Málaga Economic Theory Research Center Working Papers

The market for scoops: A dynamic approach

Ascensión Andina-Díaz, José A. García-Martínez and Antonio Parravano

WP 2017-3
September 2017

Departamento de Teoría e Historia Económica
Facultad de Ciencias Económicas y Empresariales
Universidad de Málaga
ISSN 1989-6908

The market for scoops: A dynamic approach*

Ascensión Andina-Díaz[†] José A. García-Martínez[‡] and Antonio Parravano[§]

September 20, 2017

Abstract

We present a dynamic model of competition and reputation in the media industry, in which firms compete for the publication of scoops and both the publication of scoops and their veracity determine a firm's future reputation. We study the dynamics of firms' reputations and how it relates to two issues: The consumers' preferences for information and the dispersion of the firms' editorial standards for quality. We obtain that in the case of a duopoly, there is only one stable steady state. In this equilibrium the two firms coexist and the identity of the firm that leads the market (i.e., whether it is the firm with the high editorial standard or with the low standard) depends on a combination of the two issues above. We then use numerical simulations to analyze the stochastic dynamics for a larger number of firms. We obtain that most of the insights gained for the duopoly case are robust to the consideration of a higher number of firms. We also draw predictions on the number of firms surviving in the long run, showing that the more severe consumers are with the publication of false stories and/or the more similar the firms' standards for quality are, the higher the number of firms in the stationary state.

Keywords: Media industry; Competition; Reputation; Stochastic dynamics; Deterministic dynamics

JEL: D25; L10; L82

1 Introduction

The media industry is changing faster than at any time in history. The use of online platforms and social media as a source of information has experienced an unprecedent boost in the last two decades. Accompanying the change in the habits of news consumers, traditional financial sources have proven useless for the new era.

*We gratefully acknowledge the financial support from the Ministerio de Economía y Competitividad through projects MTM2014-54199-P and ECO2014-52345-P, and the Junta de Andalucía through project SEJ2011-8065. The usual disclaimer applies.

†Dpto. Teoría e Historia Económica, Universidad de Málaga, Málaga, Spain. E-mail: aandina@uma.es

‡Dpto. Estudios Económicos y Financieros, Universidad Miguel Hernández, Elche, Spain. E-mail: jose.garciam@umh.es

§Dpto. Teoría e Historia Económica, Universidad de Málaga, Málaga, Spain and Centro de Física Fundamental, Universidad de Los Andes, Mérida, Venezuela. E-mail: parravano@uma.es

All together, news organizations have been forced to reinvent themselves and search for alternative sources of business, from investing and moving to digital formats, to catering to a more specialized audience or simply reducing their staff.

Despite global tendencies in the industry, the market for news and in particular the traditional press sector presents great heterogeneity across countries. Recent studies highlight these differences. For example, according to the "Digital News Report 2017" by *Reuters Institute*, newspaper circulation in Spain continues being dominated by two big papers, *El País* and *El Mundo*, that also dominate the market in terms of accuracy and trustworthiness.[1] In the US, the press industry is highly competitive and polarized. Despite it, it shares with the Spanish case the feature of circulation and trustworthiness going hand in hand (at least for the most prominent papers). Indeed, in 2016 the ranking of US newspaper circulation was led by *USA Today*, *Wall Street Journal* and *New York Times* (see *Agility PR*). According to Mitchell and Weisel (2014), they are also the papers trusted by a majority of consumers.[2] In contrast to this, other countries present little correlation between circulation and trustworthiness. The most stark example is the UK, where the newspapers leading the market in terms of circulation, *The Sun* and *The Daily Mail*, are far behind the first positions when talking about trust and accuracy of news, with *The Times* and *The Guardian* leading this ranking.[3]

In this paper we propose a model to study the dynamics of media firms' reputations and share of news. The objective is to try to disentangle what ingredients may play a role in explaining the observed real world differences. We acknowledge that the problem is complex and that many variables may have something to say. For example, issues like percentage of press penetration in different social classes, ideological considerations and prices, that are not considered in this paper, may have important effects. In this sense, the present work should be understood as an approach that, despite omitting possible relevant variables, identifies some general properties of the competition in the media industry.

Our contribution is to pin down the importance of two variables in explaining the dynamics. On the one hand, the consumers' preferences. In particular, how responsive and punitive consumers are when a firm publishes a story that turns out to be false. On the other hand, what the nature of competition between media firms is. Namely, whether firms are homogeneous in their editorial standards for quality and so compete for the same stories, or they are rather different in their vetting processes for stories and so offer different products.

[1] In the study people were asked to state: "What news organizations are best in 'accuracy and trustworthiness' and in 'expertise and well-founded opinions"'. The two firms leading the raking are *El País* and *El Mundo*, in this order. It is also remarkable that when assigning people to groups according to their ideology, it is also these two newspapers the ones that are trusted by most of the ideological groups. The groups consist on: Left, center and right. See Vara-Miguel et al. (2017).

[2] In the study people were grouped into five categories according to their ideology: Consistently liberal, mostly liberal, mixed, mostly conservative and consistently conservative. The results show that *The Wall Street Journal* is the only news organization (including tv, online platform and such) that is "more trusted than distrusted" by all ideological categories. *USA Today* is "more trusted than distrusted" by four out of five categories (it is the only newspaper with this rate). As for *New York Times*, it is one of the three newspapers that is "more trusted than distrusted" by three of the ideological categories.

[3] See Newman (2017) and *PRWeek/OnePoll*.

The model is as follows. We consider a media industry with N firms and a finite but large enough time horizon. At every time step, that is referred to as a day, firms compete for the publication of a scoop. No other news are published. We assume that every day one scoop is released and that only one firm can publish it. Scoops contain information on a relevant variable and the information contained can be more or less precise, i.e., accurate. We refer to the accuracy of the information as the quality of the scoop. We assume that the quality is i.i.d. across periods, and that each scoop is either true or false with a probability proportional to its quality. Each firm is characterized by an editorial standard for quality, that sets a lower bound on the quality of a scoop for the firm to be willing to publish it. Editorial standards are given and are invariant throughout the dynamics.[4] Scoops are assigned to media firms according to their reputation. In particular, we consider that the higher the reputation of a firm at a given day, the higher the probability that the firm receives the scoop.[5] A firm that receives a scoop decides whether to publish it or not. A scoop that is published is referred to as a story or a piece or news. After the publication, consumers learn whether the story is true or false. This information is used to update the firms' reputations.

We analyze the dynamics of the media firms' reputation and the frequency of scoop publication (refereed to as share of news). The objective is to understand the effect on the dynamics of two issues: The consumers' preferences and the market dispersion. To this, we take two complementary approaches. First, we propose a mean-field analysis that substitutes the stochastic dynamics by a deterministic one.[6] In particular, at every time step, we substitute the random assignation of a scoop to a firm by its expected value. This simplification allows us to obtain analytical results for the case of two media firms. Second, we perform numerical simulations on the stochastic dynamics and obtain predictions for the general case with more than two firms.[7] As expected, the insights in the deterministic mean-field approach are in accordance with the average behavior of the stochastic dynamics.

Our analytical results for the mean-field approach with two firms show that there are two stationary states: One in which the two firms coexist, i.e., they both receive positive reputation and share of news; the other one in which only the firm with the lower editorial standard is active in the market, i.e., the stringent firm ends

[4]Despite this restriction, the model is rich enough so as to cover different scenarios varying in the degree of homogeneity of the firms' editorial standards.

[5]This idea is in line with the Lewinsky and bin Laden's death stories, where the scoops were first received by renown firms (*Newsweek* and *The New Yorker*, respectively) and only after they refused to publish them were passed on to less influential media firms (*Drudge Report* and *The London Review of Books*, respectively). See Andina-Díaz and García-Martínez (2016) for a detailed discussion on both stories.

[6]The mean-field analysis is an approach commonly used in statistical physics and epidemiology that approximates the dynamics of the system by its expected motion. In economics it was not introduced until the present century, but in the last decade its use has rapidly increased. We refer the reader to the pioneer work by López-Pintado (2006, 2008) and Jackson and Rogers (2007), that use the mean-field approach to study the diffusion of behaviors in a social network. More recent examples of the use of this technique in the economic literature are Lelarge (2012) and Kreindler and Young (2013).

[7]The use of numerical simulations is also increasing in importance in the economics literature. Examples are Harrington (1999), Vega-Redondo et al. (2005), Chen and Huang (2008) and Casari (2008), all with a micro focus. In the macroeconomic analysis its use is very common, as in the analysis of the dynamic stochastic general equilibrium models.

up with zero reputation and zero share of news. We also study the stability of the two solutions and obtain that only the stationary state in which both firms coexist is stable. Accordingly, the results that follow focus on this case. Finally, we make a comparative statics exercise to study the effect of the consumers' preferences and the market dispersion on the (stable) stationary state. We obtain that in the equilibrium, whether the market is dominated by the firm with the lower editorial standard or by the firm with the higher standard deeply depends on a combination of the two issues described above. In particular, we obtain that when the punishment for publishing false stories is not very high, it is always the firm with the lower editorial standard the one with the higher reputation. In this case, it is also this firm the one with the higher frequency of scoop publication. In this sense, our results predict that societies that are specially generous with firms that break the news and do not penalize them when the stories are shown to be false, will have media industries dominated by firms with low editorial standards. On the other hand, when the cost for lying is sufficiently high and firms are similar enough in their editorial standards, there is a chance for the firm with the stricter vetting process for stories to dominate the market (both in terms of reputation and frequency of scoop publication). However, for this to occur we need firms to be sufficiently homogeneous. Otherwise, the stricter firm may be the leader in reputation but the firm with a higher share of news will always be the less stringent one.

Regarding the stochastic dynamics, the results from the numerical simulations show that the insights derived in the mean-field approach are robust. Namely, that a low punishment for publishing false stores results in media industries leaded by firms with low standards for quality (and the other way round). Beyond the robustness check, the exercise with numerical simulations yields interesting results on the relationship between consumers' preferences, market homogeneity and the number of firms surviving in the long run. Interestingly, we obtain that the more severe consumers are, the higher the number of firms surviving in the long run. In contrast, when consumers are specially generous with firms that break the news and do not penalize false stories, we obtain that only a small number of firms endure in the long run. Our simulations (run for an industry with five and ten media firms) show that the most likely result is that only two firms survive in this case. As for the effect of the market dispersion, we obtain that the more similar the editorial standards of the firms, the higher the number of firms in the long run. In contrast, the more diverse they are, the smaller the number of firms in the long run. Putting all together, our results suggest that for competition in the media industry to endure and not to be an ephemeral feature we need of either compromised consumers (that penalize false information) and/or of an homogeneous industry (where firms have similar editorial standards). Otherwise, our prediction is that in the steady state, competition will be softer than in the first periods of the dynamics.

This work relates to the literature on the economics of mass media. This is an extensive literature that has grown rapidly in the last two decades. The first papers in this literature were mostly interested in analyzing the effects of the mass media on economic policies and outcomes. See Besley and Burgess (2001), Djankov et al. (2003), Strömberg (2004*b*). A second generation of papers studied whether news provision is biased

(Groseclose and Milyo (2005), Egorov et al. (2009) and Larcinese et al. (2011)) and what the determinants of the biases are (Strömberg (2004a), Mullainathan and Shleifer (2005), Gentzkow and Shapiro (2006), Baron (2006), Petrova (2008), Ellman and Germano (2009)). At the same time, a different group of researchers focused their attention on the study of the media as a two sided-market. See Rochet and Tirole (2003), Anderson (2006), Doyle (2013). Currently, the research on the economics of mass media is extremely varied, but there is a tendency towards the analysis of online news markets (Yang and Chyi (2011), Halberstam and Knight (2016), Germano and Sobbrio (2017)) and their effects on political outcomes (Allcott and Gentzkow (2017), Boxell et al. (2017), Campante et al. (2017)), collective action (Acemoglu et al. (2017), Little (2016), Enikolopov et al. (2016), Enikolopov et al. (2017)) and more generally, social outcomes (Gentzkow and Shapiro (2011), Quattrociocchhi et al. (2014), Bakshy et al. (2015)).

Our contribution to this literature is to present a new approach to the study of competition and reputation in the media industry. Previous research shows that competition can reduce the quality of news (Zaller (1999), Cagé (2014)), can induce firms to ideologically differentiate from competitors (Gentzkow et al. (2014)) or to bias the news in an attempt to product differentiate (Anand et al. (2007)). It has also been shown that competition can reduce the possibility of media capture by the government (Besley and Prat (2006)), can increase media self-censorship on issues sensitive to advertisers (Germano and Meier (2013)) and increase giving and volunteering (Adena (2016)). In contrast to previous research, our interest is not on the effects of competition but on its determinants, from a long run perspective. To the best of our knowledge, this is new in the literature.

The rest of the paper is organized as follows. In Section 2 we propose a stochastic dynamic model of competition and reputation in the media industry consisting in N recurse equations that account for the reputation changes in a market with N firms. In Section 3 we substitute the stochastic dynamics by a deterministic dynamics and analyze the resulting mean-field model for the case of two media firms. This simplification allows us to obtain analytical solutions for the reputation values and to study the stability of the steady states. In Section 4 we perform numerical simulations to study the stochastic dynamics. The analysis is done for different number of media firms. Finally, Section 5 concludes. All the proofs are in the Appendix.

2 The model

Let us consider a market for news with N media firms and a mass of consumers. Each $t \in T = \{1, 2, ...\}$ represents a time step, that we consider to be a day. At each time t there is a scoop $s(t)$ on a random variable $\omega(t) \in \Omega$, with Ω being the set of all states of the world and $\omega(t)$ the realized state at time t, with ω being distributed according to a known distribution function. Let μ be the quality of a scoop. We consider μ to be a random variable distributed according to a Uniform distribution function in $[0, 1]$. The realization of the random variable at t is denoted by $\mu(t)$, where $\mu(t) = P(s(t) = j|\omega(t) = j)$ is the probability that the scoop at t is true.

Media firms compete for the publication of scoops according to a rule that is later specified. For simplicity, we abstract from the publication of any other type of news. Hence, we will refer to the frequency of scoop publication as the frequency of news publication or simply as the share of news.

Each media firm $i \in \aleph = \{1, 2, ..., N\}$ is characterized by a threshold $\mu_i \in [0, 1]$, that sets a lower bound on the quality of a scoop for firm i to be willing to publish it. We will refer to μ_i as the *editorial standard* of firm i. In the present work we do not explore the optimality of this rule. To this, we refer the reader to our previous work Andina-Díaz and García-Martínez (2016), where we show that in the equilibrium of a game with career concerned media firms, firms optimally choose different editorial standards when the probability that consumers learn the state depends on the firms' publication strategies. In particular, we obtain that the higher the initial reputation of a firm and/or the greater its social influence, the stricter the firm's vetting process for stories will be. Accordingly, in the present paper we consider $\mu_i \neq \mu_j$, for all $i, j \in \aleph$. Without loss of generality, we will order firms according to their editorial standards, so that $0 \leq \mu_1 < \mu_2 < ... < \mu_N \leq 1$. We will assume that a firm's editorial standard is invariant throughout the game.

Conditioned on a firm having received a scoop of quality $\mu(t)$, threshold μ_i determines whether firm i publishes it. In particular, if $\mu(t) > \mu_i$ firm i publishes the scoop and suppresses it otherwise. We will denote by $a_i(t) \in \{0, 1\}$ the action of firm i at time t, where $a_i(t) = 1$ when the firm reports the scoop and $a_i(t) = 0$ otherwise. That is, in the model media firms can suppress information but can never make up a story. A scoop that is printed/published will be referred to as a story or a piece of news.

We assume that at every t, if a story is published, consumers immediately learn the value of the random variable $\omega(t)$ with probability 1, whereas if no piece of news is published there is no learning. Thus, after a report, consumers always receive valuable information to asses both the accuracy of a story and the firm's reputation.

Let $R_i(t)$ denote the reputation of media firm i at date t. In the model $R_i(t)$ is interpreted as the subjective evaluation or belief that the general public makes of firm i at time t. In assessing this belief, both news' quantity (share of news) and news' quality (accuracy of information) aspects will be taken into account. We assume that $\forall i \in \{1, ..., N\}$, reputation evolves according to the following two steps rule: In a first step, the reputation of the firm that publishes the scoop is updated. In particular, a firm's reputation increases when it gets a story and publishes it, and decreases when the story turns out to be false. The importance of quantity versus quality aspects is modulated through parameter α, that stands for the punishment or cost from lying. We will refer to α as the consumers' preferences for information.[8] In a second step, the reputation of all N firms are re-scaled to their normalized values $\widetilde{R}(t) \in [0, 1]$, with $\widetilde{R}_i(t) = \frac{R_i(t)}{\sum_{i=1}^{N} R_i(t)}$ and $\sum_{i=1}^{N} \widetilde{R}_i(t) = 1$.[9] The two step

[8]Note that values of $\alpha \sim 1$ talk about severe societies that strongly penalize the publication of false information relatively to the mere publication of stories, whereas values of $\alpha \sim 0$ describe societies that reward the publication of scoops without caring so much about their veracity.

[9]This process tries to resemble the cognitive mechanism of consumers who first take new information into account to update

rule is subsumed in the following recursive equation.

$$R_i(t+1) = \widetilde{R}_i(t) + A\left[(1-\alpha)(1-\widetilde{R}_i(t))I_a - \alpha\widetilde{R}_i(t)I_{s\neq\omega}\right], \tag{1}$$

where $\alpha \in [0,1]$, $A > 0$ is a parameter that modulates the change or amplitude in a firm's reputation due to the publication of a single scoop, and I_a and $I_{s\neq\omega}$ are the following indicator functions:[10]

$$I_a = \begin{cases} 1 \text{ if } a_i(t) = 1, \\ 0 \text{ otherwise,} \end{cases} \quad \text{and } I_{s\neq\omega} = \begin{cases} 1 \text{ if } a_i(t) = 1 \text{ and } s(t) \neq \omega(t), \\ 0 \text{ otherwise.} \end{cases}$$

Note that the rule above considers that variations in a firm's reputation are proportional to the firm's previous normalized reputation $\widetilde{R}(t)$. In particular, it dictates that at time t a firm with normalized reputation $\widetilde{R}(t)$ can only increase its reputation in a maximum amount of $1 - \widetilde{R}(t)$ and decrease it in $\widetilde{R}(t)$. In this sense, the rule assumes bounded variations. It also implies that that the higher a firm's reputation at t, the smaller the gain from publishing a scoop and the higher the cost from lying. On the other hand, the smaller a firm's reputation, the greater the gain from breaking the news and the smaller the cost from lying. We consider this is a reasonable assumption that represents an accurate picture of the functioning of the real world media industry.

We assume that a firm's reputation at t affects the probability that the firm receives a scoop at that time step. The idea is to model a situation in which firms with higher reputation are more able to capture scoops than less reputed firms. Let $p_i(\mu(t))$ denote the probability that firm i receives a scoop of quality $\mu(t)$ at time t. Remember that there is one scoop per day and that only one media firm can publish it. We assume that probability $p_i(\mu(t))$ is given by:

$$p_i(\mu(t)) = \frac{\theta_i(t)R_i(t)}{\sum_{k=1}^{N}\theta_k(t)R_k(t)}, \tag{2}$$

where

$$\theta_i(t) = \begin{cases} 1 & \text{if } \mu_i \leq \mu(t) \\ 0 & \text{if } \mu_i > \mu(t). \end{cases} \tag{3}$$

Note that the process above determines that given a scoop of quality $\mu(t)$, only the media firms with editorial standards lower than $\mu(t)$ are *active* recipients of the scoop, i.e., have positive probability of receiving it. Note also that because the publication strategy of any firm, say i, is to publish a scoop whenever $\mu(t) > \mu_i$, the specification above determines that any media firm that receives a scoop publishes it. This simple modelling approach should be understood as a reduced form of a more general mechanism consisting in: i) At all t, any firm has a positive probability of receiving the scoop. ii) The probability that a firm receives a scoop is the same than its reputation (in relative terms). iii) Upon receiving a scoop, the firm decides whether to publish the scoop or not. iv) If the firm chooses to publish it, the mechanism at t stops. If the firm rejects it,

the firms' reputations and then normalize the obtained values and re-scale them in $[0,1]$. Note also that because of the second step, the reputation of any firm is revisited (and changed) at all t, even if the firm does not publish anything at that time step.

[10] The effect of parameter A on the results is discussed in Section 4.

the process of assigning the scoop starts again, with the firm that rejected the scoop being taken out of the pool and the rest of the firms receiving the scoop with a probability that is equal to each firm's new relative reputation (computed once we exclude the rejecting firm). The reader may note that the outcome of this more general mechanism is equivalent to that under the simpler form, where scoops are directly offered to firms that are willing to publish them. For simplicity, we stick to the proposed formulation.

Finally, note also that the process above determines a division of the scoop's quality space $[0, 1]$ in different intervals, according to which competition for a scoop is softer or tougher. In particular, ceteris paribus N, the lower the quality $\mu(t)$ of a scoop, the softer the competition, as only firms with low editorial standards are willing to publish it. In the limit, when $\mu(t) < \mu_1$ no firm publishes the scoop. On the other hand, the higher $\mu(t)$, the greater the number of firms willing to publish it, thus the tougher the competition.[11]

The model described considers a stochastic sequence of scoops and N media firms that repeatedly interact in time, with interaction determining the dynamics of reputation and news share for each firm. This process defines a system of N stochastic recursive equations, which is analyzed in Section 4. Prior to this, in Section 3 we study a mean-field approach to the problem that allows us to study the average dynamics, characterize the steady states, study the stability and perform a comparative static exercise for the case of an industry with two media firms.

3 The mean-field dynamics

This section considers a mean-field approach to the problem. It makes the following simplification: We substitute the stochastic dynamics described in equation (1) by a deterministic dynamics. In particular, for every t and firm i, we substitute the random variables I_a and $I_{s\neq w}$ by their expected values $F_{i,news}$ and $F_{i,false}$:

$$R_i(t+1) = \widetilde{R}_i(t) + \left[(1-\alpha)(1-\widetilde{R}_i(t))F_{i,news}(t) - \alpha\widetilde{R}_i(t)F_{i,false}(t)\right]. \tag{4}$$

Note that the dynamics above makes $A = 1$. This is without loss of generality.[12]

Since the quality of a scoop $\mu(t)$ is assumed to be a random variable uniformly distributed in [0,1], the expected value of the random variable I_a is:

$$F_{i,news}(t) = \int_{\mu_i}^{\mu_{i+1}} p_{i,i}(t)d\mu + \int_{\mu_{i+1}}^{\mu_{i+2}} p_{i,i+1}(t)d\mu + ... + \int_{\mu_{N-1}}^{\mu_N} p_{i,N-1}(t)d\mu + \int_{\mu_N}^{1} p_{i,N}(t)d\mu,$$

where $p_{i,k}(t) = \frac{R_i(t)}{\sum_{l=1}^{k} R_l(t)}$ is the probability that firm i receives a scoop of quality $\mu_i \leq \mu \leq \mu_k$ at time t. Note that $F_{i,news}$ represents the fraction of stories published by i in stationary conditions (over the total number

[11] Note that competition for scoops is also affected by how similar or different the firms' editorial standards are. Thus, the smaller the distance between the firms' standards, the more homogeneous firms are and so the tougher the competition. In contrast, the higher the distance the more differentiated they are and so the softer the competition in the market for scoops.

[12] As observed in Section 4, the steady state solution does not depend on A, that only affects the amplitude of the fluctuations around the solution. In particular, the higher A the greater the fluctuations. Hence, by considering $A = 1$ we analyze the stability of the system under adverse conditions.

of news received by the firms). Note also that $\sum_{i=1}^{N} F_{i,news}(t) = 1 - \mu_1$, since a fraction μ_1 of stories does not fit the minimum quality to be published by any firm and so is silenced.

Let $\mu_{N+1} = 1$. Then, we can rewrite this fraction as:[13]

$$F_{i,news}(t) = \sum_{k=i}^{N} \int_{\mu_k}^{\mu_{k+1}} p_{i,k}(t) d\mu. \tag{5}$$

Additionally, since the probability that a scoop is true is proportional to its quality μ, the expected probability $F_{i,true}(t)$ that firm i publishes a true story is:

$$F_{i,true}(t) = \sum_{k=i}^{N} \int_{\mu_k}^{\mu_{k+1}} \mu p_{i,k}(t) d\mu, \tag{6}$$

Therefore,

$$F_{i,false}(t) = F_{i,news}(t) - F_{i,true}(t) \tag{7}$$

is the expected value of the random variable $I_{s \neq w}$. Hence, $F_{i,false}(t)$ is also the fraction of stories published by firm i at t that are false.

As already mentioned, the fraction of scoops that are silenced is μ_1. Similarly, the fraction of scoops that are silenced and true is $\frac{\mu_1^2}{2}$ and the fraction of scoops that are silenced and false is $\mu_1(1 - \frac{\mu_1}{2})$. Last, a fraction $\frac{1}{2}(1 - \mu_1^2)$ of the scoops are published are true, and a fraction $\frac{1}{2} - \mu_1(1 + \frac{\mu_1}{2})$ are published and false.

3.1 Two firms

Next, we restrict the analysis to the case of two firms, 1 and 2, with $\mu_1 < \mu_2$. This simplification allows us to obtain analytical results and to identify some general properties of the system that we will later test with numerical simulations of the stochastic dynamics in Section 4.

We start analyzing the average participation of a firm on the distribution of scoops and how it relates to the firms' reputation values and editorial standards.[14] Let $\widetilde{R}_1 = \frac{R_1}{R_1 + R_2}$ denote firm 1's normalized reputation, where R_1 and R_2 are any arbitrary reputation values. From (5) we have:

$$\begin{aligned}
F_{1,news} &= (\mu_2 - \mu_1) + \widetilde{R}_1(1 - \mu_2), \\
F_{2,news} &= (1 - \widetilde{R}_1)(1 - \mu_2), \\
F_{1,false} &= \frac{1}{2}[\widetilde{R}_1(1 - \mu_2)^2 + (\mu_2 - \mu_1)(2 - \mu_1 - \mu_2)], \\
F_{2,false} &= \frac{1}{2}(1 - \widetilde{R}_1)(1 - \mu_2)^2.
\end{aligned}$$

Figure 1 represents expressions $F_{1,news}$ and $F_{2,news}$ as function of $\widetilde{R}_1$. Note that $F_{1,news}$ is increasing in $\widetilde{R}_1$ and $F_{2,news}$ is decreasing in $\widetilde{R}_1$. Additionally, note that when $1 - \mu_2 > \mu_2 - \mu_1$ both functions cross at $R_F > 0$, with $R_F = \frac{1}{2} - \frac{\mu_2 - \mu_1}{2(1 - \mu_2)} < \frac{1}{2}$. Otherwise, $F_{1,news}$ is always above $F_{2,news}$, i.e. $R_F \leq 0$. Hence, if

[13] Note that given (2), the expression in (5) can also be written as $F_{i,news}(t) = \int_{\mu_i}^{1} p_i(\mu(t)) d\mu$.

[14] Because we focus on average participation, we eliminate any reference to time t.

$\mu_2 - \mu_1 > 1 - \mu_2$, we have a situation in which the fraction of scoops published by firm 1, the less stringent firm, is always higher than the fraction of scoops published by firm 2. Since it requires $\mu_2 - \mu_1$ to be big enough, we learn that when the media industry is composed of two very different firms (in terms of their editorial standards), the more stringent firm will never be able to publish a higher share of stories. However, if μ_1 and μ_2 are not very different, then it can be that $1 - \mu_2 > \mu_2 - \mu_1$, in which case there are values of $\widetilde{R}_1$ for which firm 2 gets a higher fraction of the scoops. This is the case when $0 < \widetilde{R}_1 < R_F$.

Since $\widetilde{R}_1 \in [0, 1]$, the fact that $\widetilde{R}_1 = 1/2$ when $R_1 = R_2$ and $R_F < 1/2$ determines three regions represented in Figure 1. Region I corresponds to values $\frac{1}{2} < \widetilde{R}_1 \leq 1$. Here we have $R_1 > R_2$ and $F_{1,news} > F_{2,news}$. That is, in this region the firm with the lower standard for quality dominates the market both in terms of reputation and news share. Region II corresponds to values $R_F < \widetilde{R}_1 < \frac{1}{2}$. Here we have $R_2 > R_1$ and $F_{1,news} > F_{2,news}$. That is, in this region firm 1 still dominates the market in terms of news share but firm 2 does it in terms of reputation. Finally, Region III corresponds to values $0 \leq \widetilde{R}_1 < R_F$. Here we have $R_2 > R_1$ and $F_{2,news} > F_{1,news}$. Note that in contrast to previous regions, this region does not always exist, as R_F may be smaller than zero. When existing, it illustrates a situation in which it is the firm with the higher editorial standard the one dominating the market both in terms of reputation and news share. Finally, note that $F_{1,news} + F_{2,news} = 1 - \mu_1$.

[Figure 1 about here]

The analysis above determines the general conditions under which firm i publishes more stories than firm j, with $i, j \in \{1, 2\}$, $i \neq j$. It also determines the way the fraction of stories published by a firm relates to the firm's reputation. From this analysis we learn that in general it is the firm with the lower editorial standard the one publishing more scoops. Interestingly, this situation can occur for any pair of editorial standards, i.e., independently of whether firms are similar in their standards for quality or they are rather different. In contrast to this, for the firm with the higher standard to publish more scoops than the low standard firm we need the market to be homogenous enough. Otherwise, it will never be able to lead the market.

Next, we move into the analysis of the equilibrium of the mean-field dynamics. The stationary condition of the dynamics is $R_i(t+1) = R_i(t) = 0$ in equation (4). In the case of a media industry with two firms, it defines a system of two equations. The pairs (R_1^*, R_2^*) that satisfy the system are the equilibrium values. Next proposition presents the result.

Proposition 1. *The system has two stationary states.*

1. *In the first one,* $R_2^* = 0$. *This stationary state is unstable.*

2. *In the second one,* $0 < R_1^* < 1$ *and* $0 < R_2^* < 1$. *This stationary state is asymptotically stable.*

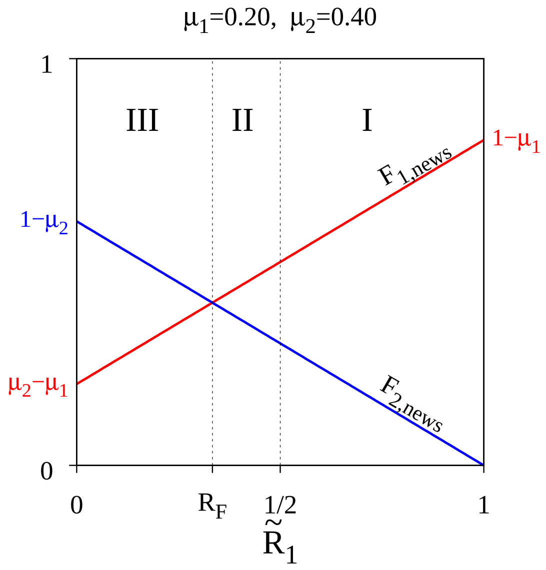

Figure 1: News share in a market with two competing media firms. The red line corresponds to the fraction of news published by the firm with the lower editorial standard (in this case $\mu_1 = 0.4$) and the blue one to the firm with the higher editorial standard (in this case $\mu_2 = 0.6$). The vertical lines at $R_F = \frac{1}{2} - \frac{\mu_2 - \mu_1}{2(1-\mu 2)}$ (a situation in which the news are shared evenly by the two firms) and at $R_1 = R_2$ (a situation in which the two firms have the same reputation) define the regions I, II and III described in the text.

Proposition 1 states that the system has two stationary states and that only one is stable. Figure 2 below presents the phase diagram of the system (left panel) and the vector field (right panel) for a particular case of the parameter values, in particular $\alpha = 0.5$, $\mu_1 = 0.4$ and $\mu_2 = 0.5$. In the left panel we represent the pairs (R_1, R_2) such that $\Delta R_1(t) = 0$ (in blue) and $\Delta R_2(t) = 0$ (in red). As observed, there are two stationary states: In the first one, $R_2^* = 0$. In the second one, $0 < R_1^* < 1$ and $0 < R_2^* < 1$. The right panel presents a graphical analysis of the stability of the system. As observed, only the stationary state with $0 < R_1^* < 1$ and $0 < R_2^* < 1$ is stable. The rest of the analysis focuses on this equilibrium.

[Figure 2 about here]

The explicit expression for R_1^* and R_2^* are in the Appendix.[15] Nevertheless, we next present a comparative static analysis of the effect of parameters α, μ_1 and μ_2 on the firms' equilibrium reputations. This is done in Figure 3 below, that represents the equilibrium value $\tilde{R}_1^*$ as a function of μ_1 and μ_2 for a high, medium and low punishment value. Note that since $\mu_1 < \mu_2$ we only represent the pair of values that satisfy this restriction.

[15] In the Appendix we give the expressions for the normalized reputations $\tilde{R}_1^*$ and $\tilde{R}_2^*$. The values for R_1^* and R_2^* are obtained by substituting $\tilde{R}_1^*$ and $\tilde{R}_2^*$ in equation (4).

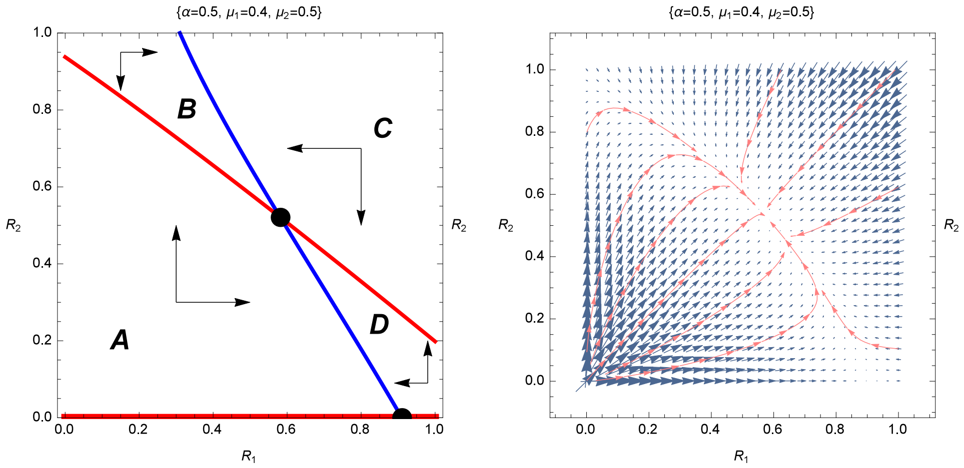

Figure 2: The left panel represents the phase diagram of the system. The blue line corresponds to the pairs (R_1, R_2) such that $\Delta R_1(t) = 0$. The red lines to the pairs (R_1, R_2) such that $\Delta R_2(t) = 0$. When $\Delta R_2(t) = 0$ for all R_1, in the steady state $R_2^* = 0$. When $\Delta R_2(t) = 0$ defines a decreasing function in R_1, in the steady state $0 < R_1^* < 1$ and $0 < R_2^* < 1$. In this case, the intersection of this function with the blue function defines four regions: A, B, C and D. The arrows in each region indicate the direction of the change of the firms' reputations. The right panel presents the vector field. In both panels we consider $\alpha = 0.5$, $\mu_1 = 0.4$ and $\mu_2 = 0.5$.

Note also that since we represent the normalized reputation $\tilde{R}_1^*$, then $\tilde{R}_1^* + \tilde{R}_2^* = 1$ always.

As expected, we observe that when $\mu_1 \sim \mu_2$, $\tilde{R}_1^* \sim \tilde{R}_2^*$. Hence, when firms are similar in their editorial standards, in the stationary state they enjoy similar reputation values. We also observe that when α is moderate, both firms receive similar equilibrium reputation values, irrespectively of whether their editorial standards are similar or not. In this sense, the choice of the optimal editorial standard is not a crucial aspect in societies where consumers are neither severe nor lenient with the media industry. Last, we observe that when α is high, firm 1 (the less stringent firm) is usually the one receiving the lower reputation, whereas when α is low it is the other way round. This result suggests that if we were to consider a situation with an incumbent (with a fixed and known editorial standard) and a potential entrant (that seeks to maximize reputation), the entrant would choose an editorial standard that is higher than the incumbent's standard if parameter α is high and the contrary if α is low.

[Figure 3 about here]

Next proposition shows the existence of a threshold $\hat{\alpha}_R(\mu_1, \mu_2)$ such that for values of the punishment higher than $\hat{\alpha}_R$, the firm with the higher editorial standard enjoys greater reputation than the low standard firm, and for values lower than the threshold it is the other way round. We also obtain the expression of this threshold.

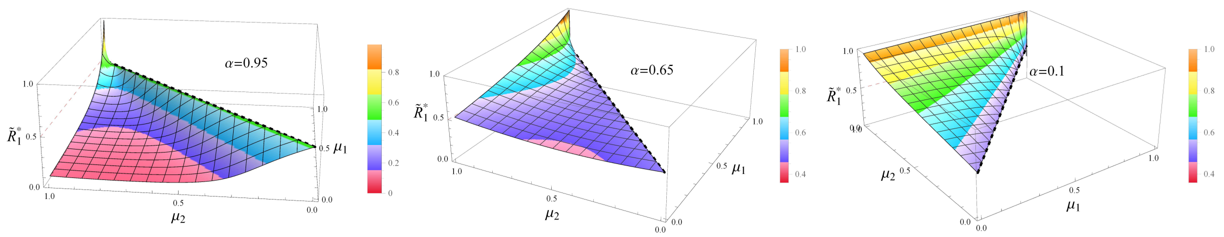

Figure 3: We represent the equilibrium reputation of firm 1, $\tilde{R}_1^*$, as a function of parameters μ_1 and μ_2 for a high ($\alpha = 0.95$), medium ($\alpha = 0.65$) and low punishment ($\alpha = 0.1$). The palette indicates the colors used to represent the different equilibrium values.

Proposition 2. *There exists threshold $\hat{\alpha}_R = \frac{2}{4-(\mu_1+\mu_2)}$ such that in the stable stationary state, $\tilde{R}_2^* \geq \tilde{R}_1^*$ if and only if $\alpha \geq \hat{\alpha}_R$ and $\tilde{R}_1^* > \tilde{R}_2^*$ otherwise.*

Note that since $\hat{\alpha}_R > 1/2$, if $\alpha \leq 1/2$ it is always the case that $\tilde{R}_1^* > \tilde{R}_2^*$ in the stationary state, independently of the firms' editorial standards. That is, if the cost for publishing false information is smaller than the reward for publishing a story, then the leader in the market (in terms of reputation) will always be the low standard firm, no matter whether firms are similar in their vetting process for stories or they are rather different. Hence, for the high quality firm to lead the market we need of compromised consumers that penalize the publication of false information. To this respect, note that since $\hat{\alpha}_R$ is increasing in $\mu_1 + \mu_2$, if μ_1 and μ_2 are small enough there exists values of α for which $\tilde{R}_2^* > \tilde{R}_1^*$. Note also that the higher the editorial standards of the firms, the more severe a society must be for firm 2 to end up with a higher reputation value than firm 1.

The next proposition defines the existence of a second threshold on the consumers' preferences such that together with an additional condition (on the firms' editorial standards), determines which firm receives and publishes a higher share of news.

Proposition 3. *There exists threshold $\hat{\alpha}_F = \frac{4-4\mu_2}{5-\mu_1^2-6\mu_2+2\mu_1\mu_2}$ such that in the stable stationary state, $F_{2,news}^* \geq F_{1,news}^*$ if and only if $\alpha \geq \hat{\alpha}_F$ and $\mu_1 \geq 2\mu_2 - 1$, and $F_{1,news}^* > F_{2,news}^*$ otherwise.*

Proposition 3 shows the existence of a second threshold $\hat{\alpha}_F$ such that for values of the punishment higher than the threshold, if the editorial standards of the two firms are not very different, namely $2\mu_2 - \mu_1 \leq 1$, the firm with the high standard receives and publishes a higher share of news than the firm with the low standard. Note that $\hat{\alpha}_F$ is decreasing both in μ_1 and μ_2, thus $\hat{\alpha}_F > 4/5$ always. Hence, if $\alpha \leq 4/5$ in the stable stationary state we always have $F_{1,news}^* > F_{2,news}^*$, independently of the firms' editorial standards. That is, unless the society is hardly punitive with the publication of false information, the leader in the market (in terms of share of news) will always be the lower standard firm.

Putting all together, the results in Propositions 2 and 3 determine which firm leads the market as a function of the consumers' preferences α and the firms' editorial standards μ_1 and μ_2. In other words, these results determine whether the equilibrium reputation of firm 1 belongs to Region I, II or III, as defined in Figure 1. Next, we develop this idea to show how the firms' equilibrium reputations and share of news depend on parameters α, μ_1 and μ_2.

To this, given a pair (μ_1, μ_2), the equilibrium reputation $\tilde{R}_1^*$ is decreasing in α, with $\tilde{R}_1^*(\alpha = 1) \geq 0$ and $\tilde{R}_1^*(\alpha = 0) \leq 1$. Let $R_{low} = \tilde{R}_1^*(\alpha = 1)$ denote the lower limit and $R_{up} = \tilde{R}_1^*(\alpha = 0)$ denote the upper limit.[16] Figure 4 below represents these ideas in two cases. In the left panel we consider $\mu_1 = 0.2$ and $\mu_2 = 0.4$; in the right panel we consider $\mu_1 = 0.4$ and $\mu_2 = 0.6$. The colored strips indicate the range of reputations for which the equilibrium of the system belongs to regions I, II and III. Note that together with values R_F and $\tilde{R}_1^* = 1/2$, thresholds R_{low} and R_{up} allow us to completely characterize regions I, II and III in equilibrium.[17]

[Figure 4 about here]

Finally, Figure 5 presents regions I, II and II as a function of the firms' editorial standards μ_1 and μ_2 for different values of parameter α. Since $\tilde{R}_1^* > \tilde{R}_2^*$ $\forall \alpha \leq 1/2$, and $F_{1,news}^* > F_{2,news}^*$ $\forall \alpha \leq 4/5$, we only consider values of α higher than 1/2. We observe that the higher α, the higher the region where in equilibrium, the firm with the higher standard receives a higher reputation than the firm with the lower standard. Note that it occurs both in regions II and III. To see why, note that from Proposition 2 we know that $\tilde{R}_2^* \geq \tilde{R}_1^*$ if and only if $\alpha \geq \frac{2}{4-(\mu_1+\mu_2)}$. This condition can be rewriting as $\mu_1 + \mu_2 \leq 4 - \frac{2}{\alpha}$. Hence, the higher the punishment, the higher the pair of values (μ_1, μ_2) for which $\tilde{R}_2^* \geq \tilde{R}_1^*$. Similarly, we observe that the higher α, the higher the region where the firm with the higher standard ends up publishing a higher share of news than the firm with the lower standard. It exclusively occurs in Region III. This result is in line with the previous one. Last, we also observe that Region III is never the whole region (defined by the pairs (μ_1, μ_2) such that $0 \leq \mu_1 < \mu_2 \leq 1$). In fact, when $\alpha = 1$ we observe that there are pairs (μ_1, μ_2) for which $\tilde{R}_2^* > \tilde{R}_1^*$ but $F_{1,news}^* > F_{2,news}^*$. This is the case when both firms have relatively different editorial standards. To see the reason for this, note that from Proposition 3 we know that for $F_{2,news}^* \geq F_{1,news}^*$ to occur we need $\mu_1 \geq 2\mu_2 - 1$, that is $2\mu_2 - \mu_1 \leq 1$. Hence, media industries with very different firms will always feature lower standard firms dominating the market in terms of share of news.

[Figure 5 about here]

[16] It can be shown that at R_{low} it is always the case that $F_{1,false} = F_{2,false}$.

[17] Substituting we obtain $R_{low} = \tilde{R}_1^*(\alpha = 1) = 1 - \frac{(1-\mu_1)^2}{2(1-\mu_2)^2}$ if $\mu_2 < 1 - \frac{1-\mu_1}{\sqrt{2}}$, otherwise, $R_{low} = 0$. We also obtain $R_{up} = \tilde{R}_1^*(\alpha = 0) = \frac{1+\mu_1-2\mu_2+\sqrt{1-6\mu_1+\mu_1^2+4(1+\mu_1)\mu_2-4\mu_2^2}}{4-4\mu_2}$, with $1/2 < R_{up} < 1$.

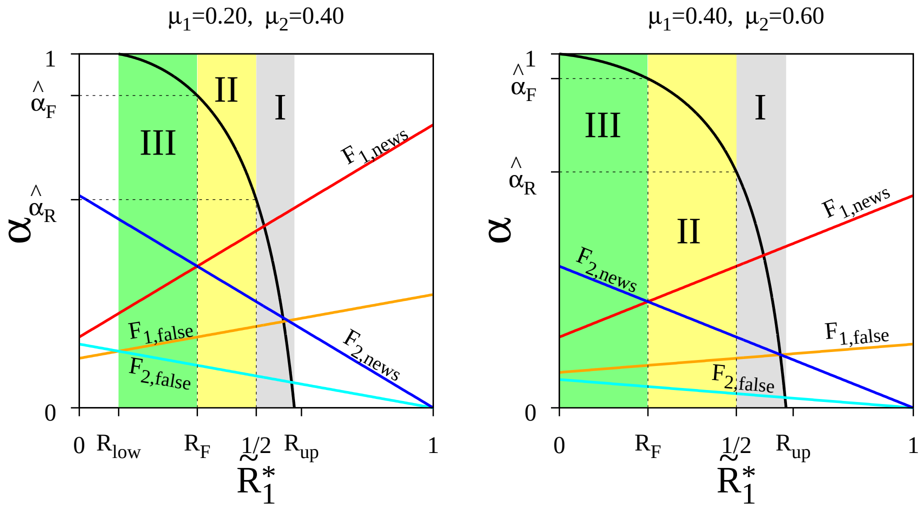

Figure 4: We represent regions I, II and III in equilibrium. The left panel corresponds to the case with $\mu_1 = 0.2$ and $\mu_2 = 0.4$; the right panel to the case $\mu_1 = 0.4$ and $\mu_2 = 0.6$. In each panel the red line corresponds to the fraction of news published by the firm with the lower editorial standard and the blue linee to the firm with the higher editorial standard. The orange and cyan lines correspond to the fraction of false stories published by the firm with the lower and higher editorial standard, respectively. The black curve gives the value of the parameter α that is required to reach a particular equilibrium reputation $\tilde{R}_1^*$. The vertical lines at R_{low}, R_F, $1/2$ and R_{up} define regions I, II and III in equilibrium. The left boundary R_{low} corresponds to the case with no reward, i.e., $\tilde{R}_1^*(\alpha = 1)$. The right boundary R_{up} to the case with no punishment, i.e., $\tilde{R}_1^*(\alpha = 0)$. Note that in equilibrium, at R_{low} the fraction of false stories published by the two firms is the same.

4 The stochastic dynamics

In this section we present numerical simulations on the evolution of the firm's reputations driven by the stochastic dynamics described in equation (1). In contrast to the mean-field analysis given by equation (4), this approach allows us to quantify the effect of the publication of a single scoop on the reputations of the firms and therefore on a firm's probability to publish the next day scoop -equations (2) and (3). Additionally, the numerical approach allows us to easily follow the evolution of a larger number of firms and so to observe behaviors that are suppressed in the mean-field approach, such as the stochastic fluctuations of the firm's reputations.

The present section is organized as follows. First, we analyze how the insights we obtained in the mean-field approach with two firms relates to the asymptotic behavior of the stochastic dynamics. This is done in Section 4.1. Then, in Section 4.2 we explore the stochastic dynamics when the number of firms competing in

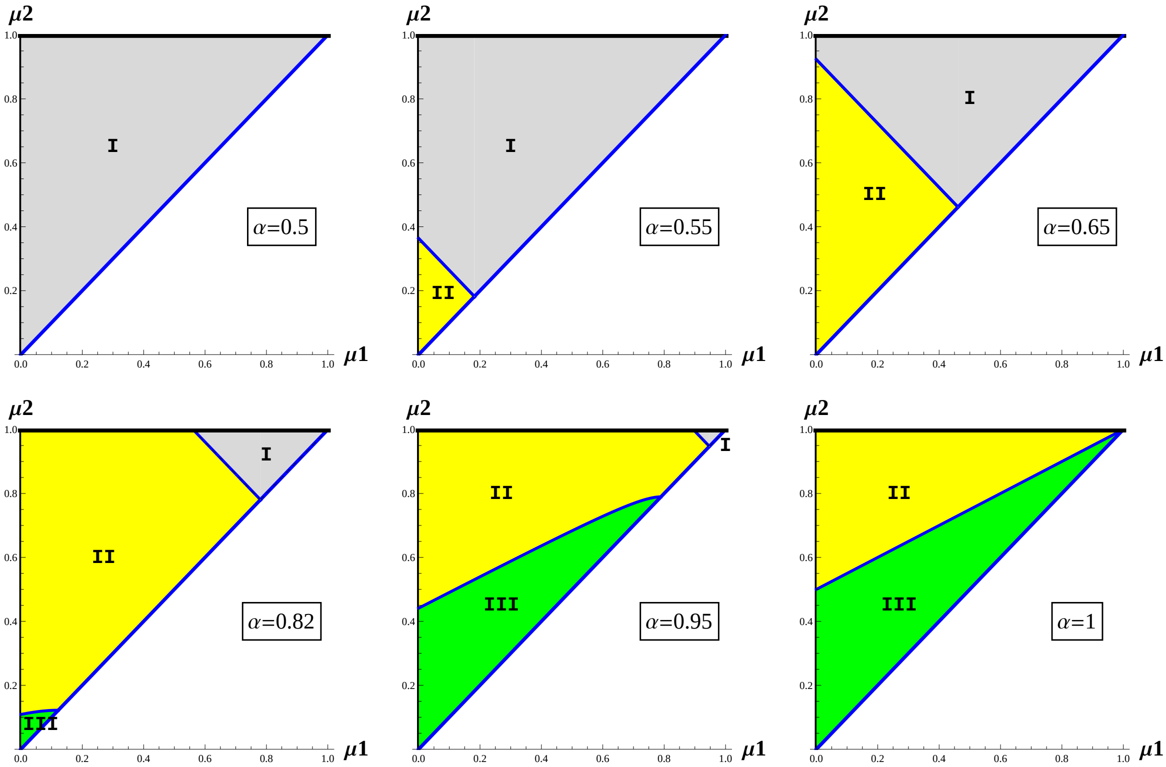

Figure 5: We represent regions I, II and III in equilibrium as a function of the pairs (μ_1, μ_2) satisfying $\mu_1 < \mu_2$, for different values of parameter α. Region I is represented in white. In this region $\tilde{R}_1^* > \tilde{R}_2^*$ and $F_{1,news}^* > F_{2,news}^*$. Region II is represented in yellow. In this region $\tilde{R}_2^* > \tilde{R}_1^*$ and $F_{1,news}^* > F_{2,news}^*$. Region III is represented in green. In this region $\tilde{R}_2^* > \tilde{R}_1^*$ and $F_{2,news}^* > F_{1,news}^*$. In blue we represent the pairs (μ_1, μ_2) for which $\tilde{R}_1^* = \tilde{R}_2^*$ and $F_{1,news}^* = F_{2,news}^*$.

the media industry is higher than two. Here we study whether the results for the case of two firms extend to the more general case, as well as what factors determine the number of firms that survive in the long run.

4.1 Two firms

We start running simulations with two firms. By construction one expects the asymptotic behavior of the stochastic dynamics to consists in a stochastic fluctuation around the equilibrium mean-field solutions $\tilde{R}_i^*$. Hereafter we refer to this fluctuating asymptotic behavior as the quasi-stationary state. We obtain that how long it takes to reach the quasi-stationary state and how large the fluctuations around the mean-field equilibrium are crucially depend on parameter A, that controls the impact of the publication of a single scoop on the reputation of the firms. In fact, note that A represents how much the society rewards and punishes, via changes of reputation, the publication of true and false information. As expected, the amplitude of the fluctuations are proportional to A, whereas the characteristic time to reach the quasi-stationary state is inversely proportional to A. The rest of parameters, namely the editorial standards μ_1, μ_2 and the punishment parameter α, were kept constant during each simulation.

To illustrate these facts Figure 6 shows the dynamics of a two firms media industry with editorial standards $\mu_1 = 0.3$ (in red) and $\mu_2 = 0.5$ (in blue) over a 20-year period in two cases: Top panels consider a severe society with $\alpha = 0.95$, bottom panels a lenient society with $\alpha = 0.2$.[18] The effect of amplitude A on the reputation dynamics is observed in the top-left and bottom-left panels of Figure 6. The left panels represent the evolution of the firms' reputations, $\tilde{R}_1(t)$ and $\tilde{R}_2(t)$. Each panel represents four curves (two for each firm). The curves with large fluctuations correspond to the case with $A = 1/10$, whereas the smooth ones correspond to the case with $A = 1/200$. The (stable) stationary states of the mean-field model are indicated with the two starred dots at the last time step, $t = 20$ years. As already said, the quasi-stationary states fluctuates around the stable steady state of the mean-field approach. It can be observed that in the top panel it takes about 20 years to reach the quasi-stationary state when $A = 1/200$, whereas in the case $A = 1/10$ the stories published during the first few months are enough to reach this state. Note that the amplitude of the stochastic fluctuations are large when $A = 1/10$ but can hardly be observed in the case with $A = 1/200$. In this last case the quasi-stationary state matches very well the stable steady state ($\tilde{R}_1^*$,$\tilde{R}_2^*$). In the bottom panel convergence is faster. The reason is twofold: The initial reputations are closer to ($\tilde{R}_1^*$,$\tilde{R}_2^*$) and, the punishment parameter α is small.[19] Finally, comparing the top and the bottom panel we observe that when the punishment for publishing false stories is high (top panel), the firm with the higher reputation is the one with the higher editorial standard, whereas when the punishment is low (bottom panel), it is the other way round. The reader may note this is the same result we obtained in the mean-field dynamics. All the simulations that follow consider $A = 1/10$; i.e. the public needs less than a year to build a quasi-stationary opinion of a media firm and simultaneously, the publication of a single scoop can have a substantial impact on a firm's reputation.[20]

The center panels show the evolution of the fraction of scoops that are published by each firm, $F_{1,market}$ and $F_{2,market}$. Note that in contrast to the deterministic dynamics, where we could define the fraction of scoops published by a firm for each t (hence $F_{i,news}(t)$), in the stochastic dynamics it is not the case. In fact, every t there is only one scoop. Hence, there is a need to define this variable over a period of time. Let $F_{i,market} = \langle F_{i,news}\rangle/\langle F_{news}\rangle)$ be the average fraction of scoops published by firm i over the previous 100 days. Now, comparing the top and the bottom panel we observe that when α is low (bottom panel), the market is dominated by the firm with the lower editorial standard. In contrast, when the punishment is high (top panel), the market is dominated by the firm with the higher editorial standard. To see the reason for this, note that the firm with the lower standard always has the advantage of being a monopoly in the range μ_1 to μ_2, as competition for scoops only occurs for qualities $> \mu_2$. Hence, the firm with the higher standard can only lead the marked when $\mu_2 - \mu_1 < 1 - \mu_2$ and $\tilde{R}_2 \gg \tilde{R}_1$. Again, this is in line with the results in the

[18] All the panels in the figure consider the same initial conditions. They were chosen far from the equilibrium values.

[19] From equation (1), note that the average change in a firm's reputation due to the publication of a scoop decreases in α.

[20] Note that when $A = 1/10$, a firm with reputation $\tilde{R} = 0.9$ that publishes one false story reduces its reputation by about 10% when $\alpha \sim 1$. Similarly, a firm with reputation $\tilde{R} = 0.1$ that publishes a true scoop almost doubles its reputation when $\alpha \ll 1$.

mean-field dynamics.

The right panels show the relation between the reputation of a firm $\langle \tilde{R}_i \rangle$ and the fraction of false scoops published by the firm, defined as $p_{i,false} = \langle F_{i,false} \rangle / \langle F_{i,news} \rangle$. Again, both $\langle \tilde{R}_i \rangle$ and $p_{i,false}$ are calculated as average values over the previous 100 days. The results show that when the punishment is high (top panel), the probability that the firm with the higher reputation publishes false stories is low. The contrary occurs when the punishment is low (bottom panel). The reason is that in this case it is the low standard firm the one that attains higher reputation. Note that the total number of false stories published in the industry only depends on μ_1, but that the way false stories are distributed between firms also depends on α and μ_2.

[Figure 6 about here]

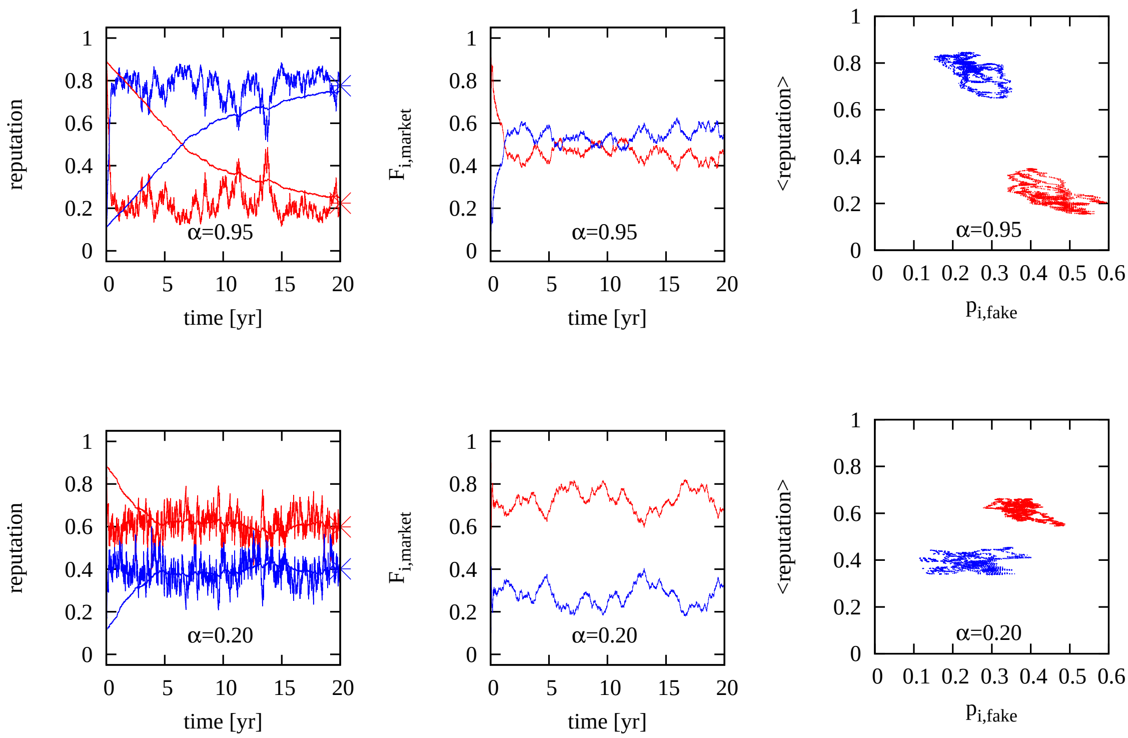

Figure 6: We represent a media industry with two firms and editorial standards $\mu_1 = 0.3$ (red) and $\mu_2 = 0.6$ (blue) over a 20-year period. Top panels correspond to a situation with a high punishment ($\alpha = 0.95$) and bottom panels to a situation with a low punishment ($\alpha = 0.2$). Left panels show the evolution of the two firms' reputations. The curves with large fluctuations correspond to the case $A = 1/10$, and the smooth ones to the case $A = 1/200$. Center panels show the evolution of the two firms' fraction of news, averaged during the previous 100 days, i.e., $F_{i,market} = \langle F_{i,news} \rangle / \langle F_{news} \rangle$. Right panels show the relation between a firm's reputation and the fraction of false stories published by the firm, averaged during the previous 100 days, i.e., $p_{i,false} = \langle F_{i,false} \rangle / \langle F_{i,news} \rangle$.

4.2 N firms

We now move to the case $N > 2$. The numerical simulations show that some of the conclusions obtained when $N = 2$ extend to the more general case. For example, it is also here the case that the speed of convergence and the amplitude of the fluctuations around the quasi-stationary state are proportional to A.

Next, we repeat the analysis in Figure 6 but for $N = 5$. The results are presented in Figure 7. Again, top panels correspond to a case with a high punishment ($\alpha = 0.95$) and bottom panels to a case with a low punishment ($\alpha = 0.2$). The editorial standards of the five media firms are 0.15, 0.30, 0.45, 0.60 and 0.75, represented in red, blue, magenta, orange and green, respectively. As in the case with two firms, we observe that when the punishment is high (top panels), reputation is positively correlated with editorial standards. The contrary occurs when the punishment is low. The result for the fraction of scoops published by a firm is also in line with those with $N = 2$. Namely, the higher the punishment the smaller the dominance of the firm with the lower standard. However, note that in the case with $N = 5$ the firm with the lowest editorial standard continues dominating the market even in the case when the punishment is very high.

[Figure 7 about here]

To better understand the effect of the consumers' preferences and the market dispersion on the equilibrium reputation values, we next present results on the final reputations of a media industry with $N = 5$ after a 20 year period for a representative combination of the editorial standards $\mu_1, ..., \mu_5$ and consumer's preferences α. The results are shown in Figure 8. The top, center and bottom panel corresponds to cases with high, intermediate and low values of the punishment parameter, respectively. Each vertical line -connecting five dots- corresponds to a single simulation, where the lowest dot corresponds to μ_1 and the highest dot to μ_5. The color of a dot represents the final reputation of the firm, as indicated in the palette. Finally, the crosses indicate the firms that lost all their reputation and went out of business. In line with the results in Figure 7, the analysis in Figure 8 shows that when α is high, there is a positive correlation between a firm's editorial standard and its final reputation. In contrast, when α is low, the correlation is negative.

[Figure 8 about here]

Beyond the discussed results, the reader may note that Figures 7 and 8 also show results on the number of firms surviving in the industry in the long run. In fact, we observe that with $N = 5$ it may occur that the dynamics drive some of the firms out of the market. Technically, it requires that at some time step t a firm (say i) reaches zero reputation $R_i = 0$. In this case, the firm no longer receives any scoop and so from then onwards it is out of business (unless it is the firm with the lowest editorial standard, in which case it always

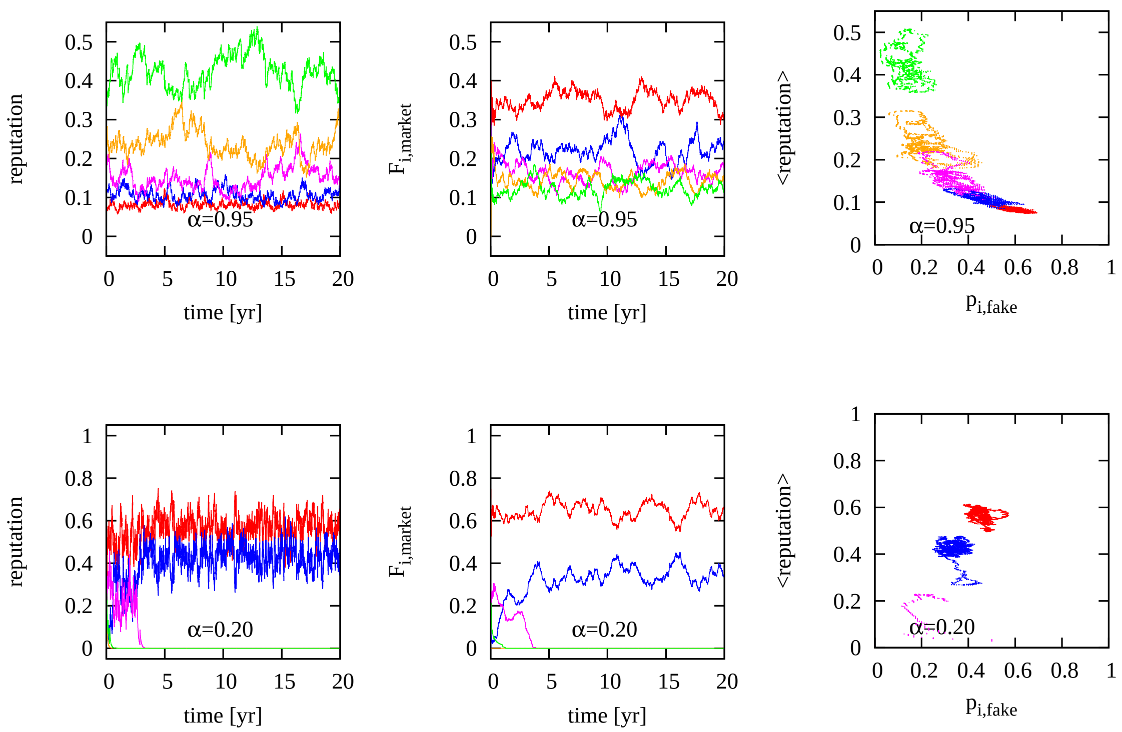

Figure 7: Same as Figure 6 but for five media firms with editorial standards $\mu_1 = 0.15$, $\mu_2 = 0.3$, $\mu_3 = 0.45$, $\mu_4 = 0.6$ and $\mu_5 = 0.75$, represented in red, blue, magenta, orange and green, respectively. Top panels correspond to the case $\alpha = 0.95$ and bottom panels to the case $\alpha = 0.2$.

publishes the scoops in the range μ_1 to μ_2.[21]

In particular, the observation of these figures suggests that both the consumers' preferences and the market dispersion may affect the degree of competition in the long run. Indeed, we observe that when α is high all firms survive in the long run, whereas when α is low it is not the case. We also observe that the more similar the editorial standards of the firms, the higher the number of firms surviving in the long run. Last, we observe that the firms that more likely run out of business are the firms with the higher editorial standards. This is in accordance with the argument discussed above.

Next, we explore in more detail the relationship between the number of firms surviving in the long run, the consumers' preferences and the market dispersion. Let us start with the effect of the consumers' preferences α. To this, Figure 9 shows the probability that $m \leq N$ firms survive in the long run for a given value of parameter α. We consider two cases: $N = 5$ and $N = 10$, in the left and right panel, respectively. For each value of α ($\alpha = 0.05,\ 0.1,\ 0.15,\ \cdots\ 1$), we performed twenty thousand simulations with random editorial standards

[21] Note that in the mean-field approach with two firms we also obtained this result, i.e., $\tilde{R}_2^* = 0$, but we showed that this stationary state is unstable.

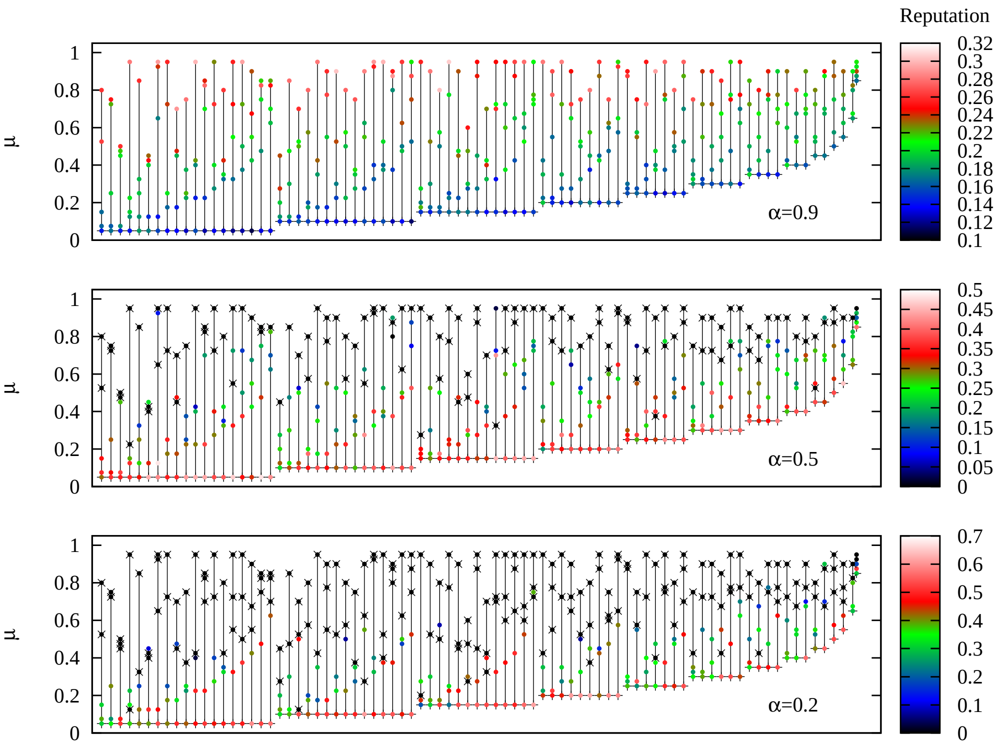

Figure 8: Final firms' reputation after 20 years of evolution for different combinations of the firms' editorial standards. The top, center and bottom panel shows the reputation values for $\alpha = 0.9$, $\alpha = 0.5$ and $\alpha = 0.2$, respectively. We consider $N = 5$. Each vertical line -connecting five dots- corresponds to a single simulation, where the lowest dot corresponds to μ_1 and the highest dot to μ_5. The color of a dot represents the final reputation of the firm, as indicated in the palette. The simulations shown are a small sample of all possible μ_i combinations, with $i = 1, \cdots, 5$. The value of μ_1 increases from left to right to facilitate the visualization. The crosses indicate the firms that lost all their reputation and went out of business.

and random initial reputations. Each simulation is run for 20 years and at the end we record the number of surviving firms. Two results are worth mentioning here, both applying to the two panels. First, when the value of α is low the number of surviving firms is very likely to be $m = 2$, with m increasing in α. For α high enough, this number is very likely to be $m = N$. Second, for a given value of α, there is a small dispersion on the number of surviving firms, with dispersion being lower when either $\alpha \sim 0$ or $\alpha \sim 1$ and higher when $\alpha \sim 0.5$.

[Figures 9 about here]

Finally, we analyze the relationship between the number m of surviving firms and the market dispersion. To this, we define the market dispersion as the standard deviation σ_μ of the N firms' editorial standards, i.e., $\sigma_\mu = \sqrt{\frac{\sum_{i=1}^{N}(\mu_i - \langle\mu\rangle)^2}{N}}$. We make the analysis for a media industry with 10 firms and we perform two hundred and fifty thousand runs with a representative sample of all $\{\mu\}$ configurations. Figure 10 presents

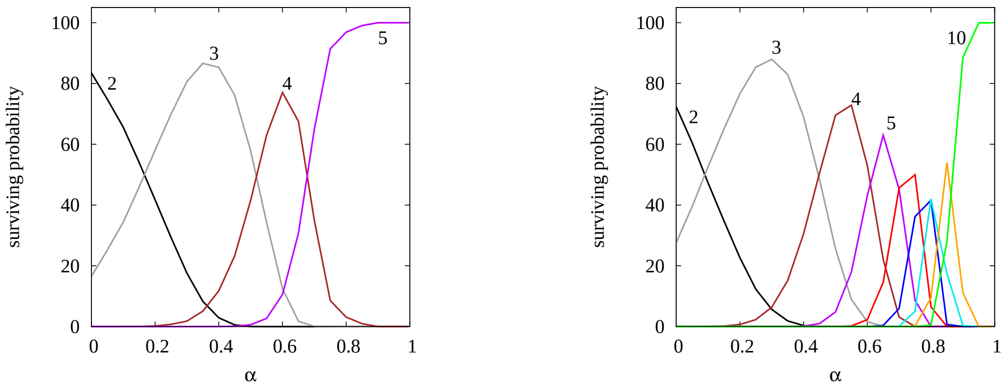

Figure 9: The left and the right panel correspond to an industry with $N = 5$ and $N = 10$ media firms, respectively. We represent the percentage of cases with m surviving firms as a function of parameter α. The label by each curve indicates the number m of surviving media firms.

the probability P_m that m firms survive as a function of σ_μ, for the case $\alpha = 0.7$. As already anticipated, we obtain that in average the number of firms surviving in the long run increases as the standard deviation of the firms' editorial standards decreases. Hence, the more homogeneous the firms in the media industry the tougher competition in the long run. Last, we also observe that compared to the effect of α, the market dispersion σ_μ is not a so powerful instrument to affect the strength of competition in the long run as it is the consumers' preferences.[22]

[Figure 10 about here]

[22] Additional simulations show that whereas small values of σ_μ may occur for a variety of mean values $\langle\mu\rangle$, those with high values of $\langle\mu\rangle$ ensure competition ($m \sim M$) and the publication of truthful information. Nevertheless, note that since small values of σ_μ and high values of $\langle\mu\rangle$ imply high values of μ_1, then the counterpart is that a relevant fraction of true stories remain unpublished. In this sense, societies that discourage the operation of firms with very low editorial standards and that heavily punish the publication of false information will enjoy high levels of competition and low levels of false stories in the long run. These result are available from the authors upon request.

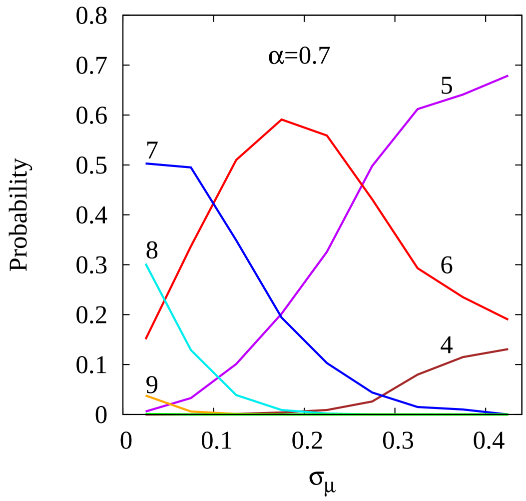

Figure 10: Probability P_m that m firms survive when the standard deviation of the editorial standards is σ_μ. We consider $\alpha = 0.7$.

5 Conclusion

This paper presents a dynamic model of competition and reputation in the media industry. Our main contribution is to pin down the importance of two variables in explaining the dynamics: The consumers' preferences for information and the market homogeneity. We obtain that the more severe consumers are with the publication of false stories, the more likely is that high standard firms receive higher reputation values in equilibrium. In contrast, when consumers are specially generous with firms that break the news and do not penalize them when the stories are shown to be false, our prediction is that the market will be dominated (both in terms of reputation and share of news) by the firms with the lower standards for quality. As for the effect of market homogeneity, we obtain that only in the case of media firms being similar enough in their vetting process for stories, there is a chance for high standard firms to lead the market in terms of share of news. Otherwise, it will be the low standard firms the ones publishing a higher share of stories.

Although moving from theory to the real world is always difficult, we consider that our results can help explain the great heterogeneity that characterizes the traditional press sector all over the world. Coming back to the cases discussed in the Introduction, we propose an argument to explain why countries like US and Spain present a positive correlation between newspaper reputation and newspaper circulation, whereas other countries like the UK present a negative correlation. The argument relies in the consumers' preferences for information and builds on the idea that the average consumer of a newspaper in the UK is quite different from the average consumer of a newspaper in the US or Spain. The reason is the strength of the tabloid press sector in the UK and the distinctive features of its audience.[23] Based on this, can we say that the average UK

[23]According to a study by ACORN, that classifies consumers according to their social status and presents a radiography of

newspaper reader is less severe with the publication of false stories and/or a more eager consumer of scoops than the average reader in the US or Spain? It is difficult and possibly controversial to argue on this respect, but if we agree on this point then the result follows.[24] Namely, we have an argument that explains why countries with severe consumers and homogeneous media firms (like the US and Spain) present media markets being dominated by high standard firms, whereas countries like the UK where consumers are less severe and the industry is more heterogeneous exhibit a clear differentiation between newspapers with high reputation and newspapers with high circulation.[25]

Beyond the traditional press sector, we consider that our model also sheds light on the use of social media as a source of information, the lack of fact-checking and editorial judgment that these platforms exhibit and the subsequent proliferation of fake news, a phenomena that took a special relevance in the US presidential election of 2016.[26] Indeed, some authors have even suggested that if not for the influence of fake news, Donald Trump would not have been elected president (Parkinson (2016), Read (2016) and Halpern (2017)). In line with the discussion above, our work contributes to this debate by highlighting the importance of the consumers' behavior on the control of false information. In particular, it suggests that as long as consumers continue rewarding the mere publication of news (with shares, likes, comments and such) and not penalizing the publication of false information, there is little to make on this respect.

Coming back to the results in the paper, our analysis also draws predictions on the number of firms surviving in the media industry in the long run. In particular, we obtain that the more severe consumers are and/or the more homogeneous firms are, the higher the number of firms surviving in the long run. Otherwise, the dynamics will drive some of the firms out of the industry and so competition will reduce in time.

To some extend, the result on the reward/punisment scheme and the number of firms operating in the market in the long run is related to Duverger's Law, that describes a nexus between electoral systems and number of parties in a political system. More precisely, Duverger's Law states that plurality systems tend to favor a two-party system, whereas proportional representation tends to favor multi party systems. In our view, the determinants for this result are not very different to that in our case. Namely, that high rewards lead to inequalities and the emergence of big players and so erode competition, whereas low rewards accommodate a higher number of players and so foster competition for longer periods. In our view, this is a conclusion that extends beyond the current setup and can help explain other relevant empirical phenomena.

their preferences, tabloids are mostly read by individuals in difficult circumstances, for example low skilled workers or unemployed people. See *The Consumer Classification*, 2014. See the *YouGovProfile* tool, an app developed by YouGov to learn more on audience profiles of US and UK newspapers.

[24] Note that the existence of a vigorous tabloid press can be understood as a support for our argument.

[25] Note that in the case of the UK, the existence of the tabloid press makes the UK newspaper industry much more heterogeneous (in terms of editorial standards) than what the media industry in other countries is.

[26] Silverman (2016) documents that in the last three months of the US presidential campaign, the 20 top-performing fake stories on Facebook generated more engagement than the 20 top-performing stories from 19 major news websites (such as the New York Times, Washington Post, Huffington Post and such).

This work makes a number of assumptions. First, we suppose media firms with a fixed editorial standard for quality that do not vary along the dynamics. Despite this limitation, the analysis allows us to derive results on a firm's optimal choice of a standard. Our results suggest that given the editorial standard of the competitor, a firm that seeks to maximize reputation would chose a standard higher than that of the competitor when consumers are severe with false stories, and lower than that of the competitor when they do not significantly penalize false information. Another assumption in the model is that there is no ideology and so everything is neutral in this respect. Additionally, there are no prices and so no possibility of differentiation in this other dimension. The reader may note that these assumptions simplify the analysis, as they get rid of both ideological and economic aspects that may affect the probability that a firm receives a scoop. A more general model that seeks to relax these assumptions should carefully reformulate this rule. We consider that these are interesting question that merit future research.

A Appendix

We first present the expression for the normalized equilibrium reputations $\tilde{R}_1^*$ and $\tilde{R}_2^*$. From equation (4), writing the expression in terms of the normalized reputation $\tilde{R}_1 = \frac{R_1}{R_1+R_2}$, we obtain:

$$\tilde{R}_1^* = \frac{2-\alpha\mu_1^2+2\alpha\mu_2^2-\alpha+2\mu_1-4\mu_2+\sqrt{\left(2-\alpha\mu_1^2+2\alpha\mu_2^2-\alpha+2\mu_1-4\mu_2\right)^2+16(\alpha-1)(\mu_2-1)(\alpha\mu_2+\alpha-2)(\mu_1-\mu_2)}}{4(\mu_2-1)(\alpha\mu_2+\alpha-2)},$$

$$\tilde{R}_2^* = 1-\tilde{R}_1^*.$$

The rest of the Appendix contains the proofs.

Proof of Proposition 1

Equation (4) can be rewritten as:

$R_1(t+1) = f_1(R_1(t), R_2(t))$,

$R_2(t+1) = f_2(R_1(t), R_2(t))$,

where $f_i : [0,1] \times [0,1] \backslash \{(0,0)\} \longrightarrow [0,1]$ for $i = 1, 2$.

After some algebra, we can obtain the explicit functions from expression (4). Using (5), (6) and (7), they are:

$$f_1(R_1, R_2) = \frac{\alpha\left(1-\mu_2^2\right)R_1^2}{2(R_1+R_2)^2} + \frac{(1-\mu_2)R_1\left(1-\alpha-\frac{R(1)}{R_1+R_2}\right)}{R_1+R_2} + \frac{\alpha\left(\mu_2^2-\mu_1^2\right)R_1}{2(R_1+R_2)} + (\mu_2-\mu_1)\left(1-\alpha-\frac{R_1}{R_1+R_2}\right) + \frac{R_1}{R_1+R_2},$$

$$f_2(R_1, R_2) = \frac{\alpha\left(1-\mu_2^2\right)R_2^2}{2(R_1+R_2)^2} + \frac{(1-\mu_2)R_2\left(-\alpha-\frac{R_2}{R_1+R_2}+1\right)}{R_1+R_2} + \frac{R_2}{R_1+R_2}.$$

Let us define

$\bar{F}_1(R_1, R_2) = f_1(R_1, R_2) - R_1$,

$\bar{F}_2(R_1, R_2) = f_2(R_1, R_2) - R_2$.

Note that the sign of $\bar{F}_i$ determines whether R_i increases, decreases or remains constant in the next period. After some algebra we get:

$\bar{F}_1(R_1,R_2) = -\frac{2R_1^3+R_1\left(\alpha(\mu_1-1)^2+4R_2-2\right)+R_1R_2(\alpha((\mu_1-4)\mu_1-(\mu_2-2)\mu_2+2)+2(\mu_1+R_2-2))-2(\alpha-1)R_2(\mu_1-\mu_2)}{2(R_1+R_2)^2}$,

$\bar{F}_2(R_1,R_2) = -R_2\frac{\left(2R_1^2+2R_1(-\alpha\mu_2+\alpha+\mu_2+2R_2-2)+R_2\left(\alpha(\mu_2-1)^2+2(R_2-1)\right)\right)}{2(R_1+R_2)^2}$.

From $\bar{F}_2$, it follows that if $R_2 = 0$, then $\bar{F}_2(R_1,R_2) = 0$.

Remark 1. *Let's define the function $R_2 = g_3(R_1)$ as $g_3(R_1) = 0$. Thus $\bar{F}_2(R_1, g_3(R_1)) = 0$.*

Note that the sing of $\bar{F}_1$ and $\bar{F}_2$ are given by the sign of their numerators. Let:

$F_1(R_1,R_2) = -2R_1^3 - R_1^2\left(\alpha(1-\mu_1)^2 + 4R_2 - 2\right) - R_1R_2(\alpha((\mu_1-4)\mu_1 - (\mu_2-2)\mu_2 + 2) + 2(\mu_1 + R_2 - 2))$
$+2(\alpha-1)R_2^2(\mu_1-\mu_2)$,

$F_2(R_1,R_2) = -\left(2R_1^2 + 2R_1(-\alpha\mu_2 + \alpha + \mu_2 + 2R_2 - 2) + R_2\left(\alpha(\mu_2-1)^2 + 2(R_2-1)\right)\right)$.

Next, Lemma 1 shows the existence of a function $g_1(R_1)$ such that $F_1(R_1, g_1(R_1)) = 0$ and Lemma 2 shows the existence of a function $g_2(R_1)$ such that $F_2(R_1, g_2(R_1)) = 0$.

Lemma 1. *The equation $F_1(R_1,R_2) = 0$ defines a strictly decreasing function $R_2 = g_1(R_1)$ such that for all $R_2 \in (0,1)$, $g_1^{-1}(R_2) \in (0,1)$.*

Proof

The roots of equation $F_1(R_1,R_2) = 0$ are too complex to work with. Hence, we take a different approach.

First, we solve the equation $F_1(R_1^0, R_2 = 0) = 0$.

There is only one solution and it is: $R_1^0 = 1 - \frac{1}{2}\alpha\left(1 + \mu_1^2 - 2\mu_1\right)$, with $0 < R_1^0 < 1$. Therefore, if it exists a function $R_2 = g_1(R_1)$, then $g_1(R_1^0) = 0$. In addition, $F_1(R_1 = 1, R_2 = 0) = -2 + \left(\alpha(1-\mu_1)^2 - 2\right) < 0$. The following remark summarizes this result.

Remark 2. *Let $R_1^0 = 1 - \frac{1}{2}\alpha\left(1 + \mu_1^2 - 2\mu_1\right) \in (0,1)$. Then $F_1(R_1 \geq R_1^0, R_2 = 0) \leq 0$.*

Second, we show that equation $F_1(R_1^1, R_2 = 1) = 0$ has only one solution.

To this, note that $F_1(R_1, R_2 = 1)$ is concave in R_1, as $\frac{\partial^2\ F_1(R_1^1, R_2=1)}{\partial R_1^2} = -2\alpha\left(1-\mu_1\right)^2 - 12R_1 - 4 < 0$. In addition,

$F_1(R_1 = 0, R_2 = 1) = 2(1-\alpha)(\mu_2 - \mu_1) > 0$ and,

$F_1(R_1 = 1, R_2 = 1) = -2\left(1-\mu_2\right) - \alpha\left(3 - \mu_2^2\right) - 4\left(\alpha\mu_2 - \left(2\alpha - 1\right)\mu_1\right) - 2\alpha\mu_1^2 < 0$.

Therefore, it exists a unique R_1^1 such that $F_1(R_1^1, R_2 = 1) = 0$. Consequently, if it exists a function $R_2 = g_1(R_1)$, then $g_1(R_1^1) = 1$. The following remark summarizes this result.

Remark 3. *It exists $R_1^1 \in (0,1)$, such that $F_1(R_1 \geq R_1^1, R_2 = 1) \leq 0$.*

Let $\hat{R}_1 = \left(\mu_2 - \mu_1\right)\left(1 - \alpha\right)$. The following result relates thresholds R_1^0, R_1^1 and $\hat{R}_1$, showing that if $R_1 < \hat{R}_1$ then $F_1(R_1,R_2)$ is convex in R_2 (it is concave if $R_1 > \hat{R}_1$).

Claim 1. *It holds that $0 < \hat{R}_1 < R_1^1 < R_1^0 < 1$.*

Proof

First, note that $\frac{\partial^2 F_1(R_1,R_2)}{\partial R_2^2} = -4R_1 + 4(\mu_2-\mu_1)(1-\alpha) \geq 0$ if $R_1 \leq (\mu_2-\mu_1)(1-\alpha)$. Let $\hat{R}_1 = (\mu_2-\mu_1)(1-\alpha)$.

Second, note that since

$F_1(R_1 = R_1^0, R_2 = 1) = -\frac{1}{2}\alpha^2(\mu_1-1)^2\left(\mu_1^2+(\mu_2-2)\mu_2\right)+\alpha\left(\mu_1^3+\mu_1+\mu_2^2-4\mu_2+1\right)-4\mu_1+2\mu_2-2<0,$

from Remark 3 it follows that $R_1^1 < R_1^0$.

Third, note that since $F_1(R_1 = \hat{R}_1, R_2 = 1) = (1-\alpha)(\mu_2-\mu_1)(4-2(1-\alpha)^2(\mu_2-\mu_1)^2-(1-\alpha)(\mu_2-\mu_1)$ $(\alpha(1-\mu_1)^2+2)-\alpha((\mu_1-4)\mu_1-(\mu_2-2)\mu_2+2)-2\mu_1)>0$, from Remark 3 it follows that $\hat{R}_1 < R_1^1$. ♦

Therefore, we can divided the set $R_1 \times R_2 = [0,1]\times[0,1]\backslash\{(0,0)\}$ in four areas and study the function $F_1(R_1,R_2)$ in each of these areas. This is done next:

1) **Area 1**$= \{(R_1,R_2)$ **such that** $R_1 < \hat{R}_1,\ R_2 \in (0,1)\}$.

Note that $F_1(R_1,R_2)$ is continuous and convex in R_2. Additionally, since $\hat{R}_1 < R_1^1$, $F_1(R_1 < \hat{R}_1, R_2 = 1) > 0$ and $F_1(R_1 < \hat{R}_1, R_2 = 0) > 0$. See Remark 3 and Claim 1. Consequently, if $\frac{\partial\ F_1(R_1,R_2)}{\partial R_2}\Big|_{R_2=0} < 0$, then $F_1(R_1,R_2)$ is always greater than zero in this area. It is straightforward to show that

$\frac{\partial\ F_1(R_1,R_2)}{\partial R_2}|_{R_2=0} = \left(4\alpha\mu_1-2\mu_1-2\alpha-2\alpha\mu_2-\alpha\mu_1^2+\alpha\mu_2^2+4-4R_1\right)R_1 > 0$ if $R_1 < \hat{R}_1 = (\mu_2-\mu_1)(1-\alpha)$.

Consequently, $F_1(R_1,R_2)$ is always greater than zero in this area.

2) **Area 2**$= \{(R_1,R_2)$ **such that** $\hat{R}_1 < R_1 < R_1^1,\ R_2 \in (0,1)\}$.

Note that $F_1(R_1,R_2)$ is continuous and concave in R_2. In addition $F_1(R_1 < R_1^1, R_2 = 1) > 0$ and $F_1(R_1 < R_1^1, R_2 = 0) > 0$. See Remark 3. Consequently $F_1(R_1,R_2)$ is always greater than zero in this area.

3) **Area 3**$= \{(R_1,R_2)$ **such that** $R_1^0 < R_1 < 1,\ R_2 \in (0,1)\}$.

In this area, $F_1(R_1,R_2)$ is continuous and concave in R_2. In addition $F_1(R_1 > R_1^0, R_2 = 1) < 0$ and $F_1(R_1 > R_1^0, R_2 = 0) < 0$. See Remark 2. Consequently, if $\frac{\partial\ F_1(R_1,R_2)}{\partial R_2}|_{R_2=0} < 0$ for any $R_1 \in \left(R_1^0, 1\right)$, then necessarily $F_1(R_1,R_2) < 0$. Note that $\frac{\partial\ F_1(R_1,R_2)}{\partial R_2}|_{R_2=0} = R_1\left(4\alpha\mu_1-2\mu_1-2\alpha-2\alpha\mu_2-\alpha\mu_1^2+\alpha\mu_2^2+4-4R_1\right) =$

$R_1\left(4\left(\left(1-\frac{1}{2}\alpha(1-2\mu_1+\frac{1}{2}\mu_1^2+\mu_2-\frac{1}{2}\mu_2^2)\right)-R_1\right)-2\mu_1\right) > 0$ if $R_1 > R_1^0 = 1-\frac{1}{2}\alpha\left(1+\mu_1^2-2\mu_1\right)$.

After analyzing Areas 1, 2, and 3, we can conclude that if the function $R_2 = g_1(R_1)$ such that $F_1(R_1,R_2) = 0$ exists, then it has to be defined in $R_1^1 < R_1 < R_1^0$. Next, we study Area 4.

4) **Area 4**$= \{(R_1,R_2)$ **such that** $R_1^1 < R_1 < R_1^0,\ R_2 \in (0,1)\}$.

First, we show that this function exists in the interval $R_1 \in \left(R_1^0, R_1^1\right)$, and second that it is decreasing and continuous.

Note that we already showed, $F_1(R_1,R_2)$ is continuous and concave in R_2 in area 4. In addition, in this area, $F_1(R_1, R_2 = 1) < 0$ and $F_1(R_1, R_2 = 0) > 0$. See Remarks 2, 3 and Claim 1. Consequently, for any $R_1 \in \left(R_1^0, R_1^1\right)$, it exists an only $\bar{R}_2$ such that $F_1(R_1, R_2 = \bar{R}_2) = 0$, $F_1(R_1, R_2 < \bar{R}_2) > 0$ and $F_1(R_1, R_2 > \bar{R}_2) < 0$. Therefore, the function $R_2 = g_1(R_1)$ such that $F_1(R_1,R_2) = 0$ exists.

To prove that this function is continuous and decreasing, it is sufficient to prove that for any $R_2 \in (0,1)$, it always exists a unique $\bar{R}_1$ such that $F_1(R_1 < \bar{R}_1, R_2) > 0$, $F_1(R_1 = \bar{R}_1, R_2) = 0$ and $F_1(R_1 > \bar{R}_1, R_2) < 0$.

First, note that $F_1(R_1, R_2)$ is a third degree polynomial, continuous and differentiable in R_1, with

$$F_1(R_1 = 0, R_2) = 2(1-\alpha)R_2^2(\mu_2 - \mu_1) > 0, \tag{8}$$

$$F_1(R_1 = 1, R_2) = -\alpha(1-\mu_1)^2 - 2R_2^2(1 + (1-\alpha)(\mu_1 - \mu_2)) - R_2\left(\alpha\left(2 + 2\mu_2 - 4\mu_1 - \mu_2^2 + \mu_1^2\right) + 2\mu_1\right) < 0.$$

Therefore, it exists at least one real root, thus the function is continuous. Next we show that there is only one root. Hence, the function must be decreasing.

Note that since $F_1(R_1 = 0, R_2) > 0$, $F_1(R_1 = 1, R_2) < 0$ and F_1 is a continuous third degree polynomial, there could be either an only root or three roots in the interval $R_1 \in (0, 1)$ for any given R_2. If there were three roots, then $\frac{\partial\ F_1(R_1,R_2)}{\partial R_1} = 0$ twice.

Since $\frac{\partial F_1(R_1,R_2)}{\partial R_1} = -6R_1^2 - 2R_1\left(\alpha(\mu_1-1)^2 + 4R_2 - 2\right) - R_2(\alpha((\mu_1-4)\mu_1 - (\mu_2-2)\mu_2 + 2) + 2(\mu_1 + R_2 - 2)) = 0$, with roots $\frac{-2\left(\alpha(1-\mu_1)^2+4R_2-2\right)\pm\sqrt{4\left((\alpha(1-\mu_1)^2-2)^2+4R_2^2+2R_2(\alpha(\mu_1(\mu_1+4)-3\mu_2(2-\mu_2)-2)-6\mu_1+4)\right)}}{12}$, it can be shown that $\frac{-2\left(\alpha(1-\mu_1)^2+4R_2-2\right)+\sqrt{4\left((\alpha(1-\mu_1)^2-2)^2+4R_2^2+2R_2(\alpha(\mu_1(\mu_1+4)-3\mu_2(2-\mu_2)-2)-6\mu_1+4)\right)}}{12} < 0$.

Consequently, the derivative is zero only once. Thus, there can only be one root. ♦

Lemma 2. *The equation $F_2(R_1, R_2) = 0$ defines a strictly decreasing function $R_2 = g_2(R_1)$ such that for all $R_1 \in (0, 1)$, $g_2(R_1) \in (0, 1)$.*

Proof

$F_2(R_1, R_2) = -\left(2R_1 + 2R_1(-\alpha\mu_2 + \alpha + \mu_2 + 2R_2 - 2) + R_2\left(\alpha(\mu_2-1)^2 + 2(R_2-1)\right)\right) = 0.$

The equation is a second degree polynomial in R_1 with two roots:

$-\frac{1}{4}\left(\alpha(1-\mu_2)^2 - 2 + 4R_1\right) \pm \frac{1}{4}\sqrt{\left(\alpha(1-\mu_2)^2-2\right)^2 + 8(\mu_2-1)R_1(\alpha\mu_2+\alpha-2)}.$

One of the roots is always negative, then the other root has to be the function $g_2(R_1)$:

$g_2(R_1) = -\frac{1}{4}\left(\alpha(1-\mu_2)^2 - 2 + 4R_1\right) + \frac{1}{4}\sqrt{\left(\alpha(1-\mu_2)^2-2\right)^2 + 8(\mu_2-1)R_1(\alpha\mu_2+\alpha-2)}.$

It satisfies $g_2(R_1 = 0) = -\frac{1}{2}\left(\alpha(1-\mu_2)^2 - 2\right)$, with $\frac{1}{2} < g_2(R_1 = 0) < 1$,

$g_2(R_1 = 1) = -\frac{1}{4}\left(\alpha(1-\mu_2)^2 + 2\right) + \frac{1}{4}\sqrt{\left(\alpha(1-\mu_2)^2-2\right)^2 + 8(\mu_2-1)(\alpha\mu_2+\alpha-2)}$, with $0 < g_2(R_1 = 1) < 1$, and $\frac{\partial g_2}{\partial R_1} = \frac{(\mu_2-1)(\alpha\mu_2+\alpha-2)}{\sqrt{(\alpha(\mu_2-1)^2-2)^2+8(\mu_2-1)R_1(\alpha\mu_2+\alpha-2)}} - 1 < 0$. ♦

Summarizing, the function $g_3(R_1)$ is an horizontal line at $R_2 = 0$, and both $g_1(R_1)$ and $g_2(R_1)$ are strictly decreasing continuous functions satisfying that for all $R_2 \in (0, 1)$, $g_1^{-1}(R_2) \in (0, 1)$ and for all $R_2 \in (0, 1)$, $g_2(R1) \in (0, 1)$. As a consequence, it follows that $g_1(R_1)$ and $g_2(R_1)$ always cross once, defining the "inner" stationary state. Moreover, $g_1(R_1)$ and $g_3(R_1)$ also cross once at the horizontal line $R_2 = 0$, defining the"edge" stationary state. See the left panel of Figure 2.

These functions divide the space $R_1 \times R_2 = [0, 1] \times [0, 1] \backslash \{(0, 0)\}$ in four regions. It is straightforward to determine the sign of functions F_1 and F_2 in these regions and to obtain the phase diagram. See the right panel of Figure 2. It follows that the "edge" stationary state in which $R_2 = 0$ is unstable. However, any point in the line $R_2 = 0$ converges to this edge stationary state. The following lemma states the stability of the "inner" stationary state.

Lemma 3. *The inner steady state is globally stable in* $R_1 \times R_2 = [0,1] \times (0,1]$.[27]

Proof

We need two preliminary results:

Claim 2. *If* $R_2(t) < (>)\, g_2(R_1(t))$*, then* $R_2(t+1) < (>)\, g_2(R_1(t))$*.*

Proof

We prove the claim with "<" (the proof with ">" is analogous). Note that for any point below the function $g_2(R_1)$, in the next period R_2 increases since $\bar{F}_2(R_1, R_2) > 0$ below $g_2(R_1)$. Claim 2 implies that in one period the increase in the vertical line never crosses the function $g_2(R_1)$.[28] This claim is equivalent to prove that for any $\bar{R}_1 \in (0,1)$ and $R_2 < g_2(\bar{R}_1)$, $f_2(\bar{R}_1, R_2) < g_2(\bar{R}_1)$. We prove it. First, note that $f_2(R_1, R_2 = 0) = 0$, and $\frac{\partial f_2(R_1,R_2)}{\partial R_2} = \frac{R_1}{(R_1+R_2)^3}\left((2-\alpha)R_1 + (R_2 - R_1)\mu_2 + \alpha\mu_2(R_1 + R_2(1-\mu_2))\right) > 0$. Because of Lemma 2, for any $\bar{R}_1 \in (0,1)$ there is a unique $\bar{R}_2$ such that $\bar{R}_2 = g_1(\bar{R}_1)$; consequently, $f_2\left(\bar{R}_1, \bar{R}_2\right) = \bar{R}_2$. As f_2 is increasing in R_2 and $f_2(R_1, R_2 = 0) = 0$, then for all $R_2 = \left(0, \bar{R}_2\right)$, $f_2\left(\bar{R}_1, R_2\right) < \bar{R}_2$. ♦

Claim 3. *If* $R_1(t) < (>)\, g_1^{-1}(R_2(t))$*, then* $R_1(t+1) < (>)\, g_1^{-1}(R_2(t))$*.*

Proof

We prove the claim with "<" (the proof with ">" is analogous). Note that for any point on the left to function $g_1(R_1)$, in the next period R_1 increases since $\bar{F}_1(R_1, R_2) > 0$. Claim 2 implies that the increase in the horizontal line never crosses function $g_1(R_1)$. This claim is equivalent to prove that for any $\bar{R}_2 \in (0,1)$ and for any $R_1 < g_1^{-1}\left(\bar{R}_2\right)$, $f_1(R_1, \bar{R}_2) < g_1^{-1}\left(\bar{R}_2\right)$. First note that $f_1(R_1 = 0, R_2) > 0$ and $\frac{\partial f_1(R_1,R_2)}{\partial R_1} = R_2 \frac{\left((2\mu_1 - \alpha\mu_1^2 + 2\alpha\mu_2 - \alpha\mu_2^2)R_1 + \left(\alpha\left(\mu_2^2 - \mu_1^2\right) + 2(2-\alpha)(1-\mu_2) + 2\mu_1\right)R_2\right)}{2(R_1+R_2)^3} > 0$.

Because of Lemma 1, for any $\bar{R}_2 \in (0,1)$ there is a unique $\bar{R}_1$ such that $\bar{R}_2 = g_1(\bar{R}_1)$; consequently $f_1\left(\bar{R}_1, \bar{R}_2\right) = \bar{R}_1$. As f_1 is increasing in R_1 and $f_2(R_1, R_2 = 0) > 0$, then for all $R_1 = \left(0, \bar{R}_1\right)$, $f_1\left(R_1, \bar{R}_2\right) < \bar{R}_1$. ♦

As mentioned above, there always exists four regions in $R_1 \times R_2 = [0,1] \times (0,1]$. See the left panel of Figure 2. Now, let us consider any initial point $R(t') = (R_1(t'), R_2(t'))$ belonging to Region A. We will show that the system always converges to the inner steady state. Note that as this region is on the left of $g_1(R_1)$ and below $g_2(R_1)$, $R_1(t') < R_1(t'+1)$ and $R_2(t) < R_2(t'+1)$, i.e., $R(t'+1)$ is on the right and above of $R(t')$. It implies that in time and as long as the system stays on in Region A, every period the firms' reputations are closer to either Region B, C, D or to the inner stationary point. Eventually, only four scenarios could occur:

1) For any $t > t'$, reputations remain in Region A. In this case, given the phase diagram, reputations necessarily converge to the inner stationary state.

[27] It is globally stable with the exception of the line in which $R_2 = 0$. In this line the dynamics converge to the edge steady state.

[28] Note that it does not imply that in the next period the dynamics does not jump over $g_2(R_1)$. It fact, it can occur because R_1 changes and $g_2(R_1)$ is decreasing.

2) At a certain $t'' > t'$, reputations jump to Region B. In this case, reputation will remain in Region B for any $t > t''$. First note that in this Region $R_1(t'') < R_1(t''+1)$ and $R_2(t') > R_2(t''+1)$, i.e., $R(t''+1)$ is on the right and below of $R(t'')$. Note that both $g_1(R_1)$ and $g_2(R_1)$ are strictly decreasing; thus, by Claims 2 and 3, a firm's reputation can never jump out of this region for any $t > t''$. Consequently, in this case reputations necessarily converge to the inner steady state.

3) At a certain $t'' > t'$, reputations jump to Region C. This case is analogous to case 2; hence reputations remain in Region C for any $t > t''$, and they eventually converge to the the inner steady state.

4) At a certain $t'' > t'$, reputation jump to Region D. In this case, the reputations at $t'' - 1$ are in Region A, and at t'' they are in Region D. Let R_1^* be the reputation of firm 1 in the inner steady state. Necessarily $|R_1(t''-1) - R_1^*| > |R_1(t'') - R_1^*|$, because of Claims 2 and 3. In this region we can apply the same argument that the one used in Region A. Therefore, reputations will converge to the inner steady state as if they jump from Region A to D and vice versa they will be closer to the inner stationary state every period and eventually, they will converge.

If we consider the initial point in any of the four regions, the dynamics is already described above. ■

Proof of Proposition 2

$F_1(R_1, R_2) = -2R_1^3 - R_1^2\left(\alpha(1-\mu_1)^2 + 4R_2 - 2\right) - R_1R_2(\alpha((\mu_1-4)\mu_1 - (\mu_2-2)\mu_2 + 2) + 2(\mu_1 + R_2 - 2)) + 2(\alpha-1)R_2^2(\mu_1 - \mu_2)$. Operating,

$F_2(R_1, R_2) = -\left(2R_1^2 + 2R_1(-\alpha\mu_2 + \alpha + \mu_2 + 2R_2 - 2) + R_2\left(\alpha(\mu_2-1)^2 + 2(R_2-1)\right)\right)$.

If we set $R_1 = R_2 = R$ and solve the equation system $\{F_1 = 0, F_2 = 0\}$ for R and α, we obtain that the solutions are $\alpha = -\frac{2}{\mu_1+\mu_2-4}$ and $R_1 = R_2 = \frac{1}{4}\frac{(\mu_1-3)(\mu_2-3)}{4-(\mu_1+\mu_2)}$. We define $\hat{\alpha}_R = -\frac{2}{\mu_1+\mu_2-4}$. ■

Proof of Proposition 3

Since $F_{1,news} = \mu_2 - \mu_1 + \frac{(1-\mu_2)R_1}{R_1+R_2}$ and $F_{2,news} = \frac{(1-\mu_2)R_2}{R_1+R_2}$, $F_{1,news} \leq F_{2,news}$ if and only if $R_1 \leq R_2\frac{(1+\mu_1-2\mu_2)}{1-\mu_1}$.

Next, we substitute $R_1 = R_2\frac{(1+\mu_1-2\mu_2)}{1-\mu_1}$ in the following two equations:

$F_1(R_1, R_2) = -2R_1^3 - R_1^2\left(\alpha(1-\mu_1)^2 + 4R_2 - 2\right) - R_1R_2(\alpha((\mu_1-4)\mu_1 - (\mu_2-2)\mu_2 + 2) + 2(\mu_1 + R_2 - 2)) + 2(\alpha-1)R_2^2(\mu_1 - \mu_2) = 0$. Operating,

$F_2(R_1, R_2) = -\left(2R_1^2 + 2R_1(-\alpha\mu_2 + \alpha + \mu_2 + 2R_2 - 2) + R_2\left(\alpha(\mu_2-1)^2 + 2(R_2-1)\right)\right) = 0$.

Solving the system for R_2 and α, we obtain $\alpha = \frac{4(1-\mu_2)}{5-\mu_1^2-6\mu_2+2\mu_2\mu_1}$, which is always greater than zero. However, it is smaller than one if and only if $\mu_1 > 2\mu_2 - 1$. ■

References

Acemoglu, Daron, Tarek Hassan and Ahmed Tahoun (2017), 'The power of the street: Evidence from egypts arab spring', *The Review of Financial Studies, forthcoming* .

Adena, Maja (2016), 'Nonprofit organizations, free media and donor's trust', *Journal of Economics* **118**(3), 239–263.

Allcott, Hunt and Matthew Gentzkow (2017), 'Social media and fake news in the 2016 election', *Journal of Economic Perspectives* **31**(2), 211–236.

Anand, Bharat, Rafael Di Tella and Alexander Galetovic (2007), 'Information or opinion? Media bias as product differentiation', *Journal of Economics & Management Strategy* **16**(3), 635–682.

Anderson, Simon P. (2006), *Handbook of the Economics of Art and Culture*, Vol. 1, Elsevier, chapter The Media and Advertising: A Tale of Two-Sided Markets, pp. 567–614.

Andina-Díaz, Ascensión and José A. García-Martínez (2016), Reputation and news suppression in the media industry. Working Paper.

Bakshy, Eytan, Solomon Messing and Lada A. Adamic (2015), 'Exposure to ideologically diverse news and opinion on facebook', *Science* **348**(6239), 1130–1132.

Baron, David P. (2006), 'Persistent media bias', *Journal of Public Economics* **90**(1-2), 1–36.

Besley, Timothy and Andrea Prat (2006), 'Handcuffs for the grabbing hand? Media capture and government accountability', *The American Economic Review* **96**(3), 720–736.

Besley, Timothy and Robin Burgess (2001), 'Political agency, government responsiveness and the role of the media', *European Economic Review* **45**(4-6), 629–640.

Boxell, Levi, Matthew Gentzkow and Jesse M. Shapiro (2017), Is the internet causing political polarization? Evidence from demographics. Working paper.

Cagé, Julia (2014), Media competition, information provision and political participation. Working paper.

Campante, Filipe R., Ruben Durante and Francesco Sobbrio (2017), 'Politics 2.0: The multifaceted effect of broadband internet on political participation', *Journal of the European Economic Association, forthcoming* .

Casari, Marco (2008), 'Markets in equilibrium with firms out of equilibrium: A simulation study', *Journal of Economic Behavior & Organization* **65**(2), 261–276.

Chen, Shu-Heng and Ya-Chi Huang (2008), 'Risk preference, forecasting accuracy and survival dynamics: Simulations based on a multi-asset agent-based artificial stock market', *Journal of Economic Behavior & Organization* **67**(3), 702–717.

Djankov, Simeon, Caralee McLiesh, Tatiana Nenova and Andrei Shleifer (2003), 'Who owns the media?', *Journal of Law and Economics* **46**(2), 341–381.

Doyle, Gillian (2013), *Understanding Media Economics*, Sage Publications.

Egorov, Georgy, Sergei M. Guriev and Konstantin Sonin (2009), 'Why resource-poor dictators allow freer media: A theory and evidence from panel data', *American Political Science Review* **103**(4), 645–668.

Ellman, Matthew and Fabrizio Germano (2009), 'What do the papers sell? A model of advertising and media bias', *Economic Journal* **119**(537), 680–704.

Enikolopov, Ruben, Alexey Makarin and Maria Petrova (2016), Social media and protest participation: Evidence from russia. Working paper.

Enikolopov, Ruben, Maria Petrova and Konstantin Sonin (2017), 'Social media and corruption', *American Economic Journals: Applied Economics, forthcoming* .

Gentzkow, Matthew and Jesse M. Shapiro (2006), 'Media bias and reputation', *Journal of Political Economy* **114**(2), 280–316.

Gentzkow, Matthew and Jesse M. Shapiro (2011), 'Ideological segregation online and offline', *Quarterly Journal of Economics* **126**(4), 1799–1839.

Gentzkow, Matthew, Jesse M. Shapiro and Michael Sinkinson (2014), 'Competition and ideological diversity: Historical evidence from us newspapers', *American Economic Review* **104**(10), 3073–3114.

Germano, Fabrizio and Francesco Sobbrio (2017), Opinion dynamics via search engines (and other algorithmic gatekeepers). Working paper.

Germano, Fabrizio and Martin Meier (2013), 'Concentration and self-censorship in commercial media', *Journal of Public Economics* **97**, 117–130.

Groseclose, Tim and Jeffrey Milyo (2005), 'A measure of media bias', *Quarterly Journal of Economics* **120**(4), 1191–1237.

Halberstam, Yosh and Brian Knight (2016), 'Homophily, group size, and the diffusion of political information in social networks: Evidence from twitter', *Journal of Public Economics* **143**, 73–88.

Halpern, Sue (2017), 'How he used facebook to win', The New York Review of Books, June 8.

Harrington, Joseph E. (1999), 'Rigidity of social systems', *Journal of Political Economy* **107**(1), 40–64.

Jackson, Matthew O. and Brian W. Rogers (2007), 'Relating network structure to diffusion properties through stochastic dominance', *B.E. Journal of Theoretical Economics* **7**(1), 1–13.

Kreindler, Gabriel E. and H. Peyton Young (2013), 'Fast convergence in evolutionary equilibrium selection', *Games and Economic Behavior* **80**, 39–67.

Larcinese, Valentino, Riccardo Puglisi and James M. Snyder (2011), 'Partisan bias in economic news: Evidence on the agenda-setting behavior of U.S. newspapers', *Journal of Public Economics* **95**(9), 1178–1189.

Lelarge, Marc (2012), 'Diffusion and cascading behavior in random networks', *Games and Economic Behavior* **75**(2), 752–775.

Little, Andrew T. (2016), 'Communication technology and protest', *The Journal of Politics* **78**(1), 152–166.

López-Pintado, Dunia (2006), 'Contagion and coordination in random networks', *International Journal of Game Theory* **34**(3), 371–381.

López-Pintado, Dunia (2008), 'Diffusion in complex social networks', *Games and Economic Behavior* **62**(2), 573–590.

Mitchell, Amy and Rachel Weisel (2014), 'Political polarization and media habits', Pew Research Center, October.

Mullainathan, Sendhil and Andrei Shleifer (2005), 'The market for news', *American Economic Review* **95**(4), 1031–1053.

Newman, Nic (2017), 'Analysis by country: United Kindom', *Digital News Report, Reuters Institute* .

Parkinson, Hannah Jane (2016), 'Click and elect: How fake news helped Donald Trump win a real election', The Guardian, November 14.

Petrova, Maria (2008), 'Inequality and media capture', *Journal of Public Economics* **92**(1-2), 183–212.

Quattrociocchhi, Walter, Guido Caldarelli and Antonio Scala (2014), 'Opinion dynamics on interacting networks: Media competition and social influence', *Scientific Reports* **4**(4938).

Read, Max (2016), 'Donald Trump won because of Facebook', New York Magazine, November 9.

Rochet, Jean-Charles and Jean Tirole (2003), 'Platform competition in two-sided markets', *Journal of the European Economic Association* **1**(4), 990–1029.

Silverman, Craig (2016), 'This analysis shows how viral fake election news stories outperformed real news on Facebook', *BuzzFeed News, November 16* .

Strömberg, David (2004*a*), 'Mass media competition, political competition, and public policy', *Review of Economic Studies* **71**(1), 265–284.

Strömberg, David (2004*b*), 'Radio's impact on public spending', *Quarterly Journal of Economics* **119**(1), 189–221.

Vara-Miguel, Alfonso, Samuel Negredo and Avelino Amoedo (2017), 'Noticias en manos de la audiencia', Digital News Report ES, June 22.

Vega-Redondo, Fernando, Matteo Marsili and Frantisek Slanina (2005), 'Clustering, cooperation, and search in social networks', *Journal of the European Economic Association* **3**(2-3), 628–638.

Yang, Mengchieh Jacie and Hsiang Iris Chyi (2011), 'Competing with whom? Where? And why (not)? An empirical study of u.s. online newspapers competition dynamics', *Journal of Media Business Studies* **8**(4), 59–74.

Zaller, John (1999), Market competition and news quality. Working paper.