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Revisiting the Battle of Midway: A counterfactual analysis

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Abstract

This paper uses a stochastic salvo combat model to study the Battle of Midway. The parameters of the model are calibrated accordingly to the historical outcome and thus, the model can be used to study alternative scenarios. Contrary to the common wisdom that the result of the Battle was an "incredible" American victory, the model shows that the probability for Japanese to win were very low and indeed close to zero. We carry on four alternative counterfactual analyses: (i) All launched American attack aircraft reach to the Japanese carriers; ii) An additional Japanese carrier; iii) Not to wait to launch Japanese attack aircraft; and iv) American carriers spotted earlier. Including the most favorable scenario for the Japanese, the Battle of Midway remains an American victory.

Keywords: Stochastic salvo combat model; Battle of Midway; Monte Carlo simulation; Counterfactual analysis.

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1 Introduction

The Battle of Midway is widely considered as the turning-point in the Pacific theater of World War II and the most important defeat of the Imperial Japanese Navy (IJN). Many authors refers to the outcome of this battle (the sinking of all four Japanese aircraft-carriers by the U.S. Navy) as an "incredible victory" or a "miracle". Furthermore, Isom (2000) pointed out that "this was a battle the American should have lost". A number of reasons have been proposed to explain the overwhelming American victory at Midway, as the "victory disease" by the Japanese, poor performance and problems with Japanese searching aircraft, Nagumo wrong decisions, etc. Nevertheless, more recent studies (Isom 2000; Parshall, Dickson and Tully 2001) based on an in-depth analysis and new information from interviews with survivors and Japanese sources, argue that the American victory was not the product of the idiocy or incompetence by the Japanese but the logical result of the course of actions. In this paper we go one step ahead, showing that the Battle of Midway was indeed a battle the Japanese never could win and that historical events were a function of the timing produced by the early Japanese attack to the Midway Island Air Base.

A widely used mathematical tool to study the outcome of battles is the Lanchester-type dynamic combat model (Lanchester 1916). A version of the standard Lanchester attrition combat model designed for naval warfare is the so-called salvo combat model developed by Hughes (1995). Although the Salvo combat model has been developed to describe modern naval warfare using missiles, it can be well applied to naval battles between aircraft-carriers as the ones occurred in the Pacific theater during World War II. Salvo model replicates some characteristics related to the pulse nature of naval combat with attacking and defensive firepower to modelling modern naval warfare as well for aircraft-carriers clash during World War II. Armstrong (2005) developed a stochastic version of the salvo model in which the number of accurate missiles fired for each ship and the number of interceptions is assumed to be an independent and identically distributed random variable that follows a binomial distribution.

Salvo combat models have been proved to be useful in studying different battles including naval fighting between aircraft-carriers. Armstrong and Powell (2005) use a stochastic salvo model to conduct a counterfactual analysis of the Battle of the Coral Sea. They studied alternative scenarios in order to calculate the effects of dispersion of USN aircraft-carriers (CVs), increasing the number of USN CVs, changing the composition of each air wing between fighter and bombers, and better air defense. They obtains that the result for the USN would have been better if the two CVs have been dispersed. Armstrong (2014) extends the stochastic salvo model to a context in which the exchange of fire is sequential rather than simultaneous. He applied the model to the Battle of the Coral Sea to show that attacking first would have given the American force a larger advantage than that provided by an extra aircraft carrier. Salvo combat models have also been applied to battles other than naval fighting. For instance, Armstrong (2015) studies the Israeli Iron Dome's performance during the Operation Pillar of Defense.

In this paper we calibrate a stochastic salvo combat model to analyze the Battle of Midway in order to calculate the likelihood of each side to win. Once the parameters of the model are correctly calibrated, the model is able to replicate the observed historical outcome. Two main historical analyses can be done. First, the model can be calibrated to reproduce the historical results and then some of the observed values for the parameters can be changed in order to conduct counterfactual experiments. For instance, Connors, Armstrong and Bonnett (2015) use a stochastic Lanchester-type model to conduct a counterfactual analysis of the charge of the light brigade in the Battle of Balaclava during the Crimean War, and Armstrong and Sodergren (2015) studied several counterfactual scenarios regarding the Pickett's Charge at the Battle of Gettysburg during the Civil War. The main problem with this approach is the fact that the historical result is assumed to be the average of all possible outcomes. An alternative approach is the one used by MacKay, Price and Wood (2016). They applies a Bayesian method for the case in which the likelihood function is unknown, the so-called Approximate Bayesian Computation (ABC) method, to the Battle of the Dogger Bank (a WWI naval battle between British and German battlecruisers) in order to study if the historical outcome were closed or not to the expected one.

In this paper we use the first approach. Using the calibrated model we carry on a counterfactual analysis of the battle by exploring almost all possible relevant alternatives. We find that "lucky" was indeed in the Japanese side and that a likelihood result could have been the sinking of all four Japanese aircraft carriers with no damage in any of the three American aircraft carriers. The counterfactual analysis is based in four possible alternatives. The first scenario assumes that the 35 dive-bombers launched by the Hornet also found the Japanese carriers. The second counterfactual analysis consider the possibility that the Zuikaku also joined the Japanese striking force to Midway. The third counterfactual experiment considers that Nagumo does not wait for a coordinate complete attack and send the reserve dive-bombers on Hiryu and Soryu. Finally, we consider the most possible favorable scenario for the Japanese, assuming that the USN fleet is discovered early and then the Japanese attack is simultaneous to the American attack. Considering together all counterfactual experiments, the extreme possible outcome for the Battle goes from the loss of all four Japanese carriers (or five in the case of an additional carrier was present) and no loss for the American, to the loss of three Japanese carriers and two American carriers.

The structure of the remainder of the paper is as follows. Section 2 describes the salvo combat type model developed to represent the Battle of Midway. Section 3 calibrates the model to the historical battle outcome. Section 4 presents the results from the stochastic model. Counterfactual experiments are collected in Section 5. Finally, Section 6 presents some conclusions.

2 Salvo combat models

The salvo combat model is an extension of the standard Lanchester's attrition model designed in particular to represent modern naval warfare between missile warships. The salvo combat model was developed by Hughes (1995) and basically it consists in a discrete version of the Lanchester model-type in which it is considered the possibility of defense to enemy attack. Although the Salvo combat model has been developed to describe modern naval warfare using missiles, it can be well applied to naval battles between aircraft-carriers as the ones occurred in the Pacific theater during World War II. Salvo combat model is able to replicate some fundamental characteristics pertaining to the pulse nature of naval combats in which are presented both attacking and defensive firepower.

The model to be used to describe the Battle of Midway consists in the following two dynamic equations:

$$\Delta CV^{USN} = -a_K^{USN} p_H^{IJN} \left[p_A^{IJN} n_A^{IJN} CV^{IJN} - p_F^{USN} n_F^{USN} CV^{USN} \right] \quad (1)$$
$$\Delta CV^{IJN} = -a_K^{IJN} p_H^{USN} \left[p_A^{USN} n_A^{USN} CV^{USN} - p_F^{IJN} n_F^{IJN} CV^{IJN} \right] \quad (2)$$

where the superscript "USN" indicates United States Navy and "IJN" indicates Imperial Japanese Navy. Equation (1) indicates the number for firepower kills (losses) suffered by American aircraft-carriers whereas equation (2) does the same for Japanese carriers. a_K^{USN} is the firepower kills per hit suffered by USN CVs and, a_K^{IJN} is the firepower kills per hit suffered by IJN CVs. p_H^{USN} is the probability of hitting an IJN carrier by surviving USN attacker and p_H^{IJN} is the probability of hitting an USN carrier by surviving IJN attacker. p_A^{USN} and p_A^{IJN} are the fraction of available attacking aircraft arriving to the target for the USN and the IJN, respectively, and p_F^{USN} and p_F^{IJN} are the fraction of intercepted attacking aircraft by the USN and the IJN defenses, respectively. n_A^{USN} and n_A^{IJN} are the total number of available attacking aircraft for each side, n_F^{USN} and n_F^{IJN} are the total number of available fighter aircraft, and finally CV is the number of aircraft-carriers.

Equations (1) and (2) are either zero or negative, showing the dynamics of the number of aircraft carriers for both sides during a battle. Armstrong (2004) studied the concept of lethality, defined as the relative balance between offensive and defensive capabilities, associated with the salvo model, depending on the values of the parameters, considering three scenarios: Low, medium and high lethality. Low lethality is related to a scenario where defenses are large enough to offset any attack. In the medium and high lethality scenarios, the combination of parameters is such that at least one side is able to inflict damage on their opponent.

Figure 1 plots the phase diagram corresponding to the salvo combat model representing a naval battle between aircraft-carriers. The two lines correspond to values for which equations (1) and (2) are zero. The positive slope for these functions are given by:

$$\frac{\Delta CV^{IJN}}{\Delta CV^{USN}} \mid_{\Delta CV^{USN}=0} = \frac{p_F^{USN} n_F^{USN}}{p_A^{IJN} n_A^{IJN}} > 0$$
$$\frac{\Delta CV^{IJN}}{\Delta CV^{USN}} \mid_{\Delta CV^{IJN}=0} = \frac{p_A^{USN} n_A^{USN}}{p_F^{IJN} n_F^{IJN}} > 0$$

The two positive zero-change lines split the space in three areas. There are two areas in which the losses of aircraft-carriers for one side are zero which corresponds to the low lethally scenario as defined by Armstrong (2004) for one of the opponents. This is the case when the number of carriers in one side is large enough compared to the number of carriers in the other side, and/or the case in which defense capacity is large enough to completely suppress attacking forces. The area on the upper-left represents a combination of the number of carriers in both sides, given the values for the parameters, in which equation (1) is negative and equation (2) is zero. That is, the number of IJN carriers remains constant and the number of USN carriers reduces. Similarly, the area down the right represents an initial number of carriers such as equation (1) is zero and equation (2) is negative. Under the assumption that defence parameters are lowers than attacking parameters, as indicates by Hughes (2000), we have that

$$\frac{p_F^{USN} n_F^{USN}}{p_A^{IJN} n_A^{IJN}} < \frac{p_A^{USN} n_A^{USN}}{p_F^{IJN} n_F^{IJN}}$$

[Insert here Figure 1]

The area between the two zero solution lines reflects a situation with losses for both sides. This is the scenario considered by Hughes (2000) to model carrier warfare during the WWII. Hughes (2000) suggests that "one carrier air wind could on balance sink of inflict crippling damage of one carrier" based on the study of carrier battles of the Pacific theater of the WWII. For the Battle of Midway, he obtained that, given the initial number of aircraft carriers in both sides, after the U.S. attack theoretical survivors are all three American carriers and only one Japanese carrier. After the Japanese counterattack, theoretical survivors are 1 and 2, for the Japanese and the American, respectively. Finally, after the U.S. second attack, theoretical survivors reduces to 0 for the Japanese and 2 for the American, just the historical result. However, it is important to note that whereas USN CVs had an air wing with 3 attack squadrons (typically composed of about 18 aircraft each, one torpedo-bombers and two dive-bombers), the IJN CVs had an air wing of only 2 attack squadrons (about 18 aircraft each, one torpedo-bomber and one dive-bombers). Armstrong and Powell (2005) assume that the mean damage per successful squadron attack is of 1 CV (of firepower kills).

The basic salvo combat model of Hughes (1995) has been extended in several ways. Armstrong (2005) developed a stochastic version of the salvo model in which the number of accurate missiles fired for each ship and the number of interceptions is assumed to be an independent and identically distributed random variable that follows a binomial distribution. Armstrong (2011) analyses the properties of the stochastic salvo combat model using Monte Carlo simulation on a large number of scenarios and under different assumptions about the distribution of the stochastic components. Armstrong (2013) developed a salvo combat model with area fire in order to study situations in which the enemy location is known only approximately. Finally, Armstrong (2014) developed a sequential salvo combat model for modeling battles in which the exchange of fire is not simultaneous but in different phases.

Salvo combat models have been applied to the study of different battlefields. Armstrong and Powell (2005) applied a salvo model to study the Battle of the Coral Sea. They performed a number of counterfactual analyses, considering larger dispersion of USN aircraft-carriers, higher number of USN carriers, changing the composition of each air wing between fighters and bombers, and better air defense. Armstrong (2014) applied a sequential salvo combat model to the Battle of the Coral Sea, obtaining that attacking first would have given USN a larger advantage than that provided by an extra aircraft-carrier. Finally, Armstrong (2015) uses a salvo model to study the case of short-range ballistic missile defense and applies it to study the performance of the Israel's Iron Dome system.

3 Calibration

The Battle of Midway took place on June 4, 1942 in the vicinity of the Midway Island. The Japanese attacking fleet was composed of four aircraft-carriers (Kaga, Akagi, Hiryu and Soryu), whereas the American fleet had three carriers (Enterprise, Hornet and Yorktown) plus the Midway Air Base. In that day, Japanese lost all four aircraft-carriers against one sunk American carrier.

Tables 1 and 2 show the availability of aircraft by type for each side. All data is taken from Naval Staff History (1952), which includes reports from US Naval Intelligence and Japanese records of the battle. Total attack aircraft for the US were 215 (154 from the three USN CVs and 61 from Midway Air Base). This implies a total of 71.66 attackers (dive-bombers and torpedo-bombers) per USN CV considering the total number of attackers, including the Midway Air Base. Total attack aircraft for Japan were 151, that is, a total of 37.5 attackers per IJN CV. The total number of fighter for the US were 107 (79 from the three USN CVs and 28 from the Midway Air Base). Excluding the Midway Air Base as land based fighter are not relevant for carriers defense, the figure is 26.33 fighters per USN CV. For the Japanese, the total fighter was 90, that is, 22.5 fighter per IJN CV.

In the calibration of the model two important points must be taken into account. First, IJN launched the first attack on Midway at 04:30 on 4 June, with a total of 107 aircraft, consisting of 36 fighters (9 from each carrier), 36 dive bombers (from Akagi and Kaga) and 35 torpedo-bombers (from Hiryu and Soryu). Therefore, these aircraft were not initially available either to defend the IJN carriers or to attack USN CVs. This implies that total available attack aircraft for the IJN when American attacked IJN CVs were in fact 151-71=80 (34 dive-bombers and 46 torpedo-bombers), and that the total number of fighters were 90-36=54. Second, the battle was done in different phases. IJN CVs were spotted at 5:52. However, USN CVs were spotted later, at 8:20. This implies that the battle was sequential rather than simultaneous (as it was the case of the Battle of the Coral Sea). As shown by Armstrong and Powell (2005) and Armstrong (2014) in their analyses of the Battle of the Coral Sea, this early spotting is crucial in WWII aircraft-carriers battles. Consistently, in the simulations we will use a sequential model as the one developed by Armstrong (2014).

[Insert here Table 1]
[Insert here Table 2]

First, we compute the probability of an attacking aircraft arriving to the target[1]. For

[1] Although we speak about "probability", strictly speaking this is just a ratio. In fact, carrier-based aircraft can be launched, recovered and again launched several times during the battle fighted in several phases. Therefore, that ratio could be larger than one.

the USN, a total of 56 aircraft attacked IJN CVs from Midway[2] and 90 from USN CVs in the first attack (as all dive bombers from Hornet, a total of 35 aircraft, didn't spotted IJN carriers as they followed an incorrect heading), and a total of 24 attack aircraft from USN CVs in the second attack. Therefore, for the USN the overall probability of attacking to enemy CVs was (56+55+24)/215=0.6140 for the whole battle. Distinguish land-based attack from CV attacks, the figures are 56/61=0.9180 and (55+24)/154=0.5130, respectively. Distinguishing between the first and the second attacks, the probability of attacking was (56+55)/215=0.5163 during the first attack and 24/215=0.1116 during the second. For the Japanese, a total of 71 aircraft attacked Midway, whereas other 28 attacked the Yorktown. In aggregate, this implies an overall probability of attacking of 99/151=0.6556. However, the actual probability of attacking to USN CVs was 28/151=0.1854.

Next, we compute the probability of a successful intercept of attacking enemy aircraft. The probability of a successful intercept depends on several factors. First, interception can be done by fighters, by AA fire or by the collaboration of both. Second, interception depends also on the number of escort fighters. This figure must represent the number of attacking aircraft destroyed before they are able to deliver their ordinance (bomb or torpedo) and can be very different from the number of attacking aircraft shooting down as this event can occur after their have launched their ordinance.

The probability of a successful intercept is calculated as follows. First, Japanese aircraft loss in the attack on Midway were 8 aircraft (of which 6 were attackers) shooting down.[3] Nevertheless, as this action does not implies the attack on a carrier, they are not considered in our analysis. In the attack from Hiryu to the Yorktown, Japanese lost were 13 dive-bombers and 3 fighters in the first attack and 5 torpedo-bombers and 2 fighters in the second attack. Total is 21 (Japanese loss all 244 aircraft during the battle as no carrier survived), but excluding fighters, the number of attackers shooting down were 18. Ignoring the loss of fighters, total Japanese attack aircraft shooting down by American interception was 5 torpedo-bombers in the Midway attack, 5 torpedo-bombers and 13 dive-bombers in the Yorktown attack, with a total of 23. Nevertheless, for calibrating the model the relevant figure is the number of successful intercepts before the attacking aircraft are able to launch their ordinance. This is a direct consequence of how the model have been constructed as the number of attacking aircraft are those who arrive to the target and can hit it. In the attack to the Yorktown, the first wave were composed of 18 Dive-bombers but only 7 where able to escape USN fighter defense. Therefore, successful intercept in this case was of 11 dive-bombers. In the second wave, 10 torpedo-bombers attacked the Yorktown but 5 were intercepted before they were able to launch their ordinance (although only 4 torpedoes were seen by the Yorktown). Therefore, total successful intercepts were 16, that is, a final probability of p_F^{USN}=16/79=0.2025.

For the Japanese, the successful intercept, p_F^{IJN}, is calculated as follows. First, in the

[2]The Midway aircraft attacks comes in different waves. At 7:10, IJN CVs were attacked by the 4 B-26 armed with torpedoes and by the 6 TBFs. At 7:55 dive bombers attacked with 18 SBD2. At 8:14 with 16 B-17, and finally at 8:20 with 12 SB2Us.

[3]Parshall and Tully (2005) indicate that a total of 11 Japanese aircrafts were shoot down during the attack to Midway. However, using information from the U.S. Naval Intelligence based on captured Japanese documents, the number of Japanese aircraft shoot down were 8 (2 Zeros, one from Akagi and other from Kaga, one dive bomber from Kaga and 5 level bombers B5N2 "Kate", four from Hiryu and 1 from Soryu).

initial attack from American bombers based on Midway, 17 bombers were shoot down. Attacking force from the USN was composed of 84 dive-bombers and 41 torpedo-bombers (a total of 125) although 35 dive bombers fail in finding IJN CVs. 15 TBDs of the VT-8, 10 of VT-6 and 10 of VT-3 (a total of 35 Devastators) were shot down with no hits on IJN CVs. The other attack squadrons were VB-6, VS-6 and VB-3, with a total of 65 dive bombers aircraft, given that VB-8 and VS-8 missed the Japanese carriers. The total number of dive bombers shooting down by Japanese defense were 12 Dauntless. Therefore, total American attacking aircraft shooting down were 70. Again, we consider as a successful interception only the case when the attacking aircraft is shooting down before having chance to launch its ordinance. First, from the 4 B-26 torpedo-bombers, three were able to launch their torpedoes and hence only one were successful intercepted. From the 6 TBF, two of them were shooting down before they could release their torpedoes. Next, from the 18 SBD2, only 7 were able to release their bombs and so successful interception of this group was of 11 attacking aircraft. Finally, from the 12 SB2U, three of them were able to release their bombs to the Kaga and other six to the battleship Haruna, selected as a secondary target given the heavy protection of the IJN CVs by fighters. Hence, in this case, we consider successful interception of 9 attacking aircraft, although this does not implies that those aircraft were shooting down.

In the attack from USN CVs, from the 15 torpedo-bombers from the Hornet, only four where able to release their torpedoes before being shooting down, and therefore successful interception of this group were 11. From the group of the Enterprise, 6 out of 14 were shot down before launching their torpedoes and from the group of the Yorktown, 7 out of 12 were shot down before launching their torpedoes. The Enterprise dive-bombers groups, a total of 32 Dauntless, attacked the Kaga and the Akagi. 12 of them were able to drop their bombs. The Yorktown dive-bomber group, composed by 17 Dauntless, attacked the Soryu with 13 aircraft being able to drop their bombs. Therefore, total successful interceptions were 21 for aircraft attacking form Midway and 48 from the USN CVs, that is, a total of 69 aircraft. The final probability of successful intercept by the Japanese is then 69/90=0.7666.

Finally, we calculate the probability of hitting a CV. USN achieved a total of 13 hits (all with bombs; Akagi with two bomb hits, Kaga with four bomb hits and Soryu with three bomb hits. The final attack to the Hiryu consisted in 24 dive bombers, with four bomb hits). IJN aircraft achieved a total of 5 hits (3 bombs and 2 torpedoes), all on the Yorktown. Hence, global probability of hitting is 13/215=0.0605 for the USN and 5/151=0.0331 for the IJN. However, excluding intercepted attackers, the probability is 13/(135-69)=0.1970 for the USN and 5/(28-16)=0.4166 for the IJN.

Following Armstrong and Powell (2005) we consider firepower kills per hit as the variable determining the change in the number of carriers. They use a value of 0.4811 IJN ships lost per hit and 0.3717 USN ships lost per hit. These values are obtained from studies by Beall (1990) and Humphrey (1992) about damage by bombs and torpedoes hits on warships during WWII. However, they are obtained assuming that all USN dive-bombers are used with a 1000 lb. bomb. In practice, a portion of dive-bombers were armed with one 500 lb. bomb and two 100 lb. bombs. In the Battle of Midway Japanese carriers received a total of 13 hits, whereas American carriers suffered a total of 5 hits. Given these figures and the loss of four Japanese carriers and one American carrier, we consider a value of 0.3077 Japanese ships lost per hit and 0.2 American ships lost per hit. If we multiply the number of firepower kills per hit by the number of hits per surviving attacker we obtain the change in the number of carriers per surviving attacker. For the USN the figure is $0.2 \times 0.4166 = 0.0833$. For the IJN the number is $0.3077 \times 0.1970 = 0.0606$. In words, IJN CVs are easier to be sunk than USN counterparts, but accuracy in hitting the target is better for IJN pilots.

[Insert here Table 3]

4 Results

Given the calibration of the parameters of the model we obtain that the slope of the zero solution for the dynamic equations (no change in the number of aircraft-carriers), is 0.5662 for the American and 2.5509 for the Japanese. This implies that the number of aircraft-carriers involved in the battle (4 IJN carriers and 3 USN carriers) lays between the two zero-change lines, that is, there will be losses in each side as predicted by Hughes (2000). Once the model have been calibrated, we conduct a Monte Carlo simulation by considering a stochastic component for each probability under the assumption that they are distributed normally. As we have no information about variability of each probability, we assume a standard deviation of 0.05. We run 100,000 simulations for each scenario.[4] The values obtained from the historical result of the battle are considered as the "average" outcome. However, as Armstrong and Powell (2005) and MacKay et al. (2016) pointed out, this assumption ignores the likelihood that the historical results were different from their "true" underlying means. That is, the observed result of the Battle of Midway could be different from the expected mean outcome, favoring either American or Japanese. In fact, the main problem with historical counterfactual experiments is the fact the historical observed outcome is considered as the average outcome of the model and thus, the model is properly calibrate to reproduce the observed outcome. But this could be not the case. Therefore, it is possible that using the historical outcome as the average, some possible simulated outcome would be impossible (with a zero probability). On the other hand, the historical observed outcome could have been the result of lucky or a very rare event, and then a result with a very low probability (far away from the average result). MacKay, Price and Wood (2016) try to overcome this shortcoming by using an Approximate Bayesian Computation approach. Nevertheless, observed result from the Battle of Midway matches exactly the prediction done by Hughes (2000) and therefore, we assume that the historical outcome is not very different from the average expected result.

Simulation of the model for the whole battle yields an average firepower kills of 1.02 suffered by the USN and 3.82 firepower kills suffered by the IJN. Only one USN carrier was attacked and then the computed figure implies the destruction of only one aircraft carrier. In the case of the IJN all four carriers were attacked and thus, the 3.82 firepower kills implies the destruction of the four Japanese carriers. Therefore, in spite that the

[4] Simulations have been done using MatLab. A copy of the code files is available from the authors upon request.

battle was fought sequentially, the standard salvo model does a good job in predicting the observed result. Figure 2 plots the probability of CVs loss for the American and Japanese, respectively, for the whole battle. The probability of losing all four carriers for the Japanese is around 50%, whereas the probability of losing one carrier for the American is around 35%. More importantly, the mode, representing the value most likely to be sampled, is four carriers loss for the Japanese and zero for the American.

[Insert here Figure 2]

Nevertheless, the Battle of Midway was fought in several phases. In order to check the robustness of the above results, we repeat the exercise but considering a sequential model given that the battle was conducted in several phases. In the sequential model we consider four events conditional to the existence of survival carriers in the previous phase. i) Initial American attack; ii) Japanese counterattack; iii) American second attack; and iv) Japanese second counterattack. However, the last phase of the battle, in which the Japanese do a second counterattack in the case of some surviving IJN carriers, could not be possible due to the timing in previous phases. First American attack aircraft where launched at 7:06 and they started the attack at 9:20. Japanese counterattack bomber-aircraft were launched at 10:58 and torpedo-bombers at 13:31, finishing this counterattack at 14:47. The second American attack wave were launched at 15:30, arriving to the last Japanese carrier position at 16:50 and starting the attack at 17:05. This timing means that, in the case of any surviving Japanese carrier after the second American attack, a second counterattack wave aircraft could not be launched before 17:30, arriving to American carriers position about one hour later, i.e., around 18:30.[5]

Figure 3 plots the histograms obtained from the simulation of the sequential model. The mode for the first American attack is two carriers lost for the Japanese, but probabilities of losing three or all four in the first attack are also high. In the Japanese counterattack, the mode is zero, but also the probability of losing one carrier for the American is high, around 35%. The simulated model produces a firepower kills of 2.89 suffered by the IJN and 1.02 firepower kills suffered by the American in the first round, very close to the historical outcome of three carriers lost by the Japanese and one carrier lost by the American. Given these results during the first phase of the battle (initial American attack and Japanese counterattack), the mode for the second American attack is one carrier lost for the Japanese, whereas remains at zero carriers lost for the American in the second Japanese counterattack. In this second phase of the battle, we obtain values of 1.34 and 0.41 for the firepower kills suffered by the IJN and USN, respectively.

[5] In the Battle of the Coral Sea, the launching of the American carrier-based attack groups commenced at 9:00 and the attack over Japanese carriers started at 10:50. Japanese launched attack aircraft a few minutes latter and they attacked American carriers at 11:13. Nevertheless, in spite of this early first simulaneous attack, the possibility of a second attack was rejected by the American given the short number of serviceable aircraft (only 8 fighters, 12 dive-bombers and 8 torpedo-bombers). Japanese took the same decision of not to do a second attack due to a shortage of fuel for aircraft on board. The day before (7th May), the Japanese tried an afternoon attack on American carriers, but at 16:30 light conditions were failing because it was getting dark at that time and attacking aircrafts decided to return to their own carriers at 17:47.

[Insert here Figure 3]

5 Counterfactual analysis

Once the model has been calibrated to fit the historical outcome we can proceed further and evaluate the likelihood of other possible outcomes performing a counterfactual analysis. Historical discussion had focused on the fact that Nagumo ordered to changes torpedoes by bombs for a second attack to the Midway Air Base; on the fact that the Tone's N^o 4 scout, the one which discovered the USN fleet, did not were in time due to engine problems; and on the decision about recovering the wave returning from the attack to Midway before to launch the attack to USN carriers, as the main factors explaining the overwhelming American victory at Midway. Our calibrated model allows us to explore the relative relevance of these elements in explaining the final result of the battle.

The counterfactual analysis is based in four possible alternatives. The first scenario assumes that the 35 dive-bombers launched by the Hornet also found the Japanese carriers. The second counterfactual analysis consider the possibility that the Zuikaku also joined the Japanese striking force to Midway (that is, the availability of five Japanese carriers). The third counterfactual experiment considers that Nagumo does not wait for a coordinate complete attack and send the reserve dive-bombers on Hiryu and Soryu. Finally, we consider the most possible favorable scenario for the Japanese, considering that the USN fleet is discovered early and then the Japanese attack is simultaneous to the American attack. Considering together all counterfactual experiments, the extreme possible outcome for the battle goes from the loss of all four Japanese carriers (or five in the case of an additional carrier was present) and no loss for the American, to the destruction of three Japanese carrier and two American carriers.

5.1 What if the Hornet's dive-bombers didn't failed to find IJN carriers?

First, we consider the case in which the probability of USN attacking aircraft were larger than the observed one. Indeed, one important aspect of the battle is that the USN carriers launched all available attacking aircraft except 17 dive-bombers from the Yorktown that were held in reserve but, fortunately for the Japanese, not all launched attacking aircraft found the Japanese carriers. The motivation of this counterfactual exercise is due to the fact that all dive-bombers from the Hornet (a total of 35 aircraft) lost in searching for the IJN carriers. This is a very likely scenario given that the torpedo-bombers from the same aircraft-carrier already found and attacked the Japanese carriers. Hence, in this scenario we consider that the probability of attacking for the American is (56+55+35)/215=0.6791 during the first attack (instead of (56+55)/215=0.5163 as in the historical benchmark scenario).

Given the rest of variables, the average outcome of this scenario is disastrous for the Japanese as they loss all four aircraft carriers in the first attack without any possibility of counterattack and thus, no losses for the American carriers. In the simulation we obtain that the mode is four CVs losses for the Japanese and zero for the American. Given the Monte Carlo experiment, the probability for the IJN of losing all four carriers during the first attack is close to 1 and that the probability of losing less than three carriers in this first attack is very close to zero. Figure 4 shows the histogram corresponding to this scenario. The probability for the American of losing one or more carriers is close to zero given that the probability for the Japanese of losing all four carriers during the first attack is close to one. Average firepower kills are 3.98 suffered by the Japanese and 0.03, practically zero, suffered by the American. Based on this exercise we can affirm that Japanese was "lucky" in sinking one American carrier (the Yorktown) as the commander of the VB-8 and VS-8 squadrons from the Hornet took a wrong decision by heading South in the first attack.

In summary, in this scenario the IJN would have lost, on average, all four CVs in the first attack conducted by the USN CVs, eliminating any possibility of counterattack by the Japanese and thus, no damage on any of the three USN CVs. This simulation shows that the Japanese were very close to the total disaster.

[Insert here Figure 4]

5.2 What if the Zuikaku was also in the Japanese striking force?

During the Battle of the Coral Sea (one month earlier) the Shokaku were putted out of action by the aircraft of Yorktown and Lexington, whereas the Zuikaku escaped without damage. Hence, in this counterfactual exercise we assume that also Zuikaku was available and joined the other four carriers striking force with 21 fighters, 21 dive-bombers and 21 torpedo-bombers. This implies a Japanese striking force of five aircraft-carriers. We simulate this scenario with both the one-shot model (the model for the whole battle) and the sequential model for comparison with the benchmark case.

Figure 5 plots the corresponding histograms for the whole battle. In this scenario the mode is 2 for the Japanese and 1 for the American, compared to the values of 4 and 0, respectively, in the benchmark case. This means that an additional carrier in one side yields a dramatic change in the outcome. Average losses are 2.74 firepower kills suffered by the Japanese carriers and 1.53 firepower kills suffered by the American. Comparing Figure 5 with Figure 2 we can clearly observe the sizable effects of an additional aircraft-carrier in the Japanese fleet. The additional carrier implies that more fighter are also available for defence, more Japanese attacking aircraft are available and a higher number of carrier must be attacked. As a consequence, the probability of losses for American carriers is also higher given the higher number of Japanese attacking aircraft whereas the probability of losing Japanese carriers is lower given the higher number of Japanese fighters. Nevertheless, in spite of the larger superiority in the number of Japanese carriers with respect to the America, firepower kills suffered by the Japanese are much higher than that suffered by the American.

[Insert here Figure 5]

Figure 6 plots the histograms for the sequential model. In the first phase (first attack by American and counterattack by the Japanese), the mode is 1 for both sides and the firepower kills average suffered values are 1.50 and 1.53 for the Japanese and American, respectively. These numbers can be interpreted as two carriers are putted out of order (not necessarily sunk) in each side, with slightly heavy damage suffered by the American. Higher damage suffered by USN carriers is because the Japanese counterattack is composed by the aircraft from at least three, and not only one, aircraft-carriers. Therefore, the American would have, on average, only one carrier available for the second attack, with the loss, on average, of an additional Japanese carrier. Average firepower kills suffered by the Japanese during the second American attack is 0.52 (mode is zero). This opens the possibility, although very unlikely, of a second Japanese counterattack. In this phase, firepower kills suffered by the American is 0.98. This would implies the loss of another USN CVs in this second counterattack. In this case, total average losses would be of two Japanese carriers and two American carriers, with the remaining of one carrier for the American and two for the Japanese.[6] Excluding the possibility of a second Japanese counterattack, losses would have been two carriers for the Japanese and one for the American.

[Insert here Figure 6]

The implications of an additional Japanese carrier presents in the battle can be easily observed by comparing Figure 6 with Figure 3. The additional carrier has important implications for the battle but not enough to produces a Japanese victory. Probability of success of the first American attack reduces but attacking first is still a significant advantage. Reduced losses for Japanese carriers during the first American attack combined with a large number of attacking aircraft increases the probability of success for the Japanese counterattack.

5.3 What if Nagumo decided not to wait for a coordinate attack?

This third scenario considers the possibility that Nagumo would have decided not to wait for the torpedo-bombers to be ready and would have ordered an attack on American carriers with the available dive-bombers from Hiryu and Soryu (a total of 34 aircrafts). In fact, as pointed out by Isom (2000), the commander of the Second Carrier Division (Hiryu and Soryu) requested Nagumo to do just that. Thus, this is also a very probable scenario that could had happen and a direct test about if the decision taken by Nagumo to wait for torpedo-bombers to be ready to attack American carriers was correct or not. In reality, Nagumo decided not to do that and wait for a coordinate attack for two reasons: First, the

[6] Additionally, we consider the combination of this scenario with the first one that all American attacking aircrafts launched found Japanese carriers. This alternative scenario would have been catrastrophic for the Japanese with the possibility that all five carriers were destroyed in the first USN carrier-based attack and eliminating any possibility of counterattack by the Japanse.

probability of success increases with a coordinate attack of dive-bombers and torpedo-bombers, but only dive-bombers were available at that moment. Second, all available fighters were used in the defense of the carriers and therefore, a very small number or even none was available at that time for escorting attacking aircrafts. These two elements together reduces dramatically the probability of success in the attack to American carriers and by sure Nagumo did that calculus.

But what if Nagumo decided to attack the American carriers following the recommendation of the commander of the Second Carrier Division? Given that in this scenario the Japanese counterattack was earlier than in reality, the likelihood of a second Japanese attack is large. In this case, in the first almost simultaneous attack, the Japanese lose three aircraft carriers (an average of 2.89 firepower kills as in the benchmark case), whereas the American lose one carrier (an average of 1.12 firepower kills), with a mode of 2 for the Japanese and zero for the American, as in the benchmark case. Average firepower kills suffered by American in this scenario is larger than in the benchmark case during the Japanese counterattack but not significantly different. In this scenario we also consider the possibility of a second almost simultaneous attack. This means that for the second attack, on average, the Japanese only have one CV, whereas the American have two. Mode is zero for carrier losses for both sides in this second phase of the battle, but average values are 1.03 firepower kills suffered by the Japanese, which implies, in average, the loss of the remaining Japanese carrier. Average firepower kills suffered by the American in this second attack is 0.81 which, at least, implies another American carrier severely damage.

Figure 7 plots the histogram corresponding to the two attack phases considered in this scenario. It can be observed that in the first attack, probability distribution for IJN carriers losses does not change with respect to the benchmark case, whereas the histogram for USN carriers losses reflects a higher probability of damage for American carriers compared to the benchmark case.

[Insert here Figure 7]

In summary, we find that the results in this scenario are not so different from the historical ones. On average, the Japanese losses all four carrier against one or two for the American, as actually it happened. A remaining question is related to the number of Japanese fighters available to escort the dive-bombers. In the simulation we assume that the escort of fighter was similar to the observed in the Hiryu attack to the Yorktown. But given that from 7:10 to 8:30 IJN carriers were under attack by American aircrafts based on Midway, all Japanese fighters were busy in the defense of the carriers. Therefore, probability of sinking an American carriers in the Japanese attack could be even lower then estimated figure. What we learn from this counterfactual exercise is that Nagumo's decision to wait was not decisive for the final result of the Battle.

5.4 Earlier discovery of American carriers

Finally, we consider the best possible scenario for the Japanese. In the Battle of Midway, the IJN CVs fleet was spotted by American search aircrafts at 5:52. However, USN CVs were not spotted by the Japanese search aircrafts until 8:20.[7] Here, we consider a scenario where USN carriers are discovered before Nagumo ordered to rearms aircrafts with bombs for a second attack to Midway. This implies the assumption that the American carriers are discovered before 7:15 (this could likely be the case of the *Chikuma 5*).[8] This is the most possible favorable scenario for the Japanese and the worse for the American.

The Japanese attack group to Midway were launched at 4:30. It arrived at Midway at 6:30 and the attack lasted by about 17 minutes (Naval Staff History, 1952). At 7:10 started the attack to the IJN carriers by the aircraft from the Midway Air Base. This was an uncoordinated attack with several waves, the last starting at 8:20 by eleven SB2Us, just at the same time that the Japanese attack group returning from Midway were arriving to the base carriers. The attack to IJN carriers by the Hornet's torpedo squadron started at 9:20. This implies the existence of a window of about 1 hour between attacks. However, Nagumo decides to recover the aircraft returning from attacking Midway before attacking USN carriers. Parshall, Dickson and Tully (2001) show that the Akagi started the recovery of the Midway attack group at 8:37 and finished at 9:20. Therefore, the window to launch an attack over American carriers reduces from about 20 minutes for the Akagi to zero minutes for the rest. Therefore, we consider the case that Nagumo decided to attack USN carriers before to recover the Midway attack group. In this case the availability of attack aircraft reduces to 70-36=34 dive-bombers and 81-35=46 torpedo-bombers, that is, a total of 80 attacking aircrafts.

Therefore, we consider an almost simultaneous attack on carriers in both sides by eliminating the historical American advantage of early spotting the enemy carriers. The probability of attacking for the Japanese is calculated by assuming all available aircraft are launched. Nevertheless, as noted by Isom (2000), on Kaga, only 18 aircraft were armed with torpedoes and nine remained unarmed due to less experienced crews. Therefore, we exclude those nine torpedo-bombers and assume that the Japanese attacking wave consisted of 34 dive-bombers and 35 torpedo-bombers (69 in total), that is, only 86.25% of available attacking aircraft. Therefore, probability of attacking for the Japanese in this scenario is 69/151=0.4569. We do not consider any assumption regarding the escort of fighters. In the simulation, we also consider the possibility of a second simultaneous attack, depending on the results from the first attack.

Figure 8 plots the histograms from the simulation of this scenario. In the first simultaneous attack, the average firepower kills is 1.68 suffered for the USN and 2.89 suffered for the IJN. Mode is 2 for the Japanese and zero for the American as in the benchmark case, although the probability of losing two or three carriers is higher for the American (around 20% each). For the Japanese, this new scenario does not change the results during the first attack as they loss on average three carriers as in the historical scenario. For the American this is a worse scenario as they loss one or two carriers. Given that in the first attack total destruction of carriers is not achieved in any side, the sequential model derives to an additional phase involving surviving carriers (one Japanese. Considering the possibility of a second attack, average firepower kills are 0.62 and 0.72 suffered by the

[7]The Tone 4 reconnaissance aircraft sighted and reported the position of the American fleet at around 7:28, reporting 10 enemy surface ships even though carriers were not identified.

[8]The Chikuma 5 aircraft passed over the American carriers at 6:30. Nevertheless, bad weather prevented the Japanese to discover the American fleet.

USN and the IJN, respectively.

In summary, the average outcome from this scenario is the loss of three or four carriers for the Japanese and two for the American. This implies only one survival aircraft-carrier would remain afloat in each side. This result cannot be considered as a Japanese victory but still would remain an American victory although with heavy losses. This counterfactual analysis shows that the historical argument explaining Japanese defeat related to the fact that IJN carriers were discovered much earlier than USN carriers is not the principal reason for the American victory as our results, including this very favorable scenario for the IJN, seem to indicate that the Battle of Midway was a battle the Japanese never could win.

[Insert here Figure 8]

6 Concluding remarks

This paper calibrates a simple stochastic salvo combat model to better understand some key questions regarding the Battle of Midway. We show that the American victory in the Battle of Midway was not a "miracle", neither caused by "lucky" in the American side, by the "victory disease" by the Japanese or by wrong decisions taken by Nagumo. Indeed, we show that Midway was a battle the Japanese never could win and the final result was conditioned by the timing imposed by the earlier attack to the Midway Air Base. The model replicates, in average, the historical results from the battle. Using the calibrated model we consider four counterfactual exercises: i) All American attacking aircraft find Japanese carriers; ii) An additional Japanese carrier; iii) Not to wait for a Japanese coordinate attack; and iv) Earlier discovery of American carriers. Counterfactual analyses show a final outcome in the range from losing all aircraft carriers to the Japanese and none American, to the loosing of three carriers for the Japanese and two for the American in the most favorable scenario for the IJN.

The result of the Battle of Midway is heavily conditioned by the earlier attack to the Midway Air Base. This initial operation reduced the force to the equivalent of two Japanese aircraft-carriers, a force clearly inferior to the three American aircraft-carriers plus the Midway Air Base. Only in the unlikely case in which the IJN fleet were not discovered by American searching and the American carriers spotted earlier, that is, Japanese attacking American carriers first, the Japanese would have a chance to win the battle.

All results obtained from the counterfactual analyses are subject to the assumption that the observed historical outcome is the average and this could be a caveat if observed results were unlikely to occur. Nevertheless, the result of the battle is consistent with the predictions by Hughes (2000) and hence, the observed result would be not so different from what we consider the average outcome. Future research will consist in applying the methodology developed by MacKay, Price and Wood (2016) to confirm that the historical outcome of the Battle of Midway was close to the expected average result.

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Table 1: Number of USN aircraft

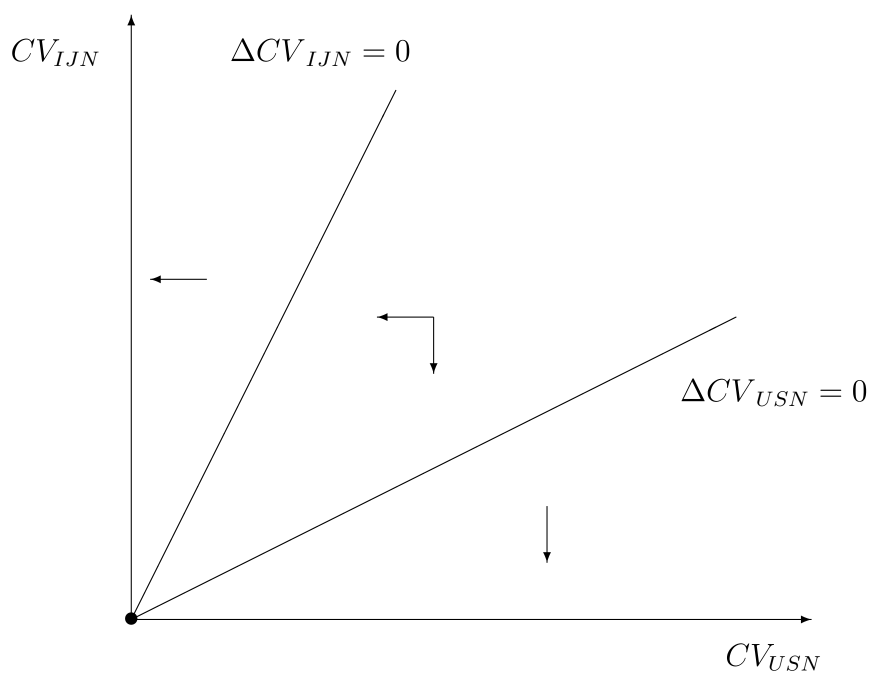
Aircraft	USN CVs				Total
	Yorktown	Enterprise	Hornet	Midway Air Base	
F4F Wildcat	25	27	27	7	86
SBD Dauntless	37	37	36	18	128
TBD Devastators	15	14	15		44
F2A Buffalo				21	21
TBF Avenger				6	6
SB2U Vindicators				16	16
B-17 Fortress				17	17
PBY Catalina				30	30
B-26 Marauder				4	4
Total	77	78	78	119	347

Table 2: Number of IJN aircraft

Aircraft	IJN CVs				Total
	Kaga	Akagi	Hiryu	Soryu	
A6M Zero	24	24	21	21	90
D3A "Val"	18	18	18	16	70
B5N "Kate"	27	18	18	18	81
D4Y "Judy"	2			1	3
Total	71	60	57	56	244

Table 3: Calibration of the parameters of the model

Parameter	Definition	Calibrated value
CV^{USN}	Number of USN aircraft carriers	3
CV^{IJN}	Number of IJN aircraft carriers	4
a_K^{USN}	Firepower kills per hit of USN CV	0.2000
a_K^{IJN}	Firepower kills per hit of IJN CV	0.3077
p_H^{USN}	Probability of hit per shot on IJN CV	0.1970
p_H^{IJN}	Probability of hit per shot on USN CV	0.4166
p_A^{USN}	Probability of USN attacking aircraft	0.6140
p_A^{IJN}	Probability of IJN attacking aircraft	0.1854
n_A^{USN}	Number of USN attack aircraft per CV	71.66
n_A^{IJN}	Number of IJN attack aircraft per CV	37.50
p_F^{USN}	Probability of intercept by USN fighters	0.2025
p_F^{IJN}	Probability of intercept by IJN fighters	0.7666
n_F^{USN}	Number of USN fighters per CV	26.33
n_F^{IJN}	Number of IJN fighters per CV	22.50



CV_{IJN}
$\Delta CV_{IJN} = 0$
$\Delta CV_{USN} = 0$
CV_{USN}

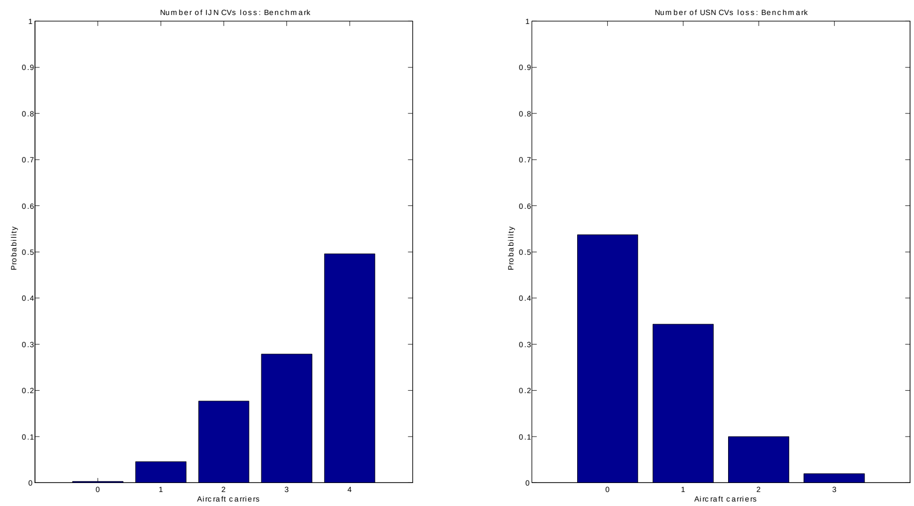


Figure 1: Benchmark case: Full battle

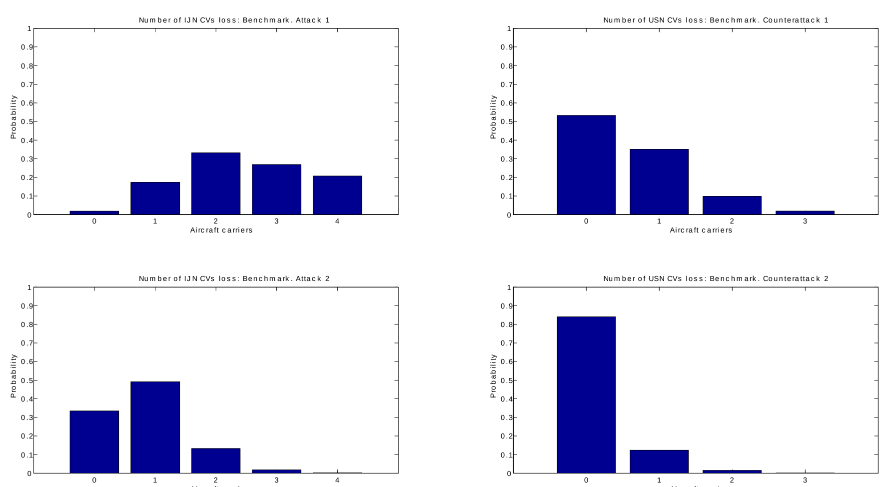


Figure 2: Benchmark case: Sequential model

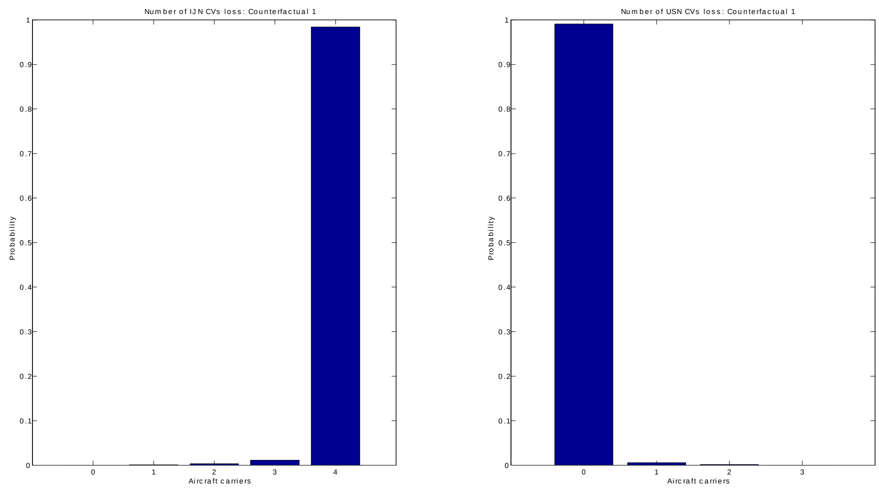


Figure 3: Counterfactual 1: Hornet's dive-bombers didn't failed to find IJN carriers

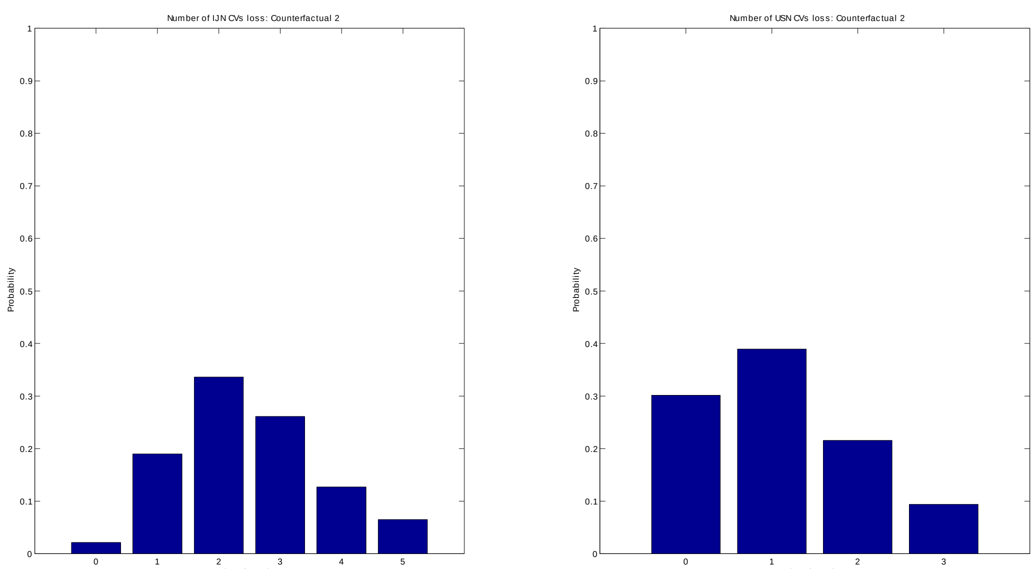


Figure 4: Counterfactual 2: Additional Japanese carrier. Full battle

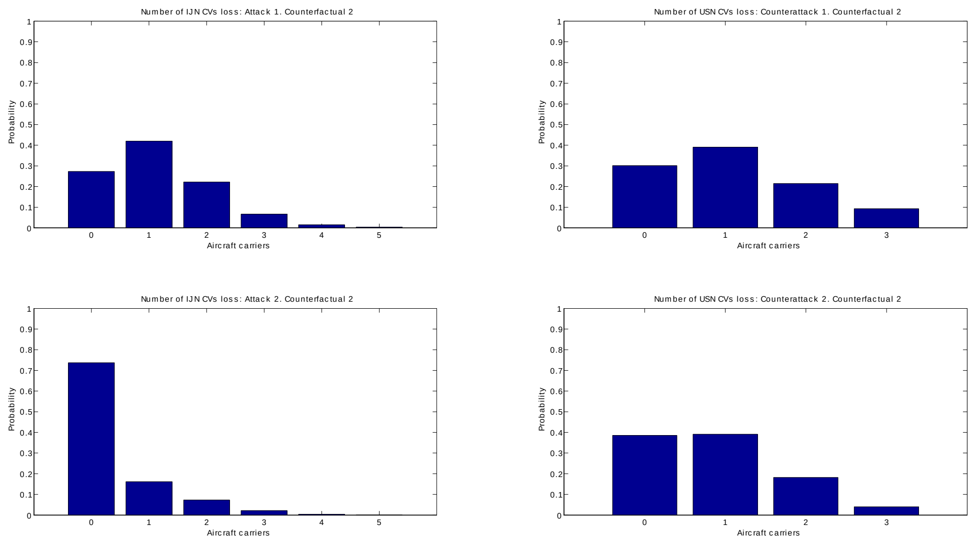


Figure 5: Counterfactual 2: Additional Japanese carrier. Sequential battle

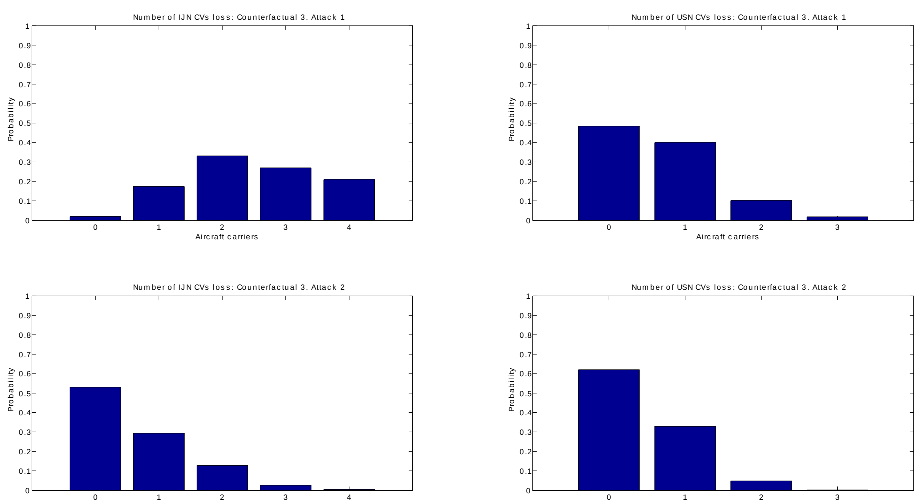


Figure 6: Counterfactual 3: Not to wait for a coordinate attack

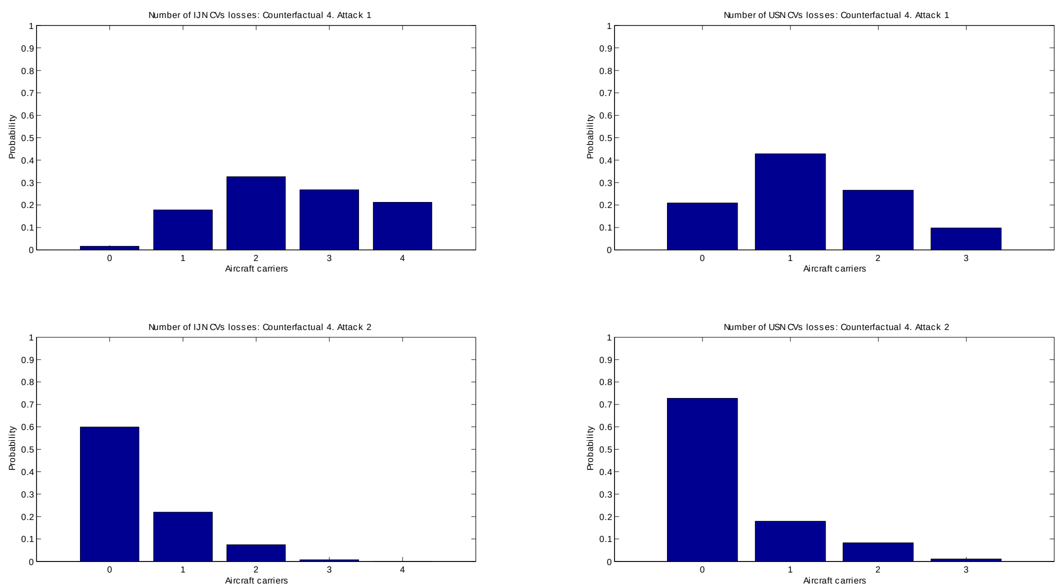


Figure 7: Counterfactual 4: Earlier discovery of American carriers.