uncertain about the socially correct policy and believe that R is more likely to be pragmatic. This situation is represented in Figure 2.

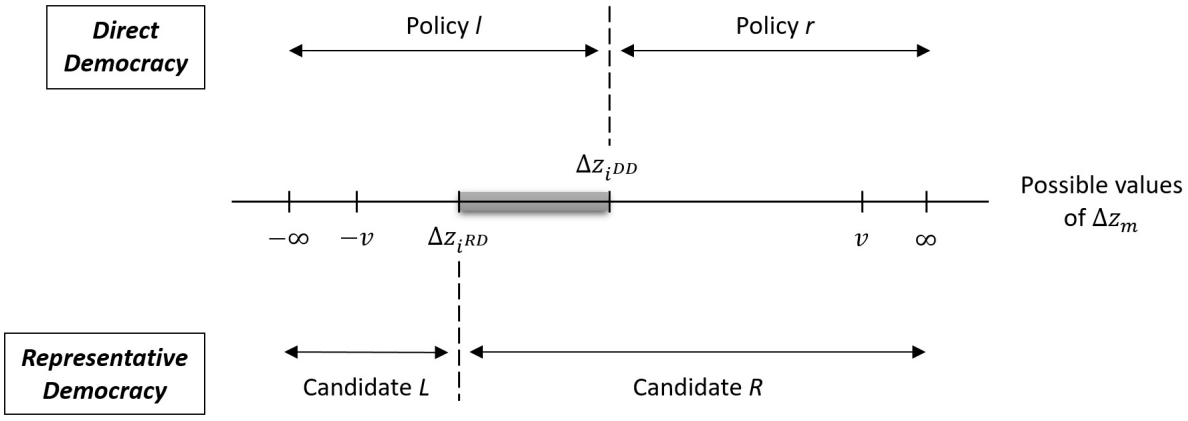

Figure 2 – Comparison of DD and RD when $p \in (0,1)$ and $\mu_L < \mu_R$

Suppose that $\triangle z_m \in [\triangle z_{iRD},\ \triangle z_{iDD}]$ (see shaded region in Figure 2). We know then that the median voter is pragmatic since $|\triangle\ z_m| \leq v$ holds from Lemma 4. Under direct democracy, the median voter casts her ballot for policy l, whereas she votes for candidate R under representative democracy, even though R's private bias is not the policy l. This behaviour might indicate interest in the implementation of the socially correct policy: the median voter decides to vote for candidate R since she is more likely to be pragmatic. Note that this is in line with the fact that the considered median voter is pragmatic.

4. Electorate's preference on regimes: Direct Democracy versus Representative Democracy

Will voters choose direct democracy or representative democracy? The purpose of this section is to determine which is the system preferred by the majority of voters given the uncertainty about both the socially correct policy and the type of each candidate. The answer hinges on the median voter. We compare the median voter's expected utility under direct democracy (*i.e.*, the utility before a *policy* is selected) with her expected utility under representative democracy (*i.e.*, the utility before a *candidate* is selected). We say that representative democracy is preferred to direct democracy by the majority of voters if $\mathbb{E}u_m(x^{DD}, \theta) \leq \mathbb{E}u_m(c^{RD}, \theta)$.

Theorem 2 shows that the preferred voting system depends on both the type of the electorate and the type of the median voter of the distribution of voters' biases intensity. The proof of this Theorem is in *Appendix B*.

Theorem 2. *If the electorate is pragmatic, then representative democracy is the system preferred by the majority of voters. If the electorate is ideological, then:*

i. if the median voter is ideological, direct democracy is the system preferred by the majority of voters, while

ii. if the median voter is pragmatic, representative democracy is the system preferred by the majority of voters.

We illustrate the three combinations exposed in Theorem 2.

Situation 1. *Pragmatic electorate.*
A majority of voters are pragmatic, so representative democracy is the preferred regime. The reason is simple: given that the majority of voters prefer that the socially correct policy be implemented and voters are aware of the candidates' informational advantage, they prefer that the decision on which policy to implement be made by a candidate. Note that in this case the median voter is necessarily pragmatic.

Situation 2. *Ideological electorate and ideological median voter.*
A majority of voters are ideological and ideological voters who are biased for the same policy constitute a majority by themselves. Since at least 50% of voters are interested in the implementation of their (same) bias regardless of the socially correct policy, direct democracy is the preferred regime. This system allows voters to implement whatever policy they desire, thus preventing a candidate from ending up implementing a different policy.

Situation 3. *Ideological electorate and pragmatic median voter.*
A majority of voters are ideological but neither ideological voters biased for l nor ideological voters biased for r constitute a majority by themselves. This is what we have called a **polarized electorate**. Representative democracy is the preferred regime, which may seem counterintuitive: why might a society where the majority of voters are not concerned about

the socially correct policy end up preferring to delegate decisions on a candidate? The key to answer this question is the electorate's polarization between l and r. A group of ideological voters may fear that, as a result of the ideological polarization of society, a policy different to their private bias be chosen under direct democracy. Faced with the risk that their preferred policy would not be chosen, they may prefer representative democracy. The reason is that these ideological voters believe that the implementation of their private bias is more likely under this voting system: with certain probability, their preferred policy is socially correct and the candidate selected under representative democracy is pragmatic, so she would end up implementing such policy. Therefore, in this situation there would exist a coalition of voters supporting the representative democracy composed by, not only the group of pragmatic voters but also a group of ideological voters.

5. Conclusion

When a decision has to be made, voters may choose between directly deciding or delegating to informed representatives. We have proposed a model to study why voters would opt for one or another option. We find that the choice depends on both the type of the electorate and the type of the median voter of the distribution of voters' biases intensity. As long as the electorate is pragmatic, representative democracy is the preferred system. The informational advantage of representatives is enough for an electorate concerned with the common interest to have incentives to delegate. When the electorate is ideological, though, we have to look at the type of the median voter in order to determine the preferred regime. If both electorate and median voter are ideological, then direct democracy is the preferred system. This is a society in which there is a majority of voters who do not care about what is socially correct and agree on what decision should be made. Such an electorate prefers a regime that guarantees the implementation of the policy that they desire, rather than running the risk of allowing a representative to choose. However, if the electorate is ideological and the median voter is pragmatic, then representative democracy is the preferred system. The striking thing about this result is that even with a majority of ideological voters, a system of political representation might be preferred. Although there is again a majority of voters who do not care about what is socially correct, in this case there is no consensus among all of them on what action should be carried out. Instead, there exists a clear division of the electorate into two groups defending

opposing opinions. Since neither of these two groups constitute a majority by themselves, we say that the electorate is polarized. As a consequence of this polarization, any of the two proposed alternatives could be the outcome under direct democracy. Thus, whatever the result, there will always be a group of ideological voters who oppose such a policy. Trying to avoid that something opposed to their interests is chosen, they prefer to delegate their vote on representatives. Those ideological voters who fear that their preferred policy will not be selected under direct democracy believe that its implementation might be more likely under representative democracy: with certain probability, such policy is socially correct and the selected candidate is pragmatic, which would guarantee the implementation of their preferred policy. These ideological voters, along with the pragmatic voters that could be in the society, constitute a majority coalition in favour of the representative democracy.

An interesting implication of our results is that instruments of direct democracy, such as referendums and popular consultations, should be used to make decisions only by those societies in which there is a consensus among a majority sector of the population about what decision should be taken with respect to a particular issue. However, political representation would be convenient in those societies that show a segregation of their citizens into two different sides with opposing positions about which choice should be made. In recent times, we have witnessed a growing demand for referendums around the world. The United Kingdom European Union membership referendum (June, 23rd 2016), Colombian peace agreement referendum (October, 2nd 2016), and Italian constitutional referendum (December, 4th 2016) are just some of the most well-known cases of important decisions that were made through referendum. A very enriching exercise would be to study whether such a referendum boom is motivated by the emergence of non-polarized ideological societies. Achieving this objective involves identifying measurable criteria to determine if a society should be considered as pragmatic or ideological, as well as its possible polarization in the latter case. This would allow us to verify with real data whether the predictions of the model conform to reality. This analysis is left for future research.

Appendix A

Appendix A.1

Figure 3 provides a graphical representation of all possible combinations between private biases and types for each voter $j \in N$.

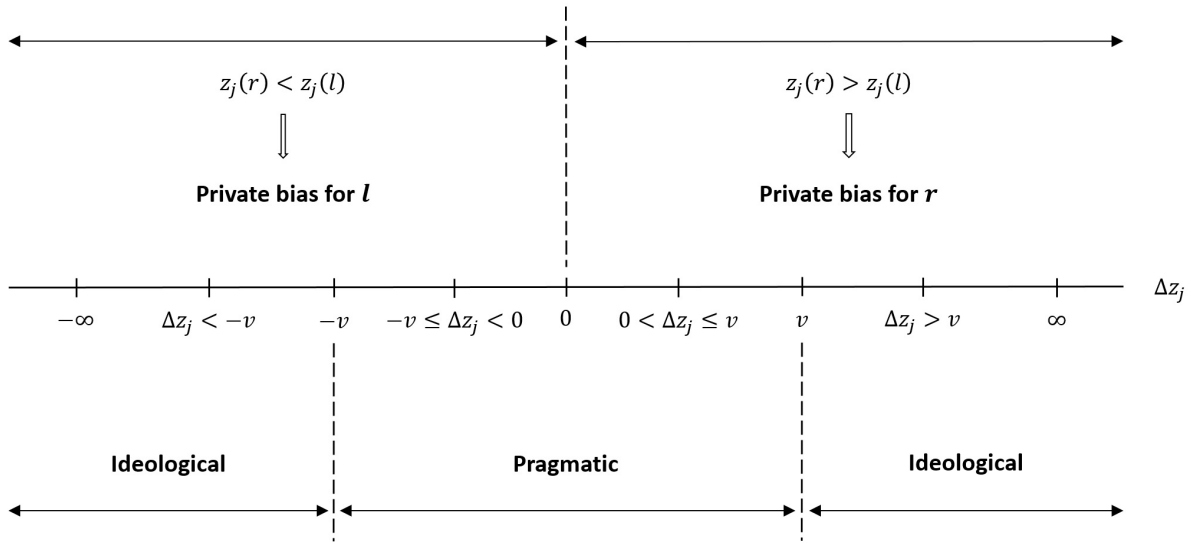

Figure 3 – Private bias and type of voter j

Consider a voter j such that $\triangle\ z_j < -v$. Since $\triangle\ z_j < 0$ (*i.e.*, $z_j(r) < z_j(l)$), we know that voter j has a private bias for policy l. In addition, if $\triangle\ z_j < -v$, then the utility that j obtains from policy l is so high compared to the utility that she receives from policy r that, even when policy r may be the socially correct one, j would still prefer policy l. The difference between the private component evaluated in l and the private component evaluated in r is so large that it cannot be compensated by the public component of the utility function. Thus, voter j is concerned about the implementation of policy l, regardless what the socially correct policy is. Note that, by symmetry, a voter j such that $\triangle\ z_j > v$ will be concerned about the implementation of policy r above all things. For this reason, in both cases we say that voter j is ideological. Consider now a voter j such that $-v \leq \triangle\ z_j < 0$. In this case, voter j still prefers policy l when she only cares about the private component. However, the difference between the utility that j receives from policy l and the utility that she obtains from policy r is not as large as in the case where $\triangle\ z_j < -v$. In fact, the value of the additional utility received

in case the socially correct policy is implemented (*i.e.*, v) is larger than the aforementioned difference for voter j. This implies that, although voter j has a private bias for policy l, she ends up preferring policy r when it is the socially correct policy. Put differently, the public component of the utility function is large enough to compensate the difference between the private component evaluated in l and the private component evaluated in r. By symmetry, a voter j such that $0 < \triangle\, z_j \leq v$ will prefer policy l when it is socially correct to implement it, even though she has a bias for policy r (*i.e.*, $\triangle\, z_j > 0$). Consequently, we refer to voter j as pragmatic voter in both cases.

Appendix A.2

$$\mathbb{E}u_j(l,\theta) = p \cdot u_j(l,l) + (1-p) \cdot u_j(l,r) = p\Big(z_j(l) + v\Big) + (1-p)\Big(z_j(l)\Big) = z_j(l) + pv$$

$$\mathbb{E}u_j(r,\theta) = p \cdot u_j(r,l) + (1-p) \cdot u_j(r,r) = p\Big(z_j(r)\Big) + (1-p)\Big(z_j(r) + v\Big) = z_j(r) + (1-p)v$$

Appendix A.3

$$\begin{aligned}
\mathbb{E}u_j(L,\theta) &= p\Big(\mu_L \cdot u_j(l,l) + (1-\mu_L) \cdot u_j(l,l)\Big) + (1-p)\Big(\mu_L \cdot u_j(r,r) + (1-\mu_L) \cdot u_j(l,r)\Big) \\
&= p\bigg(\mu_L\Big(z_j(l) + v\Big) + (1-\mu_L)\Big(z_j(l) + v\Big)\bigg) + (1-p)\bigg(\mu_L\Big(z_j(r) + v\Big) + (1-\mu_L)\Big(z_j(l)\Big)\bigg) \\
&= (1-p)\mu_L\Big(v + z_j(r) - z_j(l)\Big) + pv + z_j(l)
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}u_j(R,\theta) &= p\Big(\mu_R \cdot u_j(l,l) + (1-\mu_R) \cdot u_j(r,l)\Big) + (1-p)\Big(\mu_R \cdot u_j(r,r) + (1-\mu_R) \cdot u_j(r,r)\Big) \\
&= p\bigg(\mu_R\Big(z_j(l) + v\Big) + (1-\mu_R)\Big(z_j(r)\Big)\bigg) + (1-p)\bigg(\mu_R\Big(z_j(r) + v\Big) + (1-\mu_R)\Big(z_j(r) + v\Big)\bigg) \\
&= p\mu_R\Big(v - z_j(r) + z_j(l)\Big) + (1-p)v + z_j(r)
\end{aligned}$$

Appendix A.4

Example 1. *i^{DD} and i^{RD} with private biases.*

Consider the distribution function $F(\cdot)$ proposed in Figure 4. Suppose that $v = 11$ and $p = 0.4$. From Lemma 1, we have that $\triangle z_{iDD} = -2.2$. Note that i^{DD} has a private bias for policy l since $\triangle z_{iDD} < 0$.

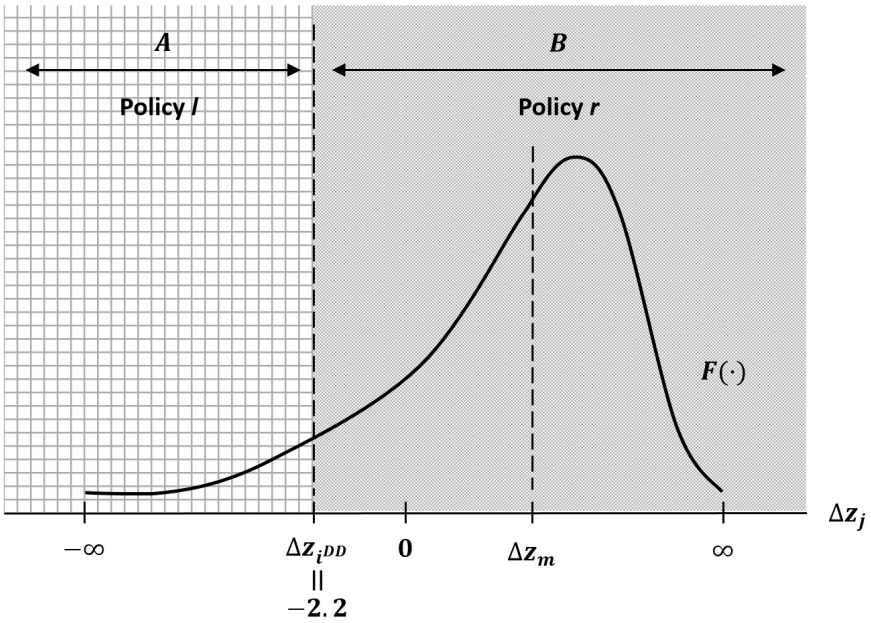

Figure 4 – Example of i^{DD} biased for l

Under direct democracy, every voter $j \in N$ in region A casts her ballot for l, while every voter $j \in N$ in region B casts her ballot for r. The policy r has the support of at least a majority of voters, which implies that such policy is implemented under this regime.

Consider again the distribution function $F(\cdot)$ proposed in Figure 4. Suppose that $v = 11$, $p = 0.4$, $\mu_L = 0.3$, and $\mu_R = 0.9$. From Lemma 2, we have that $\triangle z_{iRD} = -9.1$. Note that i^{RD} has a private bias for policy l since $\triangle z_{iRD} < 0$. Furthermore, we find that i^{RD} is not indifferent between policies under direct democracy. From Lemma 1, we have that i^{RD} prefers the policy l under direct democracy since $\triangle z_{iRD} = -9.1 < -2.2 = \triangle z_{iDD}$. Figure 5 offers a graphical representation of that particular example.

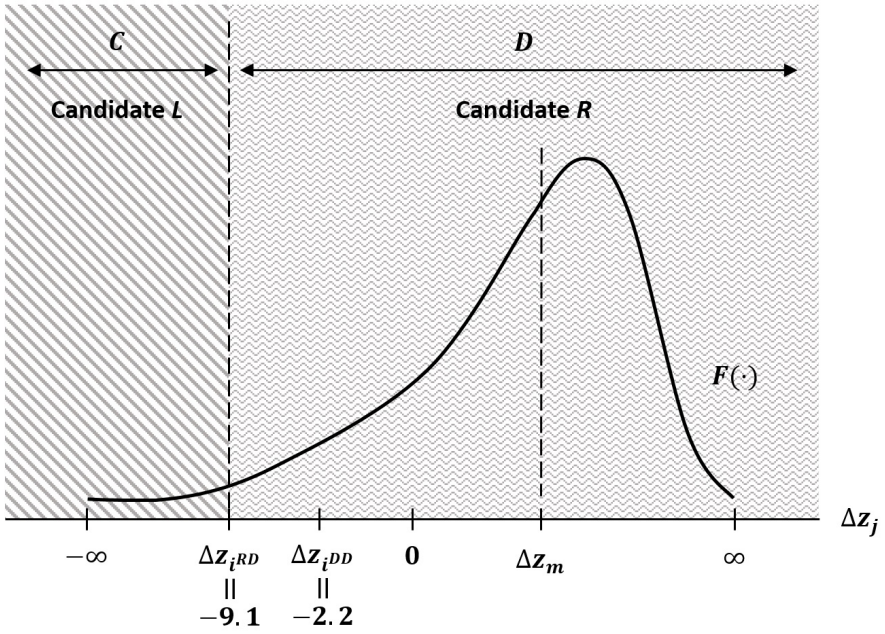

Figure 5 – Example of i^{RD} biased for l and preferring l under DD

Under representative democracy, every voter $j \in N$ in region C votes for L, while every voter $j \in N$ in region D votes for R. The candidate R has the support of at least a majority of voters, which implies that such candidate is selected under this regime.

Appendix B

PROOF OF LEMMA 1. Let i^{DD} be such that:

$$\mathbb{E}u_{i^{DD}}(l,\theta) = \mathbb{E}u_{i^{DD}}(r,\theta) \tag{8}$$

From Equation (4) we have that the expected utility of i^{DD} when policy l is implemented is:

$$\mathbb{E}u_{i^{DD}}(l,\theta) = z_{i^{DD}}(l) + pv \tag{9}$$

and, from Equation (5), the expected utility of i^{DD} when policy r is implemented is:

$$\mathbb{E}u_{i^{DD}}(r,\theta) = z_{i^{DD}}(r) + (1-p)v \tag{10}$$

By substituting (9) and (10) in (8) such condition can be rewritten as:

$$\begin{aligned} z_{i^{DD}}(l) + pv &= z_{i^{DD}}(r) + (1-p)v \\ \Leftrightarrow z_{i^{DD}}(r) - z_{i^{DD}}(l) &= v(2p-1) \\ \Leftrightarrow \triangle\ z_{i^{DD}} &= v(2p-1) \end{aligned} \tag{11}$$

Consider now a voter $j \in N$ such that $\triangle z_j < \triangle z_{i^{DD}}$. Note that this condition is equivalent to:

$$\begin{aligned} \triangle\ z_j &< v(2p-1) \\ \Leftrightarrow z_j(r) - z_j(l) &< v(2p-1) \\ \Leftrightarrow \mathbb{E}u_j(l,\theta) &> \mathbb{E}u_j(r,\theta) \end{aligned} \tag{12}$$

which implies that the expected utility of j when policy l is implemented is higher than her expected utility when policy r is implemented. Therefore, voter j prefers policy l. By symmetry, if $\triangle z_j > \triangle z_{i^{DD}}$, then voter j prefers policy r.

■

PROOF OF LEMMA 2. Let i^{RD} be such that:

$$\mathbb{E}u_{i^{RD}}(L,\theta) = \mathbb{E}u_{i^{RD}}(R,\theta) \tag{13}$$

From Equation (6) we have that the expected utility of i^{RD} when the candidate L is elected is:

$$\mathbb{E}u_{i^{RD}}(L,\theta) = (1-p)\mu_L\Big(v + z_{i^{RD}}(r) - z_{i^{RD}}(l)\Big) + pv + z_{i^{RD}}(l) \tag{14}$$

and, from Equation (7), the expected utility of i^{RD} when the candidate R is elected is:

$$\mathbb{E}u_{i^{RD}}(R,\theta) = p\mu_R\Big(v - z_{i^{RD}}(r) + z_{i^{RD}}(l)\Big) + (1-p)v + z_{i^{RD}}(r) \tag{15}$$

By substituting (14) and (15) in (13) such condition can be rewritten as:

$$\begin{aligned}(1-p)\mu_L\Big(v + z_{i^{RD}}(r) - z_{i^{RD}}(l)\Big) + pv + z_{i^{RD}}(l) &= p\mu_R\Big(v - z_{i^{RD}}(r) + z_{i^{RD}}(l)\Big) + (1-p)v + z_{i^{RD}}(r)\\ \Leftrightarrow z_{i^{RD}}(r) - z_{i^{RD}}(l) &= \frac{v(2p-1-p\mu_R+(1-p)\mu_L)}{1-p\mu_R-(1-p)\mu_L}\\ \Leftrightarrow\triangle\ z_{i^{RD}} &= \frac{v(2p-1-p\mu_R+(1-p)\mu_L)}{1-p\mu_R-(1-p)\mu_L}\end{aligned} \tag{16}$$

Consider now a voter $j \in N$ such that $\triangle z_j < \triangle z_{i^{RD}}$. Note that this condition is equivalent to:

$$\begin{aligned}\triangle\ z_j &< \frac{v(2p-1-p\mu_R+(1-p)\mu_L)}{1-p\mu_R-(1-p)\mu_L}\\ \Leftrightarrow z_j(r) - z_j(l) &< \frac{v(2p-1-p\mu_R+(1-p)\mu_L)}{1-p\mu_R-(1-p)\mu_L}\\ \Leftrightarrow \mathbb{E}u_{i^{RD}}(L,\theta) &> \mathbb{E}u_{i^{RD}}(R,\theta)\end{aligned} \tag{17}$$

which implies that the expected utility of j when candidate L is elected is higher than her expected utility when candidate R is elected. Therefore, voter j prefers candidate L. By symmetry, if $\triangle z_j > \triangle z_{i^{RD}}$, then voter j prefers candidate R.

■

PROOF OF LEMMA 3. From Lemma 1 we know that the relationship between $\triangle z_{i^{RD}}$ and $\triangle z_{i^{DD}}$ reveals the policy preferred by i^{RD}. We divide this proof into two parts:

- First, we prove that the equality $\triangle z_{i^{RD}} = \triangle z_{i^{DD}}$ is trivial in the three following cases:
  - If $p = 0$, then $\triangle z_{i^{RD}} = \triangle z_{i^{DD}} = -v$ for every $\mu_c \in [0,1]$ where $c \in C$.
  - If $p = 1$, then $\triangle z_{i^{RD}} = \triangle z_{i^{DD}} = v$ for every $\mu_c \in [0,1]$ where $c \in C$.
  - If $p \in (0,1)$ and $\mu_L = \mu_R$, then $\triangle z_{i^{RD}} = \triangle z_{i^{DD}} = v(2p-1)$.
- Second, we prove that if $p \in (0,1)$, then the relationship between $\triangle z_{i^{RD}}$ and $\triangle z_{i^{DD}}$ depends on the values of μ_L and μ_R. Assume that $\triangle z_{i^{RD}} < \triangle z_{i^{DD}}$. By Lemmas 1 and 2, this condition can be rewritten as:

$$\frac{v(2p-1-p\mu_R+(1-p)\mu_L)}{1-p\mu_R-(1-p)\mu_L} < v(2p-1) \tag{18}$$

which holds when:

$$\mu_L < \mu_R \tag{19}$$

By symmetry, $\triangle z_{iRD} > \triangle z_{iDD}$ when $\mu_L > \mu_R$.

■

PROOF OF LEMMA 4. We divide this proof into two parts:

- First, we prove that $|\triangle\ z_{iDD}| \leq v$, or equivalently, $|v(2p-1)| \leq v$. Note that this is equivalent to prove that conditions (1) and (2) hold:

  (1) $v(2p-1) \leq v$, which holds since $p \leq 1$ is always the case.

  (2) $v(2p-1) \geq -v$, which holds since $p \geq 0$ is always the case.

  Hence, we have that $|v(2p-1)| \leq v$ for all $p \in [0,1]$ and $v \geq 0$.

- Second, we prove that $|\triangle\ z_{iRD}| \leq v$, or equivalently, $\left|\frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L}\right| \leq v$. Note that this is equivalent to prove that conditions (3) and (4) hold:

  (3) $\frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L} \leq v$

  (4) $\frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L} \geq -v$

  We distinguish three cases depending on the values of μ_L and μ_R. We show that conditions (3) and (4) hold for each of these cases:

  a. Suppose that $\mu_L = \mu_R$. Then, conditions (3) and (4) are equivalent to conditions (1) and (2) respectively. Therefore, they hold for all $p \in [0,1]$ and $v \geq 0$.

  b. Suppose that $\mu_L < \mu_R$. Then, condition (3) requires that $\mu_L \leq 1$, and condition (4) requires that $\mu_R \leq 1$, which are always the case. Therefore, they hold for all $p \in [0,1]$ and $v \geq 0$.

  c. Suppose that $\mu_L > \mu_R$. By symmetry to case *b.*, conditions (3) and (4) also hold for all $p \in [0,1]$ and $v \geq 0$.

  Hence, we have that $\left|\frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L}\right| \leq v$ for all $p \in [0,1]$, $v \geq 0$ and every $\mu_c \in [0,1]$ where $c \in C$.

■

PROOF OF LEMMA 5. We divide this proof into two parts:

- **Direct democracy.**[9] The selected policy depends on the preferences of the median voter of the distribution $F(\cdot)$. By Lemma 1, the median voter prefers the policy l when $\triangle z_m < v(2p-1)$. If the median voter prefers the policy l, then a majority of voters also prefer the policy l. Thus, policy l will be selected under a majority rule voting system. By symmetry, policy r will be selected when $\triangle z_m > v(2p-1)$.

- **Representative democracy.**[10] The selected candidate depends on the preferences of the median voter of the distribution $F(\cdot)$. By Lemma 2, the median voter prefers the candidate L when $\triangle z_m < \frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L}$. If the median voter prefers the candidate L, then a majority of voters also prefer the candidate L. Thus, candidate L will be selected under a majority rule voting system. By symmetry, candidate R will be selected when $\triangle z_m > \frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L}$.

■

PROOF OF THEOREM 2. We divide this proof into two parts.

- First, we study which is the system preferred by the majority of voters. We distinguish four cases, which correspond to the cases identified in Theorem 1. For each of these cases, we compare the expected utility of the median voter under direct and representative democracy.

Case 1. Suppose that $\triangle z_m \in \left(-\infty, \min\{\triangle z_{iDD}, \triangle z_{iRD}\}\right)$. Then DD $\precsim$ RD if:

$$\begin{aligned} & \mathbb{E}u_m(l,\theta) \leq \mathbb{E}u_m(L,\theta) \\ \Leftrightarrow\ & z_m(l) + pv \leq (1-p)\mu_L\Big(v + z_m(r) - z_m(l)\Big) + pv + z_m(l) \\ \Leftrightarrow\triangle\ & z_m \geq -v \end{aligned} \tag{20}$$

[9] In case the median voter is indifferent we assume that the status quo policy (l) remains.

[10] In case the median voter is indifferent we assume that the status quo candidate (L) remains.

Case 2. Suppose that $\triangle z_m \in \big(\max\{\triangle z_{iDD}, \triangle z_{iRD}\}, \infty\big)$. Then DD $\precsim$ RD if:

$$\begin{aligned} &\mathbb{E}u_m(r,\theta) \leq \mathbb{E}u_m(R,\theta) \\ \Leftrightarrow\ & z_m(r) + (1-p)v \leq p\mu_R\Big(v - z_m(r) + z_m(l)\Big) + (1-p)v + z_m(r) \\ \Leftrightarrow\triangle\ & z_m \leq v \end{aligned} \tag{21}$$

Case 3. Suppose that $\triangle z_m \in [\triangle z_{iRD},\ \triangle z_{iDD}]$. Then DD $\precsim$ RD if:

$$\begin{aligned} &\mathbb{E}u_m(l,\theta) \leq \mathbb{E}u_m(R,\theta) \\ \Leftrightarrow\ & z_m(l) + pv \leq p\mu_R\Big(v - z_m(r) + z_m(l)\Big) + (1-p)v + z_m(r) \\ \Leftrightarrow\triangle\ & z_m \geq \frac{v(-1 + p(2-\mu_R))}{1-p\mu_R} \end{aligned} \tag{22}$$

which always holds in the considered interval $[\triangle z_{iRD},\ \triangle z_{iDD}]$ since $\frac{v\left(-1+p(2-\mu_R)\right)}{1-p\mu_R} \leq \triangle z_{iRD}$ for all $p \in [0,1]$, $v \geq 0$, and $\mu_c \in [0,1]$ where $c \in C$. Therefore, we have that DD $\precsim$ RD throughout the interval $[\triangle z_{iRD},\ \triangle z_{iDD}]$.

Case 4. Suppose that $\triangle z_m \in [\triangle z_{iDD},\ \triangle z_{iRD}]$. Then DD $\precsim$ RD if:

$$\begin{aligned} &\mathbb{E}u_m(r,\theta) \leq \mathbb{E}u_m(L,\theta) \\ \Leftrightarrow\ & z_m(r) + (1-p)v \leq (1-p)\mu_L\Big(v + z_m(r) - z_m(l)\Big) + pv + z_m(l) \\ \Leftrightarrow\triangle\ & z_m \leq \frac{v(-1 + 2p + (1-p)\mu_L)}{1-(1-p)\mu_L} \end{aligned} \tag{23}$$

which always holds in the considered interval $[\triangle z_{iDD},\ \triangle z_{iRD}]$ since $\triangle z_{iRD} \leq \frac{v\left(-1+2p+(1-p)\mu_L\right)}{1-(1-p)\mu_L}$ for all $p \in [0,1]$, $v \geq 0$, and $\mu_c \in [0,1]$ where $c \in C$. Therefore, we have that DD $\precsim$ RD throughout the interval $[\triangle z_{iDD},\ \triangle z_{iRD}]$.

Note that, from Lemma 4, $|\triangle\ z_{iDD}| \leq v$ and $|\triangle\ z_{iRD}| \leq v$. Thus, from *Cases 1*, *2*, *3*, and *4* it is derived that representative democracy is the system preferred by the majority of voters as long as $|\triangle\ z_m| \leq v$. Equivalently, representative democracy is the system preferred by the majority of voters if the median voter is pragmatic, while direct democracy is the system preferred by the majority of voters if the median voter is ideological.

- Second, we study the preference for one or another system depending on the type of the electorate.

  i. *Pragmatic electorate.* By definition, the electorate is pragmatic if the majority of voters are pragmatic. This is equivalent to say that $\beta \geq 0.5$. Consequently, $\alpha + \gamma \leq 0.5$. Given that $\alpha, \gamma > 0$, the previous condition implies that $\alpha, \gamma < 0.5$. Therefore, the median voter of the distribution will necessarily belong to the proportion of voters denoted by β. In other words, if the electorate is pragmatic, then the median voter is necessarily pragmatic. From *Cases 1*, *2*, *3*, and *4* above we know that representative democracy is the system preferred by the majority of voters if the median voter is pragmatic. Thus, we conclude that if the electorate is pragmatic, then representative democracy is the system preferred by the majority of voters.

  ii. *Ideological electorate.* By definition, the electorate is ideological if the majority of voters are ideological. This is equivalent to say that $\alpha + \gamma > 0.5$. Consequently, $\beta < 0.5$. Note that, in this case, the median voter is not necessarily either pragmatic or ideological. In fact, the type of the median voter depends on the specific configuration of parameters α and γ as follows:

     - Assume that $\alpha + \gamma > 0.5$. Therefore, the electorate is ideological. If either $\alpha \geq 0.5$ or $\gamma \geq 0.5$, then the median voter belongs to the proportion of voters denoted by either α or γ respectively. In other words, the median voter is ideological. From *Cases 1*, *2*, *3*, and *4* above we know that direct democracy is the system preferred by the majority of voters if the median voter is ideological. Thus, we conclude that if the electorate is ideological and the median voter is ideological, then direct democracy is the system preferred by the majority of voters.

     - Assume that $\alpha + \gamma > 0.5$. Therefore, the electorate is ideological. If $\alpha < 0.5$ and $\gamma < 0.5$, then the median voter belongs to the proportion of voters denoted by β. In other words, the median voter is pragmatic. From *Cases 1*, *2*, *3*, and *4* above we know that representative democracy is the system preferred by the majority of voters if the median voter is pragmatic. Thus, we conclude that if the electorate is ideological and the median voter is pragmatic, then representative democracy is the system preferred by the majority of voters.

■

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