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Learning and Forgetting in the Jet Fighter Aircraft Industry

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Abstract: A recent strategy carried out by the aircraft industry to reduce the total cost of new generation fighter has consisted in the development of a single airframe with different technical and operational specifications. This strategy has been designed to reduce costs in the Research, Design, and Development phase with the aim of reducing the final unitary price of aircraft. This is the case of the F-35 Lightning II, where three versions, with significant differences among them, are produced simultaneously based on a single airframe. Whereas this strategy seems to be useful to reduce pre-production sunk costs, remains key to study their effects on production costs. This paper shows that this strategy can imply larger costs in the production phase by reducing learning acquisition and hence, the total effect on the final unitary price of the aircraft is indeterminate. Learning curves are estimated based on the flyaway cost for the latest three fighter aircraft models: The A/F-18E/F Super Hornet, F-22A Raptor, and the F-35A Lightning II. We find that learning rates for the F-35A are significantly lower (an estimated learning rate around 9%) than for the other two models (around 14%).

Keywords: Learning curve, organizational forgetting, jet fighter aircraft, flyaway cost.

1. Introduction

The final acquisition cost of fighter aircraft depends on three main factors. First, procurement price is related to embodied technology of the equipment. As the technical and performance characteristics of an aircraft increase, the cost of such an aircraft will also increases (see Bongers and Torres, 2014). Second, the unitary acquisition cost is related to average fixed costs. Average fixed cost depends on two variables: Total sunk costs, and on the total number of aircraft manufactured. Production of aircraft implies an important pre-production fixed cost (due to design, experimentation, prototypes, etc.) and thus, fixed average cost can be an important component of the final price. Finally, the unitary acquisition cost depends on the intensity of the learning-by-doing process during production, which implies a reduction in average cost as accumulated production increases. Ziemer and Kelly (1993) noted that jet aircraft prices are affected by a number of factors, such as the number of units to be ordered, the production rate, the learning curve, or differences in price due to different producers.

One key well-known phenomenon in a wide range of industries is the fact that the production cost is a decreasing function of cumulative production. Experience in production translates to higher productivity. Such phenomenon is represented by the so-called learning curve, also known as progress curve. The learning curve was first studied by Wright (1936) in the aircraft industry. Wright observed that the number of man-hours inputs needed for the production of an airframe was a decreasing function of the total number of airframes of the same type previously produced. Learning curve models were first used to estimate aircraft production cost prior to the WWII. Later, this model has been applied to a large variety of industries turning into a key instrument for strategic management. Learning or progress functions become highly popular among management consultants and engineers after the World War II. This popularity was due mainly to the fact that progress functions were a simple tool very useful to be applied to a complex phenomenon.[1]

The basic idea behind learning-by-doing is simple. Workers and organizations "learn" doing a repeated task and, consequently, labor efficiency depends on the number of units previously produced. When decreasing production costs are observed, several factors can explain this

[1] See, for instance, Yelle (1979) and Dutton, Thomas and Butler (1984) for a description of the origins of the learning curve and its applications.

phenomenon, being learning by doing one of them. Learning by doing can be interpreted as a sort of dynamics economies of scale. The learning process can arise at an individual worker level (labor learning) or at a plant level (organizational learning), see Hirschmann (1963). One key characteristic of the learning curve is that the rate of learning is greater at the beginning of the production and hence, it is a decreasing function of the production. Learning means that potential economies of scale can be exploited, resulting in average or unit cost decreases as the level of output accumulates. In general terms, the learning curve states that as the total quantity of units produced doubles, the cost per unit decreases at a constant rate.

In the literature we find several alternative approaches to study the implications of learning-by-doing from a theoretical point of view. A branch of the literature had focused on incorporating the learning by doing process in the neoclassical production cost theory, in order to develop a dynamic cost approach. Seminal papers in this area are Alchian (1959), Hirschleifer (1962), and Preston and Keachie (1964), among others. A second branch of the literature studies the implications of learning by doing for industry structure and pricing. Cost reductions due to learning effects can have important implications for market structure and economic welfare, by creating barriers to entry and protecting early entrants from effective market competition. Seminal papers of this branch of the literature are Rosen (1972), Lee (1975), Spence (1981) and Fudenburg and Tirole (1983). In this context, the existence of the learning curve can provide a rationale for a pricing and marketing strategy in which producers initially price low in order to expand sales, thereby quickly accumulating experience and exploiting the costs reducing effect of the production learning. Indeed, this is the basis for the recommendations by the Boston Consulting Group (1973). A concept related to learning by doing is that of organizational forgetting, which implies that the accumulated stock of production experience can depreciate over time. Empirical evidence shows that experience depreciation can be very important in a wide range of industries, not only in the case of interruptions in production, but as a simultaneous process implicit in the learning-by-doing process. Finally, at an aggregate level, learning-by-doing is considered as one of the factors producing endogenous growth (Arrow, 1962).

Estimation of learning curves is a key element for pricing determination of military aircraft. Usually, procurement processes are not competitive at the production stage. On the other hand, learning curves are fundamentals for government decision about how much units of a particular aircraft will be procured. This is particularly important as the price of fighter aircraft has soared dramatically during the last decades. In this context, a recent strategy pursued by the military aircraft industry has consisted in the development of a single airframe with different technical and operational specifications. This strategy have been designed in order to reduce costs in the Research, Design and Development phase, which are pre-production sunk costs with the aim of reduce the final unitary price of the aircraft. This is the case of the F-35 Lightning II, where three versions, A, B, and C, with significant differences among them but based on the same airframe, are produced simultaneously. Whereas this strategy can be useful to reduce pre-production sunk costs, remains key to study their effects on production costs and on the final price of the aircraft.

This paper studies the implications of that new strategy on the dynamics economies of scale associated to learning and forgetting processes. For that purpose, this paper estimates learning curves for the three more recent fighter aircraft models (the F/A-18E/F Super Hornet, the F-22A Raptor and the F-35A Lightning II) using flyaway cost as a proxy for production cost. We find an 85% learning curve for both the Super Hornet and the Raptor. Nevertheless, results for the F-35A show a lower learning rate, with an estimated learning curve around 90%. Two main conclusions can be derived from these results. First, learning by doing process does not change as technological complexity of airframes increases. Indeed, we find that learning in the production of the F-22A, a fifth generation aircraft, is of similar magnitude to the one observed in the production of the F/A-18E/F, a fourth generation less advanced aircraft. Second, we cast doubts about the strategy of developing a single airframe with different versions to be produced simultaneously in the same assembly line. Whereas this strategy can be useful in reducing fixed costs, our results detect a lower learning process during the production phase, which implies a lower rate of reduction of production costs in the assembly of the aircraft as accumulated production increases. Our results are consistent with the ones found by Kleiner, Nickelsburg and Pilarski (2012) who studied learning and forgetting processes in the production of the DC-9 and MD-80 aircrafts which shared the same fuselage, and found that the simultaneous production of both models in the same assembly line provoked a rise in production costs. This could also be the case of the F-35 Lightning II.

2. The learning curve

Learning curve models have been developed to explain the observed phenomenon of increasing productivity as production accumulates in a variety of industries. Learning curves was developed initially in the seminal paper by Wright (1936) given the observation that the unit labor requirement in airframe manufacturing declined at a constant rate as cumulative output doubles, which implies that unit cost of production decreases at a decreasing rate. The pioneer work of Wright (1936) developed the so-called cumulative average model, was confirmed by Crawford (1944) who developed the so-called "unit" learning model. The observed reduction in production costs, without varying other productive elements, is explained by the learning process in practice during production. Learning effects appears mainly in the assembly process and therefore it is related to the human production factor not only at individual level but also at the level of the organization. Hirschmann (1963) states two ways of learning: labor learning and organizational learning which lead to a reduction in costs during production.

In general, we assume that output of an industry is produced with the following technology:

$$Y_t = A_t F(K_t, L_t)$$

where Y_t is output, K_t is capital inputs and L_t is labor. $F(\cdot)$ is a production function and where A_t is a measure of Total Factor Productivity (multi-factor productivity or neutral technology) defined as:

$$A_t = A(E_t, t)$$

where E_t represents cumulated experience and t is the time. That is, learning-by-doing is a component of Total Factor Productivity. Following Irwin and Klenow (1994), A_t can be defined as:

$$A_t = A_0 E_t^{\beta} e^{\gamma t}$$

where A_0 and β are constants. That is, productivity depends on cumulated experience (interpreted as a type of dynamic returns to scale) and on calendar time, representing the standard measure of exogenous neutral or Hicks technical change. Cumulated experience is proxied by cumulated production.

Learning curve can be defined in terms of the following mathematical function:

$$P_i = aE_i^{\beta}$$

where P_i is the number of labor hours requires to produce the i unit (or the price), a is the number of labor hours or the cost required to produce the first unit, E is the cumulative number of units produced reflecting "experience", and β is the learning index which defines the slope of the learning curve. The basic formulation for the learning curve can be extended by including additional factors to cumulative experience, such as industry spillovers, production rate, organizational forgetting, etc.

In spite of the widely empirically observed learning effects, no theoretical model have been developed yet to ground this phenomenon, although learning-by-doing have been incorporated into several existing models. From a theoretical point of view, learning curves make reference to a context in which the production process is dynamic. This contrast with standard neoclassical cost function theory, which it is developed in a static framework and thus does not take into account the acquisition of knowledge through experience. As a consequence, researchers initially tried to incorporate learning-by-doing in the classical production cost theory, using alternative approaches. First extensions were done by Alchian (1959) and Hirschleifer (1962), who developed a dynamic theory of production bases on the progress function, linking learning with classical static cost theory. Contributions in the same direction are Preston and Keachie (1964), Rapping (1965), Oi (1967), and Womer (1979), all of them developing models with a production function augmented by learning in order to integrating learning processes with the neoclassical production cost theory, consisting in the specification of a dynamic cost function where learning-by-doing can be understood as a kind of dynamic return-to-scale.

An alternative attempt to develop a theoretical framework for learning-by-doing, at a macroeconomic level, was Arrow (1962), which it is considered as the pioner work of the endogenous growth literature. Although learning curves consider the cumulative output as the index of experience, Arrow (1962) used cumulative gross investment as the variable representing experience, which makes possible the existence of a continuous learning process. The idea is similar to the standard learning curve but replacing cumulative output by cumulative investment as measure of experience. Levhari (1966a, b) and Sheshinski (1967) extended Arrow's model. Stokey (1988) develop a dynamic general equilibrium model in which goods are valued according to the characteristics they contain and where learning by doing is the force behind sustained growth. Young (1991) studied the dynamic effects of international trade under the existence of learning by doing.

Another branch of the literature dealing with the effects of learning curves has focused on the study of their strategic implications in industrial organization. In this context, learning processes may have important implications for market structure and economic welfare, since can create barrier to entry and reduce market competition. Indeed, one of the main implications from the work of the Boston Consulting Group (1970) was the advice to produce a lot early on, in order to moving down the learning curve faster than rivals firms as a strategy for gaining strategic advantage. Rosen (1972) studied the implications of learning-by-doing for a competitive firm. Lee (1975) shows that learning would rise entry barriers in a dynamic pricing model. Spence (1981) studied the case of a Cournot quantity-setting model showing that learning curves creates entry barriers against late entrants. Fudenberg and Tirole (1983) and Dasgupta and Stiglitz (1988) both conclude than is better to set a price below marginal cost, particularly at the beginning of the introduction of the new product. More recently, Cabral and Riordan (1994) studied the strategic price-setting implications of the learning curve in a differentiated duopoly setting.

2.1 Organizational forgetting

Learning-by-doing implies a continuous fall in marginal cost as experience in production is accumulated and hence, it is assumed to be a persistent process over time. Related to the learning-by-doing process is the notion of organizational forgetting. The concept of organizational forgetting was first introduced by Argote, Beckman and Epple (1990). Whereas standard learning curve approach assumes that learning is cumulative and persistent over time, Argote *et al*. (1990) showed that knowledge acquired via learning by doing may depreciate rapidly. Forgetting can only occurs in a context of learning. Depreciation of experience can be easily understood in the case of interruptions in production. After a period of interruption, when production is restarted, productivity is observed to be inferior to the previous periods (see, for instance, Hirsch, 1952). This is the case, for instance, of a strike or a stopping in the production process due to a negative shock on the demand or to parts and subassembly shortages. Nevertheless, as pointed out by Argote *et al*. (1990), forgetting can occur even when there are not interruptions in production or other factors limiting production but as a process directly related to learning.

Under the organizational forgetting hypothesis, experience, E, is assumed to depreciate at a constant exponential rate δ and to accumulate as a result of production experience:

$$E_1 = 0,$$
$$E_i = 1 + e^{-\delta t} E_{i-1}$$

where t is the calendar time at which unit i is produced. Organization forgetting has been incorporated to the definition and estimation of learning curves. At a theoretical level, Besanko *et al*. (2010) extend the learning-by-doing model of Cabral and Riordan (1994) by including organizational forgetting. They show that learning by doing and organizational forgetting are empirically important for the industry dynamics and that forgetting can offset the effects of learning by increasing competition.

3. Empirical literature

There is a vast empirical literature estimating learning curves in a number of industries. Although the concept of the learning curve was initially developed in the aircraft industry, it has rapidly extended to other products. Examples are Wright (1936), Asher (1956), Alchian (1963), Sturmey (1964), Hartley (1965, 1969), Reinhardt (1973), Womer and Patterson (1983), Frischtak (1994), Mishina (1999) and Benkard (2000, 2004) for the aircraft industry; Searle (1945), Rapping (1965), Argote, Beckman and Epple (1990) and Thompson (2001, 2007) for shipbuilding; Joskow and Rozanski (1979) and Zimmerman (1982) for energy power plants; for the automobile industry; Gruber (1992) and Irwing and Klenow (1994) for semiconductors, etc. Empirical studies including organizational forgetting are, for instance, Argote, Beckman and Epple (1990), Darr, Argote and Epple (1995), Benkard (2000), Shafer, Nembhard and Uzumeri (2001) and Thompson (2003).

On the other hand, learning curve has been widely used as a planning instrument popularized by the Boston Consulting Group. One of the main applications of the learning curve by the Boston Consulting Group (1970) was the advice to firms to produce as much as possible early on in order to moving down the learning curve faster than rivals firms to gaining strategic advantage. Furthermore, learning-by-doing curve is a planning tool used by the Department of Defense (DoD) in the acquisition process of airplanes.

Perhaps the more well-known empirical application of learning curves has been for the shipbuilding industry, particularly thank to the Liberty program.[2] The Liberty-type ships were constructed in 16 different yards during the World War II. The first study based on this case study was done by Searle (1945) who estimated a model relating man-hours per output with accumulated output using data

[2] The Liberty program was launched by the U.S. Maritime Commission in 1941, calling for a massive expansion of the merchant fleet. A total 2,708 ships were produced under this program. This program was the largest ever production of a single ship design (a total of 2,580 units of the same vessel-type were produced) in a number of shipyards. Availability of cost and production data from this program and the fact that all ships were of the same design converted this case study in the most popular in the learning-by-doing empirical literature.

from 10 yards and found that during December 1941 and December 1944, man-hours required to produce a Liberty vessel fell from an index value of 100 to 45, that is, an impressive increase in productivity of 122 percent during the whole period, roughly a 40 percent by year. Searle estimated that each doubling of cumulative output reduced labor hours per ship by between 12 and 24 percent. Lane (1951) studied productivity growth in the Liberty program at individual yards, arriving to similar results.

Rapping (1965) estimated a production function in which output (number of ships) depends on labor inputs and physical capital services using data from 15 yards, obtaining the existence of increasing returns to scale. He also included the accumulated output and found that each doubling of accumulated output is accompanied by a 29 percent increase in output, keeping inputs constant. Rapping (1965), also at individual yards, arrived to similar results with an average productivity growth of 23 percent (between 11 and 34 percent at individual yards). Argote, Beckman and Epple (1990) used the same dataset to study the dynamics of learning and its depreciation. They showed that the measured learning in terms of accumulated output significantly overstates the persistence of learning. Using data from 13 yards they estimated that from the stock of knowledge at the beginning of a year, only 3.2 percent would remain one year later, that is, knowledge depreciates rapidly. Nevertheless, Thompson (2001) studied the Liberty ships case in the Calship, showing that the increases in output per worker are explained by an increase in capital intensity and a reduction in quality, where other factors as changes in production technology and capacity utilization also pay a significant role. Thornton and Thompson (2001) focused on the study of knowledge spillovers effects between different shipyards, showing that they are an important source of productivity growth, even more than learning itself. Their results contrast with the ones obtained by Argote et al. (1990) which found only very limited evidence of spillover effects among the different shipyards involved in the Liberty program.

The other industry which has received special attention by the empirical literature is the aircraft industry. In fact, the learning curve was first applied to aircraft production. The seminal paper is Wright (1936) who observed that the number of man-hours inputs needed for the production of an airframe were a decreasing function of the total number of airframes the same type previously produced. Wright estimated a learning curve of about 80%. Based on these results, learning curves were used by the U.S. Air Force and the industry for estimating the cost of producing airframes, as shown by Asher (1956).

Alchian (1963) studied direct man-hours in the production of 22 military aircraft, including bombers, fighters, trainers, and transports, during the World War II, focusing on the prediction error derived from the estimated progress curve. Sturmey (1964) estimated learning curves for 18 British aircraft. Reinhardt (1973) studied costs and revenues in the Lockheed Tri-Star L-1011 program, estimating a value for the learning curve of 77.4%. Womer and Patterson (1983) estimated learning curves for the C-141 cargo aircraft. They use different specifications and find a learning curve of 76-81%. Frischtak (1994) computed the learning curve for the Brasilia EMB-120 aircraft produced by Embraer and find a 72.7-81.8% curve for the airframe and a 74.7-82.9% for the fuselage. Mishina (1999) studied the Boeing B-17 Flying Fortress program and found a decrease of 27.9% in the number of working hours per machine whenever production doubled, noting that productivity gains were mainly from improvements in the production system rather than learning process.

Argote *et al*. (1990) also studied the L-1011, focusing on the persistence of learning. They analyzed the importance of adjustment costs of the rate of output. In fact, the L-1011 TriStar program was characterized by wide variations in the rate of output over time. Production costs reported by Lockheed seem to suggest that depreciation of learning was an important factor and that costs will rise when the rate of production fell. Benkard (2000) estimated several alternative specification of the learning curve for the Locked L-1011 TriStar aircraft, considering forgetting, adjustment cost, input prices and spillovers. He found a learning rate between 18% and 53%. Benkard (2004) estimates a dynamic oligopoly model for the commercial aircraft industry to analyze industry pricing, industry performance, and optimal industry polity, computing a dynamic model for the market of wide-bodied commercial aircraft including the learning curve as a key element. More recently, Kleiner, Nickelsburg and Pilarski (2012) studied learning and forgetting processes in the McDonnell-Douglas MD-80 analyze the case of the production of the MD-80, concluding that organizational forgetting is virtually non-existent in this case. Which analyze cases of production of DC-9 and MD-80. As these authors point, these two models share the same fuselage, although the wings, electronics and systems integration are different.

Empirical evidence is not restricted to the shipbuilding and aircraft industries but learning curves have been estimated in a large variety of industries, such as power generation, automobile industry, and semiconductor, among others. Joskow and Rozanski (1979), study the learning process in practice in the construction and the increase in electricity production as experience is gained and, on the other, in the process of own nuclear plants. Zimmerman (1982) finds that the production cost of nuclear power plants not only decrease as a result of companies to increase their level of knowledge, but also because of the experience gained by industry, again highlighting the importance of spillover effects. Joskow and Rose (1985) on the contrary, they found that the transfer of experience between coal generating industries is very limited. Irwin and Klenow (1994) studied 32 firms from various countries and producing various kinds of semiconductor chips, including also external learning to the firms. They estimated learning rates in the range 14-28.7. Meanwhile, Gruber (1994) studies the learning processes in the industry of semiconductor memory chips, getting that learning processes are very different for different chips, although they are very similar. Empirical literature has extended the analysis of learning by including organizational forgetting. Darr, Argote and Epple (1995) studied the acquisition, transfer, and depreciation of knowledge in pizza stores. They found that learning by doing depreciates rapidly in these stores. David and Brachet (2011) studied organizational learning and forgetting for a dataset of ambulance companies and found that organizational forgetting is an important phenomenon due to both labor force turnover and skill decay.

4. Data and variables

The main focus of this paper is to quantify learning process in the latest fighter aircraft produced by the aerospace industry. In particular, we estimate learning and forgetting processes for three recent fighter: The F/A-18E/F Super Hornet produced by Boeing, and the F-22A Raptor and the F-35A Lightning II, both models produced by Lockheed Martin. The objective is twofold. First, we want to study whether learning processes in the industry have changed in the production of the new more technological advanced aircraft. In particular, how learning process is different or not for 5th generation fighter compared to 4th generation. Second, we are interesting in studying the impact of the last strategy designed by the industry to reduce the total cost of new generation fighter, by developing a single airframe with different technical and operational specifications, on the learning process in the industry.

In the literature, learning curves have been estimated using alternative variables: man-hours, labor cost, production cost, and final price, depending on data availability and industry characteristics. Irwing and Klenow (1994) showed that under Cournot competition, there is a strict relationship between prices and marginal costs. In our case, given data limitations, learning curves are estimated using unit flyaway cost for each aircraft. The flyaway cost values the price of an aircraft in terms of its marginal cost, and thus can be consider as a good proxy for the production cost. Furthermore, when pricing an aircraft, it is usually used the flyaway cost of the 100th unit, as indicated by Knaack (1978) and Ziemer and Kelly (1993), in order to take into account the learning curve and its effect on the production costs. In the literature it is assumed that, by the hundredth unit of production of a new fighter aircraft, additional learning is relative minor (Ziemer and Kelly, 1993). This is because most of the improvement takes place during the early units of production, and assuming a 80% learning the curve will eventually become almost flat, and thus, the flyaway cost of additional units remains almost constant. As it is pointed out by Ziemer and Kelly (1993) "*the price of the hundredth unit, expressed in dollars of the time period when the first production contract is signed, represents the best estimate of the actual resource cost*". Therefore, estimation of the learning curve is crucial to determine the price of military aircraft and in fact is a tool used in the procurement process of these equipment.

The data for the F/A-18E/F refer to acquisitions during the fiscal period 1997-2013, with a total of 554 units purchased. Data from the F-22A correspond to purchases made by the USAF for the fiscal period 2000-2009, with a total of 182 units purchased (from a total of 195 units produced, of which 9 were test aircraft in stage I and 6 test aircraft in stage II, an airplane to replace a lost test aircraft and two EMD accident, Engineering and Manufacturing Development, see Gertler, 2013). Finally, with respect to the F-35 Lightning II, the data correspond to the version A, since it is still a very limited production of versions B and C, including a total of 108 units. In this last case, data correspond to acquisitions for fiscal years 2007 to 2014.

Data have an annual structure and are taken directly from the budgets of the Department of Defense of the United States (DoD). The flyaway price in current dollars has been converted to constant dollars, using the DoD procurement deflator. Moreover, output is determined in terms of number of units acquired by the DoD. This approach is correct in the case of the F-22A Raptor and the F/A-18E/F Super Hornet, models for which all production has been acquired by the United States. However, in the case of the F-35A Lightning II, not all units produced of this model have been acquired by the United States as some units produced have been acquired by different countries.

Our dataset has some particular characteristics and limitations. First, we use price data as an alternative to production cost data. In the literature we find learning curves estimated using both production cost, mainly labor cost, and product price data. Indeed, initially learning curves refers to cost and only under certain conditions learning curves can be applied to the price of the product.

This is the case when the relationship price/cost remains constant. In our case, the flyaway cost is a direct valuation of production cost. Another characteristic of the dataset is that available data refers to lots which are procured by the DoD in an annual basis. In the case of discrete production, i.e., when data refers to lots or batch, the unitary cost or price does not refers to the average unit of the lot as learning occurs continuously within the lot production. In this case, mid-unit formula must be used (see Smunt, 1999). The lot or batch midpoint equation can be defined as:

$$q_{midpoint} = \left[\frac{(q_f - q_i + 1)(1+\beta)}{(q_f + 0.5)^{1+\beta} - (q_i - 0.5)^{1+\beta}} \right]^{-1/\beta}$$

Where q_f is the cumulated quantity at the end of the lot, q_i is the cumulated quantity until previous lot and β is the learning rate. The main problem for the use of the lot midpoint formula is that we need previously the value for β. This can be done using iterative estimation methods. Alternatively, Loerch (2013) proposed the following approximation for estimate lot midpoint:

$$q_{midpoint} = \frac{q_f + q_i + 2\sqrt{q_f q_i}}{4}$$

Estimations have been done using original data and corrected data using Loerch (2013) method. All estimations have been done using raw data and lot-adjusted data using Loerch (2013) approach. Nevertheless, given that the size of lots is small with a short number of units, results do not change significantly.

5. Results and Discussion

We begin by estimating the standard specification for the learning curve for each aircraft model, where the log of the flyaway cost is regressed against the log of cumulative production as proxy for experience:

$$\log L_t = \alpha + \beta \log E_t + \varepsilon_t \qquad (6)$$

where $\alpha > 0$ and $-1 < \beta < 0$. Estimation of expression (6) is done by using Ordinary Least Square. The basic specification is estimated also using as additional explanatory variables a time trend and the production rate proxied by the annual acquisition number. A summary of the main results froom the estimation is shown in Table 1.

Using the basic specification were only cumulative production is used as the explanatory variable, we obtain en estimate of the learning curves of 86% for the Super Hornet; 85.4% for the Raptor; and 91% for Lightning II. This implies that learning rates are around 14% for the first two aircraft, but around 9% for the last one. That is, for the first two models, the estimated learning rates are roughly similar, but we find evidence of a less intensive learning process is the case of the F-35A. In spite that learning rates as roughly similar for the Super Hornet and the Raptor, total produced quantities for these two models are very different. Estimated constants reflect the price of the first production unit. We found that the price for the first production unit is fairly similar for the Super Hornet and the Lightning II, whereas is much more higher for the Raptor.

When time is included in the regression (column 2), the estimated coefficient is only significant for the case of the Super Hornet, but with a positive value, contrary to the expected sign. This may be due to the fact that use data from annual batches, which cannot properly collect the negative theoretical relationship between time and production costs. Column (3) shows the results including the production rate as an additional explanatory variable. Only in the case of the Super Hornet, the estimated parameter is significant. Finally, column (4) shows the results considering both the time and the production rate. In this case the time trend estimated parameter is significant and with the correct sign for F-35A, while in the case of the F-22A is significant at 10%, but with the wrong sign. However, in this case of the F-35A the estimation of the learning curve is no consistent, as the estimated learning parameter is positive, which implies a rise in costs one flyaway cost has been controlled by the time and the acquisition rate. Note than in the case of the F-35A, learning in expected to be lower than reported, given that not all produced units are included in the data and also units procured by the DoD are considered. This means that real production is higher than the figure used in the estimation, and therefore, learning rates are overestimated for this aircraft.

Table 1: Learning curve estimation

	(1)	(2)	(3)	(4)
F/A-18E/F Super Hornet				
Constant	5.700 (0.119)***	6.107 (0.123)***	6.341 (0.110)***	6.344 (0.117)***
Learning	-0.218 (0.022)***	-0.347 (0.033)***	-0.180 (0.012)***	-0.173 (0.057)***
Time	-	0.03 (0.007)***	-	-0.001 (0.011)
Production rate	-	-	-0.245 (0.035)***	-0.254 (0.076)***
θ (%)	85.98	78.62	88.27	88.70
R^2	0.867	0.943	0.969	0.969
F-22A Raptor				
Constant	13.099 (0.053)***	13.109 (0.064)***	13.081 (0.075***	13.012 (0.072)***
Learning	-0.228 (0.013)***	-0.237 (0.032)***	-0.241 (0.037) ***	-0.501(0.134)***
Time	-	0.004 (0.015)	-	0.292 (0.146)*
Production rate	-	-	0.025 (0.068)	0.064(0.032)*
θ (%)	85.40	84.85	84.62	70.66
R^2	0.975	0.976	0.975	0.985
F-35A Lightning II				
Constant	5.752 (0.061)***	5.706 (0.066)***	5.774(0.082)***	5.765(0.018)***
Learning	-0.135 (0.017)***	-0.064 (0.056)	-0.106(0.071)	0.257(0.041)***
Time	-	-0.042 (0.031)	-	-0.117(0.012)***
Production rate	-	-	-0.049(0.115)	-0.325(0.038)***
θ (%)	91.07	-	-	-
R^2	0.911	0.934	0.914	0.996

Standard error in parenthesis. ***, **,* implies significance at 1%, 5% and 10%, respectively.

Next we proceed to re-estimate the three learning curves but considering depreciation of experience. In this case, the stock of knowledge in a period is defined in terms of the stock of knowledge in the previous period plus the added experience between the two periods, such that:

$$E_t = \lambda E_{t-1} + q_t$$

where E_t the cumulated experience and q_t is the experience between the time t and the time *t-1*, equivalent to the number of units produced in that time period. As the experience gained so far and q_t is the experience between the time *t-1* and time *t*, equivalent to the number of units produced in that space of time. The parameter λ $(0 < \lambda < 1)$ (measured depreciation experienced by the stock of experience from period to period. If $\lambda = 1$, all previously acquired experience is transmitted to the next period without suffering any depreciation. On the contrary if $\lambda = 0$ this means that depreciation is total and there is no transmission of experience between periods and, therefore, learning depreciation would be complete.

In order to introduce the depreciation of experience within the learning function, we estimate, using nonlinear least squares, the following specification:

$$\log P_i = \alpha + \beta \log(\lambda E_{i-1} + q_i) + \gamma T_i + \varepsilon_i$$

Table 2: Learning and forgetting estimation

	F/A-18E/F Super Hornet		**F-22A Raptor**		**F-35A Lightning II**	
	(1)	(2)	(1)	(2)	(1)	(2)
Constant	6.481	6.464	13.065	13.135	5.965	5.894
	(0.168)***	(0.188)***	(0.098)***	(0.292)***	(0.086)***	(0.721)***
Learning	-0.504	-0.498	-0.189	-0.229	-0.239	-0.165
	(0.046)***	(0.054)***	(0.072)**	(0.192)	(0.041)***	(0.924)
Time	-	-0.002	-	0.012	-	-0.040
		(0.011)		(0.034)		(0.551)
Forgetting	0.072	0.064	2.514	1.861	0.179	-0.019
	(0.010)***	(0.0351)*	(5.458)	(5.120)	(0.144)	(2.921)
Learning curve (%)	70.51	70.81	87.72	-	84.73	-
R^2	0.941	0.941	0.941	0.943	0.960	0.970

Standard error in parenthesis. ***, **, * implies significance at 1%, 5% and 10%, respectively.

The estimation results are shown in Tables 2 where the variable "Forgetting" refers to the estimated value of the parameter λ. Column (1) estimated the standard learning curve but including forgetting and column (2) controls for a time trend. As we can see, the results show that this parameter is not significantly different from zero for the case of the F-22A and F-35A. This means that the depreciation of experience in the production of these units is total, on an annual basis. The estimated value for the case of the Super Hornet is 0.072, indicating that, on an annual basis, only 7.2% of experience is maintained. Benkard (2000) obtained a depreciation rate of 0.96, using monthly data, for the L-1011 TriStar, which implies that 61% of the stock of experience existing at the beginning of the year survives at the end of the year, using data for a total of 250 L-1011s produced between 1970 and 1984. He argues that the low aircraft production rate is the main factor explaining the observed high forgetting.

Now learning rates cannot be interpreted as in the standard learning curve estimation as learning is not longer reflecting cumulative production. Learning rates are expected to be higher as cumulative experience is allowed to depreciate. Learning rates are about 30% for the Super Hornet and 15% for the Lightning II. Estimated model for the Raptor cannot be considered as estimated learning rates with cumulated experience depreciation are lower than in the standard learning model and thus, estimation is not consistent.

From the results of the estimation of learning curves can be drawn from a broad set of conclusions that are relevant to determining the learning process in the production of these aircraft, as well as to determine the final price of them. First, we obtain that learning curve for a fifth-generation aircraft (such as the F-22A) is similar to that of a fourth-generation aircraft (such as the A/F-18E/F). Therefore, higher complexity of fifth-generation aircraft with respect to previous generation seems not to be a factor affecting learning rates in the assembly production phase. Moreover, the number of produced units for the A/F-18E/F is much higher than the produced units for the F-22A. The finding of a similar learning curve for Super Hornet and the Raptor can be interpreted as evidence that increasing technological burden on fighter aircraft is not a significant variation in the learning process in practice.

Second, learning rates estimated for these two aircraft (the F-22A and the A/F-18E/F) are significantly lower than previous estimations for older aircraft. For instance, Wright (1936) estimated a value of $b = -0.322$, which corresponds to a 80 percent learning curve. [3] Similar

[3] The learning rate is calculated as $1-2^{\beta}$. For a β=-0.322, the corresponding learning rate is 0.20 or, equivalently, a learning curve of 80%.

estimates are obtained for a number of commercial aircraft.

Third, we find that rates of learning in the production of F/A-18E/F Super Hornet and-22A Raptor are much higher than that for the F-35A Lightning II. This finding has important implications to study some elements concerning the strategy of the industry use a single platform with different variants. Compared to the other two aircraft, the estimate for the F-35A yields different results, obtaining a learning curve of 91%, reflecting the existence of a minor learning process in practice for this model in relation to the above. This difference may be due to certain specific characteristics associated with the production process of the F-35A. The F-35 is assembled in the Lockheed plant in Fort Worth, Texas. The distinguishing feature over previous two models is that the assembly produces three different versions simultaneously (A, B and C), with significant differences between them. The idea for the design of the F-35 is to use the same airframe to produce three variants with different characteristics: a vertical standard version, a takeoff and landing version and a version capable of operating from aircraft carriers. However, the simultaneous production of three versions can cause major problems regarding the acquisition of learning and depreciation. The alternation of different models on the same assembly line, involves the manufacture of a non-homogeneous product, so each version is exposed to periods when production is stopped. If the differences between the versions are not very high, negative effects on learning could be of little significance. However, if there are significant differences between the different versions, so this strategy could lead to a lower learning process and to further depreciation thereof.

The results cast doubt on the creation of platforms with different specifications to be produced simultaneously, in the house where they raised significant differences as a strategy for reducing production costs and the purchase price in the industry fighters. In the case of F-35, the C version has a greater magnitude than the other two versions, while the motor B is very different from the other two versions. This makes the simultaneous production of three versions on the same assembly line, may adversely affect the learning process in practice slowing it down in time and even increasing depreciation. While this strategy seems appropriate to reduce fixed costs during the development stage of the aircraft, it may entail a less intensive process of learning by doing, so that unit production costs will not decrease to the expected rates, so not clear that this strategy will result in a decrease in the unit price of this aircraft.

Interestingly, Kleiner et al. (2012) reported the cases of the DC-9 and MD-80 aircraft production. They pointed out that the DC-9 and the MD-80 shared the same fuselage cross section, but the wing, electronics, and systems integration were different. The mixing of the two models on the same production line dramatically increases the production cost of the DC-9. Our results can be interpreted as evidence in the same line with respect to the F-35A Lightning II.

To illustrate the question consider the following example. Suppose that the DoD need two types of aircraft. Suppose the existence of two aerospace firms, *i* and *j*. The strategy of firm *i*, is to develop a single airframe for two differentiated (technical and operational) specifications to the produced in the same assembly line. Firm *j* develops two airframes for two different aircraft models in two different assembly lines. Assume that the total number of aircraft to be produced by both firms is the same, *N*. As a consequence, fixed costs (*FC*) in the development and research pre-production phase (RDT&E, Research, Design, Test and Evaluation costs) for firm *j* is higher than the fixed cost for firm *i*. This means that fixed cost per production unit will be lower for the two aircraft models derived from the same airframe produced by firm *i* compared to the fixed cost per unit for the two aircraft models derived from different airframes by firm *j*. However, learning-by-doing in the production phase will be different. Given the learning-by-doing process, average variable cost (*AVC*) can be defined as:

$$AVC = \frac{a\sum_{k=1}^{N} Q_k^{\beta}}{Q_N}$$

whereas average total cost (*ATC*) would be:

$$ATC = \frac{FC}{Q_N} + \frac{a\sum_{k=1}^{N} Q_k^{\beta}}{Q_N}$$

For the new strategy to be the best option in reducing the final price of the aircraft, the following condition must holds:

$$\frac{FC_i}{Q_N} + \frac{a\sum_{k=1}^{N} Q_k^{\beta_i}}{Q_N} < \frac{FC_j}{Q_N} + \frac{a\sum_{k=1}^{N} Q_k^{\beta_j}}{Q_N}$$

Under the assumption that the number of total production units will be the same under both strategies, the above condition can be written as:

$$\sum_{k=1}^{N} Q_k^{\beta_i - \beta_j} < \frac{FC_j - FC_i}{a}$$

From that expression, we find that the optimality of each strategy depends on the following factors. First, lower the price of the first unit (lower *a*), better is the new strategy for producing aircraft. This is because when the first unit to be produced of a particular aircraft model is very expensive, learning effect becomes more important in pricing produced units. Second, higher the difference between fixed costs, better is the new strategy. It is clear that when fixed costs are very high and represent a significant fraction of the final price, the new industry strategy has an advantage. Third, the best strategy depends on the total number of units to be produced. As the number of total units increases, better is the new strategy. This is because when the number of units to be produced is very large, fixed costs per unit represent a small quantity. Finally, the optimal strategy depends on the difference between the learning rates ($\beta_i - \beta_j$). Given that learning in the first strategy is lower than in the second, the greater the difference between learning rates, the worse the new strategy.

6. Concluding remarks

This article studies learning curves and persistence of experience for the three most recent fighter aircraft. Learning process is particularly important in the aircraft industry, where skilled labor in the assembly process involves a high percentage of total costs and accumulated experience in the manufacture of a particular aircraft model leads to a progressive reduction in production costs as experience increases productivity. Furthermore, the final price of an aircraft depends on the number of previously produced aircraft of the same model. In the case of military aircraft, the learning-by-doing process observed during the manufacturing process has important implications in determining the final procurement price of the equipment, given the intensity of learning acquisition in the assembly phase of this equipment. In fact, the estimation of the learning curve is one of the main tools for fixing price of the aircraft acquired by the U.S. DoD.

Given the dramatic increases in the prices of more advanced fighter, a recent strategy pursued by the industry to reduce the total cost of new generation fighter is the development of a unique airframe with different technical and operational specifications to meet the requirements of the DoD. This is the case of the F-22 Lightning II produced by Loockeed Martin, where three versions, A, B, and C, have been developed simultaneously, sharing a similar airframe. Whereas this strategy can be useful in reducing the costs in the pre-production phase, remains open the question on its effects on the costs in the assembly phase. This is the main question this paper tries to answer.

We find that the learning curve of the F-22 Raptor is similar to the F/A-18E/F Super Hornet, reaching approximately 85%, a value slightly higher (lower learning rates) than that found in the literature for other aircraft models. In the case of the F-35A, the observed learning progress is smaller, with a learning curve above 90%, which may be due to the special features in the production of this aircraft, using a unique platform to produce three different versions, with different technical and operational specification, simultaneously in the same assembly line. Based on these results, the design and development of a single model with very different conversions does not seem to be the solution, according to the results obtained for the case of the F-35A. The simultaneous production of different versions on the same assembly line can hamper significantly the learning processes in practice, while can facilitate the depreciation of previously accumulated experience, so that the reduction in production costs as cumulative production increases would be lower.

The estimated depreciation on experience is very high for all three models. On an annual basis, the depreciation is complete for cases of F-22A and F-35A. This may be due to the small number of units produced and the annual nature of the data used. In the case of F/A-18E/F, the estimated depreciation is over 90% in annual terms. Finally, further analyses of the learning curves for each of the variants of the F-35, may be very important to determine the best strategy to follow by the aircraft industry in the development of new fighter aircraft. Additionally, is should be also important to quantify the impact of this strategy over the expected life cycle costs of the whole program. Finally, it would be of interest to study how higher technological complexity of fighter aircraft has affected both learning and forgetting in this particular industry.

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