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Subgame perfect implementation of the deserving winner of a competition with natural mechanisms*

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Abstract

A jury has to decide the winner of a competition among a group of contestants. All members of the jury know who the deserving winner is, but this contestant is unknown to the planner. The social optimum is that the jury select the deserving winner. Each individual juror may be biased in favor (friend) or against (enemy) some contestant, and therefore her goal does not necessarily coincide with the social objective. We analyze the problem of designing extensive form mechanisms that give the jurors the right incentives to always choose the deserving winner when the solution concept is subgame perfect equilibrium. We restrict the class of mechanisms considered to those which satisfy two conditions: (1) the jurors take turns to announce the contestant they think should win the competition, and (2) telling the truth is always part of a profile equilibrium strategies. A necessary condition for these mechanisms to exist is that, for each possible pair of contestants, there is at least one juror who is impartial with respect to them. This condition, however, is not sufficient. In addition, the planner must know the friend or the enemy of at least one juror.

Key Words: mechanism design; jury; subgame perfect equilibrium.
J.E.L. Classification Numbers: C72, D71, D78.

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1 Introduction

A jury must choose one winner among the contestants involved in a competition. All jurors know who the best contestant is: the "deserving winner". The social optimum is that the jury select the deserving winner. However, the objective of each particular juror may be different from the social objective. For example, a juror may be biased in favor of one contestant (her friend) and want this contestant to win the competition even if she is not the deserving winner. Similarly, a juror may be prejudiced against one contestant (her enemy) and prefer any other one to win the competiton.

The fact that jurors are partial does not necessarily imply that the social objective is unattainable. Depending on the specific bias of the jury, the social planner might be able to design a mechanism that induces the jurors to always choose the deserving winner, whoever she is. When such a mechanism exists, we say that the social optimal choice rule (SOCR) is implementable. A necessary condition for the implementation of the SOCR is that, for each possible pair of contestants, there is some juror who is impartial with respect to them in the sense that, whenever one of them is the deserving winner, the juror prefers that contestant to the other one (Proposition 1). When this condition is satisfied we say that the jury is minimally impartial. If the jurors take their decisions according to the Nash or subgame perfect equilibrium concepts, minimal impartiality of the jury is also sufficient for the implementation of the SOCR (Proposition 2). The reason is that, under this condition, the canonical mechanism for Nash implementation (Maskin, 1999; Danilov, 1992) implements the SOCR in Nash and subgame perfect equilibrium.

The canonical mechanism for Nash implementation, however, has been widely criticized for having unnatural features such as too complex message spaces, employing integer games or modulo games, etc. (see Jackson, 1992). This type of mechanisms are designed to characterize what can be implemented, and therefore they have to apply a broad range of environments and social choice rules. As argued by Jackson (1992) and Serrano (2004), we would hope that for particular settings and social choice rules we could find simple and natural mechanisms with desirable properties. This is precisely the goal of the present paper: we study the problem of implementing the SOCR when we restrict the class of mechanisms considered to those that are simple and satisfy desirable properties. In particular, we are interested in studing subgame perfect implementation, and therefore we focus on extensive form mechanisms.

When choosing the winner of a competition, some of the simplest mechanisms are those where each juror only has to announce the contestant she thinks should win the competition. We call these mechanisms straightforward mechanisms. Furthermore, if we ask the jurors to tell us who should win the competition, it is natural to require that telling the truth (announcing the deserving winner) is always part of some profile equilibrium strategies. We say that the SOCR is naturally implementable in subgame perfect equilibrium if there exists a straightforward extensive form mechanism (*i.e.*, a mechanism where the jurors take turns to announce the contestant they think should win the competition) that implements the SOCR and which is such that telling the truth is always part of an equilibrium.

The restriction on the class of mechanisms considered and the truth-telling equilibrium condition imposed makes the problem more difficult. As a consequence, minimal impartiality of the jury is not a sufficient condition to ensure that the jury will choose the deserving winner: Proposition 3 shows that having one impartial juror for each possible pair of contestants does not guarantee that the SOCR can be naturally implemented in subgame perfect equilibrium (although it is still a necessary condition).

In order to naturally implement the SOCR, we need to impose some extra conditions on the configuration of the jury besides minimal impartiality. Sometimes the planner has more information about the jury and knows that some jurors have friends or enemies among the contestants. Knowing that a juror wants to favor or harm a given contestant reduces the size of admissible preferences for that juror (compared with the case in which we have no information about her preferences on that particular contestant), which facilitates implementation. If, for example, the planner knows that a given contestant is the friend of a juror, she can design a mechanism that works only when this juror wants to favor that contestant. If, on the contrary, the planner has no information about the preferences of this juror on that contestant, then the same mechanism must work when the juror wants to favor the contestant, when she wants to harm the contestant, etc. Following this reasoning (and given a minimally impartial jury), the higher the number of jurors with friends or enemies the planner knows, the "easier" it will be to naturally implement the SOCR. It turns out, however, that the planner only needs to know that one of the jurors has a friend or an enemy to naturally implement the SOCR in subgame perfect equilibrium, at least in the three contestants case. To prove these results, we propose two different straightforward extensive form mechanisms. The first one, Γ_3^f, works when there is at least one juror with a known friend (Proposition 4). The second mechanism, Γ_3^e, works when there is at least one juror with a known enemy (Proposition 5). Both mechanisms are such that the first juror to speak is the juror with the known friend/enemy. We prove that this is a requirement to be met by any straightforward mechanism naturally implementing the SOCR in subgame perfect equilibrium (Remarks 1 and 2).

Related literature

Amorós (2015) analyzes natural implementation of the SOCR in Nash equilibrium and shows that, as in the subgame perfect equilibrium case studied in the present paper, minimal impartiality is a necessary but not sufficient condition for that. When naturally implementing in Nash equilibrium, however, there is an asymmetry between the cases in which jurors have friends and those in which they have enemies: the number of jurors with friends that the planner needs to know to naturally implement the SOCR in Nash equilibrium is less than the number of jurors with enemies that she would need to know for it. Amorós (2013) studies necessary and sufficient for the implementability of the SOCR in Nash and dominant strategies equilibrium. This paper, however, does not study natural implementation (the mechanism proposed for Nash implementation employs modulo games). Amorós (2011) studies the case where the jurors are the contestants themselves (so that each juror has one friend) and proposes a natural extensive form mechanism that implements the SOCR in subgame perfect equilibrium. Moskalenko (2013) proposes a variation of that mechanism. There is a series of papers dealing with the problem in which the jury has to provide a full ranking of contestants; *i.e.*, there is a true ranking of contestants instead of just one deserving winner. Amorós et al. (2002) analyze this problem when each juror wants to favor a different contestant. Amorós (2009) provides necessary and sufficient conditions on the jury for the implementation of the true ranking in Nash and dominant strategies. Adachi (2014) analyzes the problem of implementing the socially optimal ranking in subgame perfect equilibrium when jurors may have friends. Ng and Sun (2003) investigate the problem of excluding the self-awarded marks in the calculation of the ranking when each contestant is biased in favor of itself. Finally, our paper is also related to the literature on the effects of having honest agents on the general implementation problem (Matsushima, 2008; Dutta and Sen; 2011).

The rest of the paper is organized as follows. Section 2 presents the model and states the necessary and sufficient conditions for subgame perfect implementation. Section 3 presents the results on natural implementation. Section 4 concludes. The Appendix provides the proofs of some of the results.

2 The model and preliminaries

Let $N = \{a, b, ...\}$ be a set of contestants in a competition. A group $J = \{1, 2, ...\}$ of jurors must choose one winner from the contestants. All jurors know who the best contestant is. We call this contestant the **deserving winner**, $w_d \in N$. The socially optimal outcome is that the deserving winner wins. General elements of N are denoted by x, y, etc., and general elements of J are denoted by j, k, etc.

Let $\Re$ be the class of preference relations defined over N. Each juror $j \in J$ has a **preference function** $R_j : N \longrightarrow \Re$ which associates with each deserving winner, $w_d \in N$, a preference relation $R_j(w_d) \in \Re$. Let $P_j(w_d)$ denote the strict part of $R_j(w_d)$. Let $\mathcal{R}$ denote the class of all possible preference functions. Table 1 shows an example of preference function when $N = \{a, b, c\}$ (higher contestants in the table are preferred to lower contestants).

R_j

$w_d =$	**a**	**b**	**c**
	a	b	ab
Preferences	b	a	c
	c	c	

Table 1 Example of preference function.

Let 2^N_2 denote the set of all possible pairs of contestants. A juror j is **impartial** with respect to a pair of contestants $xy \in 2^N_2$ if, whenever one of the two contestants is the deserving winner, j prefers that contestant to the other one. A contestant x is a known **friend** of juror j if x is always the most preferred alternative for j, regardless of who is the deserving winner. A contestant x is a known **enemy** of juror j if x is always the less preferred alternative for j, regardless of who is the deserving winner. Each juror $j \in J$ is characterized by a triple (I_j, x^f_j, x^e_j) where $I_j \subset 2^N_2 \cup \{\emptyset\}$ is a set of pairs of contestants with respect to whom j is impartial, $x^f_j \in N \cup \{\emptyset\}$ is a known friend of j, and $x^e_j \in N \cup \{\emptyset\}$ is a known enemy of j. The cases (i) $I_j = \emptyset$, (ii) $x^f_j = \emptyset$, and (iii) $x^e_j = \emptyset$, correspond to the situations where the planner does not know (i) if j is impartial with respect to some pair of contestants, (ii) if j has a friend or not, and (iii) if j has an enemy or not. Let $\mathcal{E} \equiv 2^N_2 \cup \{\emptyset\} \times N \cup \{\emptyset\} \times N \cup \{\emptyset\}$.

Definition 1 *A preference function $R_j \in \mathcal{R}$ is **admissible** for juror j at $(I_j, x^f_j, x^e_j) \in \mathcal{E}$ if and only if:*

(1) for each $xy \in I_j$, whenever $x = w_d$, then $x\ P_j(w_d)\ y$ (i.e., j is impartial with respect to every pair of contestants in I_j),

(2) if $x^f_j \neq \emptyset$ then, for each $w_d \in N$ and each $y \in N \backslash \{x\}$, $x^f_j\ P_j(w_d)\ y$ (i.e., x^f_j is a known friend of j),and

(3) if $x^e_j \neq \emptyset$ then, for each $w_d \in N$ and each $y \in N \backslash \{x\}$, $y\ P_j(w_d)\ x^e_j$ (i.e., x^e_j is a known enemy of j).

For each $(I_j, x^f_j, x^e_j) \in \mathcal{E}$, let $\mathcal{R}(I_j, x^f_j, x^e_j)$ be the class of all preference functions that are admissible for j at (I_j, x^f_j, x^e_j). Suppose, for example, that $I_j = \{b, c\}$, $x^f_j = a$, and $x^e_j = \emptyset$. Then, $R_j \in \mathcal{R}(I_j, x^f_j, x^e_j)$ if and only if $b\ P_j(b)\ c$, $c\ P_j(c)\ b$, and $a\ P_j(w_d)\ x$ for every $w_d \in N$ and $x \in N \backslash \{a\}$ (see Table 2).

R_j		
a	**b**	**c**
a	a	a
$\vdots$	$\vdots$	$\vdots$
$\vdots$	b	c
$\vdots$	$\vdots$	$\vdots$
$\vdots$	c	b
$\vdots$	$\vdots$	$\vdots$

Table 2 Admissible preference functions when $I_j = \{b, c\}$, $x^f_j = a$, and $x^e_j = \emptyset$.

A **jury configuration** is a profile $(I, x^f, x^e) \equiv (I_j, x^f_j, x^e_j)_{i \in J} \in \mathcal{E}^{|J|}$. A **state** is a tuple $(R, w_d) \in \mathcal{R}^{|J|} \times N$, where $R \equiv (R_j)_{j \in J}$ is a profile of preference functions. A state (R, w_d) is **admissible** when the jury configuration is (I, x^f, x^e) if $R_j \in \mathcal{R}(I_j, x^f_j, x^e_j)$ for every $j \in J$. Let $S(I, x^f, x^e)$ denote the set of all states that are admissible when the jury configuration is (I, x^f, x^e). Given a jury configuration (I, x^f, x^e), the **socially optimal**

choice rule (SOCR) is the function $\varphi : S(I, x^f, x^e) \to N$ such that, for each $(R, w_d) \in S(I, x^f, x^e)$, $\varphi(R, w_d) = w_d$ (*i.e.*, for each state admissible at (I, x^f, x^e), φ selects the deserving winner).

We are interested in analyzing implementation of the SOCR via extensive form mechanisms. An **extensive form mechanism** is a dynamic mechanism in which jurors make choices sequentially and it is denoted by $\Gamma \equiv (\Theta, \gamma)$, where $\Theta = \times_{j \in J} \Theta_j$, Θ_j is the set of possible strategies for juror j and $\gamma : \Theta \to N$ is the outcome function. The contestant selected by mechanism Γ when jurors play the profile of strategies $\theta \equiv (\theta_j)_{j \in J} \in \Theta$ is denoted $\gamma(\theta)$.

For each extensive form mechanism Γ and each state (R, w_d), a profile of strategies θ is a **subgame perfect equilibrium** if it induces a Nash equilibrium in every subgame. Let $E(\Gamma, R, w_d)$ denote the set of subgame perfect equilibrium strategies of mechanism Γ at state (R, w_d). An extensive form mechanism **implements the SOCR in subgame perfect equilibrium** when the jury configuration is (I, x^f, x^e) if, for each admissible state $(R, w_d) \in S(I, x^f, x^e)$, the only subgame perfect equilibrium outcome is w_d; *i.e.*, (i) $E(\Gamma, R, w_d) \neq \emptyset$, and (ii) $\theta \in E(\Gamma, R, w_d)$ if and only if $\gamma(\theta) = w_d$. The SOCR is **implementable** in subgame perfect equilibrium when the jury configuration is $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ if there exists an extensive form mechanism that implements it.

We say that a jury configuration is minimally impartial if, for each pair of contestants, there is at least one juror who is impartial with respect to them.

Definition 2 *A jury configuration $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ is **minimally impartial** if, for each $xy \in 2^N_2$, there is some $j \in J$ with $xy \in I_j$.*

Minimal impartiality is a necessary condition for the implementability of the SOCR in any equilibrium concept, including subgame perfect equilibrium.

Proposition 1 *Suppose that the SOCR is implementable in subgame perfect equilibrium when the jury configuration is $(I, x^f, x^e) \in \mathcal{E}^{|J|}$. Then, (I, x^f, x^e) is minimally impartial.*

Proposition 1 can be deduced as a corollary from Amorós (2013, Proposition 1) and we include the proof in the Appendix for completeness. The intuition of this result is simple. If there is a pair of contestants with respect to whom no juror is impartial, then the preferences of the jurors (and therefore the contestant chosen by them) might not change with the deserving winner (regardless of what is the equilibrium concept considered). From now on, we only consider jury configurations that are minimally impartial. It turns out that this condition is also sufficient for the implementability of the SOCR in subgame perfect equilibrium, at least three jurors.

Proposition 2 *Suppose that there are at least three jurors. Let* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ *be a minimally impartial jury configuration. Then, the SOCR is implementable in subgame perfect equilibrium when the jury configuration is* (I, x^f, x^e).

Proposition 2 follows as a corollary from Amorós (2013, Proposition 2) and we include the proof in the Appendix. The idea behind this result is that, if for each pair of contestants there is at least one juror who is impartial with respect to them, then the SOCR satisfies essential monotonicity, a sufficient condition for Nash implementation when there are at least three agents (see Danilov, 1992). Essential monotonicity is also a sufficient condition for subgame perfect implementation, since the mechanism à la Maskin (Maskin, 1999) proposed by Danilov (1992) to prove his result is a one-shot-mechanism (and therefore a profile of messages is a Nash equilibrium if and only if it is a subgame perfect equilibria). This type of mechanisms, however, have received criticism for being unnatural, having too complex message spaces, and making use of extraneous devices such as integer games or modulo games (see Jackson, 1992).

In this paper, we are interested in implementing the SOCR through simple and natural mechanisms. Some of the simplest mechnaisms in this setting are those where jurors only have to announce who they think should win the competition. Since we are interested in extensive form mechanisms, we will consider mechanisms in which the jurors take turns when announcing the contestant they think should win the competition. We call these mechanisms straightforward-extensive-form mechanisms.

Definition 3 *An extensive form mechanism* $\Gamma \equiv (\Theta, \gamma)$ *implementing the SOCR in subgame perfect equilibrium is* ***straightforward*** *if:*

(1) it consists of $|J|$ *stages,*

(2) at each stage one different juror moves,

(3) each juror knows the movements of all jurors who precede him, and
(4) the set of possible choices available for each juror is N.

Figure 1 shows an example of a straightforward-extensive-form mechanism for the case in which $N = \{a, b, c\}$ and $J = \{1, 2, 3\}$. By abuse of notation, for every straightforward extensive form mechanism $\Gamma \equiv (\Theta, \gamma)$ and every $x, y, z \in N$, let $\gamma(x, y, z)$ denote the contestant selected by Γ when the jurors moving at the first, second, and third stages choose x, y, and z, respectively. For example, in the mechanism represented in Figure 1, $\gamma(b, a, b) = b$, and $\gamma(c, a, a) = a$. By abusing the notation again, let $\Gamma(.)$ denote the initial node of mechanism Γ. For every $x \in N$, let $\Gamma(x, .)$ denote the node at the second stage of mechanism Γ that is reached after the juror moving at the first stage chose x. Similarly, for every $x, y \in N$, $\Gamma(x, y, .)$ denotes the node at the third stage of mechanism Γ that is reached after the juror moving at the first stage chose x and the juror moving at the second stage chose y, etc.

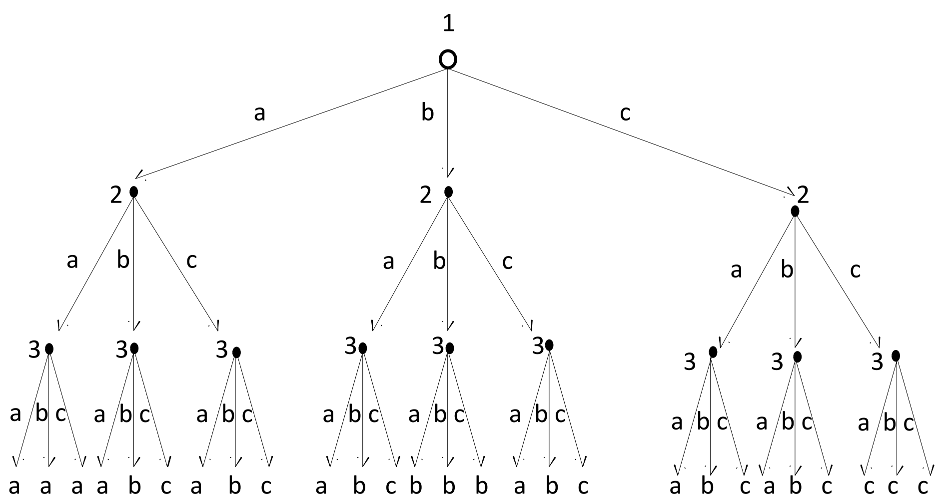

Figure 1 Example of straightforward-extensive-form mechanism.

Since, in a straightforward mechanism, we are asking each juror to reveal who she thinks should win the competition, it is natural to require that telling the truth (announcing the deserving winner) is always part of some profile of subgame perfect equilibrium strategies.

Definition 4 *We say that a straightforward-extensive-form mechanism* $\Gamma \equiv (\Theta, \gamma)$ ***naturally implements*** *the SOCR in subgame perfect equilibrium when the jury configuration is* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ *if:*

(1) Γ *implements the SOCR in subgame perfect equilibrium when the jury configuration is* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$*, and*

(2) for each $(R, w_d) \in S(I, x^f, x^e)$*, there exists* $\theta \in E(\Gamma, R, w_d)$ *such that the jurors moving at nodes* $\Gamma(.)$*,* $\Gamma(w_d, .)$*,* $\Gamma(w_d, w_d, .)$*, etc., choose* w_d*.*[1]

The SOCR is **naturally implementable** in subgame perfect equilibrium when the jury configuration is (I, x^f, x^e) if there exists a straightforward-extensive-form mechanism that naturally implements it.

3 The results

Our first result shows that the necessary and sufficient condition for the implementation of the SOCR stated in Propositions 1 and 2 is not sufficient for its natural implementation: knowing, for each possible pair of contestants, at least one juror who is impartial with respect to them does not guarantee that the SOCR is natural implementable in subgame perfect equilibrium.

Proposition 3 *Suppose that there are at least three jurors. Minimal impartiality of the jury configuration* (I, x^f, x^e) *is a necessary but not sufficient condition for the natural implementation of the SOCR in subgame perfect equilibrium, at least in the three contestants case.*

Proof. That minimal impartiality is a necessary condition follows immediately from Proposition 1 and the fact that natural implementation implies implementation. Suppose now that there are three contestants, $N = \{a, b, c\}$, and that, for each pair of contestants, there is at least one juror who is impartial with respect to them. Suppose, without loss of generality, that there are three jurors, $J = \{1, 2, 3\}$, and (I, x^f, x^e) is such that $I_1 = \{bc\}$, $I_2 = \{ac\}$, $I_3 = \{ab\}$, and, for each $j \in J$, $x_j^f = \emptyset$ and $x_j^e = \emptyset$ (so that minimal impartiality is satisfied). Then, any profile of preference functions $R \in \mathcal{R}^{|J|}$ satisfying $b\ P_1(b)\ c$, $c\ P_1(c)\ b$, $a\ P_2(a)\ c$, $c\ P_2(c)\ a$, $a\ P_3(a)\ b$, and $b\ P_3(b)\ a$ is

[1] Note that, if Γ naturally implements the SOCR then, if all jurors choose the same contestant $x \in N$, the alternative selected by Γ must be x (i.e., for every $x \in N$, $\gamma(x, x, x) = x$).

admissible when the jury configuration is (I, x^f, x^e) (see Table 3). Suppose, by contradiction, that there is a straightforward-extensive-form mechanism $\Gamma \equiv (\Theta, \gamma)$ that naturally implements the SOCR in subgame perfect equilibrium when the jury configuration is (I, x^f, x^e). Suppose, without loss of generality, that Γ is such that juror 1 moves at the first stage, juror 2 moves at the second stage, and juror 3 moves at the third stage.

R_1			R_2			R_3		
a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	b	c	a	$\vdots$	c	a	b	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	c	b	c	$\vdots$	a	b	a	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Table 3 Admissible preference functions when no juror has a friend or enemy.

Claim 1. For every $x \in N$, we have $\gamma(c, c, x) = c$, $\gamma(c, a, x) \neq b$, and $\gamma(c, b, x) \neq b$.

Suppose by contradiction that $\gamma(c, c, x) \neq c$ for some $x \in N$. Note that there exists a preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ such that c is the unique worst alternative when $w_d = c$. Therefore, there exists $(R, c) \in S(I, x^f, x^e)$ such that $\gamma(c, c, x)\ P_3(c)\ c$. Hence, any profile of equilibrium strategies at state (R, c), $\theta \in E(\Gamma, R, c)$, must be such that, if jurors 1 and 2 choose c at the first and second stage, respectively, then juror 3 does not choose c at the third stage. This contradicts that Γ naturally implements the SOCR.

Suppose now that, for some $x \in N$, either $\gamma(c, a, x) = b$ or $\gamma(c, b, x) = b$. Note that there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that b is the unique best alternative when $w_d = c$. Similarly, there exists a preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ such that b is the unique best alternative when $w_d = c$. Therefore, there exists $(R, c) \in S(I, x^f, x^e)$ such that either (i) $\gamma(c, a, x)\ P_2(c)\ y$ and $\gamma(c, a, x)\ P_3(c)\ y$ for every $y \in N$, or (ii) $\gamma(c, b, x)\ P_2(c)\ y$ and $\gamma(c, b, x)\ P_3(c)\ y$ for every $y \in N$. Hence, since $\gamma(c, c, y) = c$ for every $y \in N$, any profile of equilibrium strategies at state (R, c), $\theta \in E(\Gamma, R, c)$, must be such that, if juror 1 chooses c at the first stage, then juror 2 chooses either a or b at the second stage (and juror 3 chooses x at the third stage). This contradicts that Γ naturally implements the SOCR.

Claim 2. Either $\gamma(c,a,x)=a$ for every $x\in N$, or $\gamma(c,b,x)=a$ for every $x\in N$.

Suppose by contradiction that (i) $\gamma(c,a,x)\neq a$ for some $x\in N$ and (ii) $\gamma(c,b,y)\neq a$ for some $y\in N$. Then, by Claim 1, $\gamma(c,a,x)=c$ for some $x\in N$ and $\gamma(c,b,y)=c$ for some $y\in N$. Note that there exists a preference function $R_3\in\mathcal{R}(I_3,x_3^f,x_3^e)$ such that $c\ P_3(a)\ a$. Similarly, there exists a preference function $R_1\in\mathcal{R}(I_1,x_1^f,x_1^e)$ such that a is the unique worst alternative when $w_d=a$. Let $(R,a)\in S(I,x^f,x^e)$ be an admissible state where the preference functions of jurors 3 and 1 are as just described. Then, there is no $\theta\in E(\Gamma,R,a)$ such that jurors 1, 2, and 3 choose a at the first, second, and third stage, respectively. The reason is that, if juror 1 chooses c at the first stage, then the movements of jurors 2 and 3 at the second and third stages will never result in a, which is the less preferred alternative for 1 (if juror 2 chooses c then, by Claim 1, the winning contestant will be c, no matter what juror 3 chooses; if juror 2 chooses a then juror 3 will not choose any movement resulting in a, since she can choose x and make c winning the competition, which she likes most; if juror 2 chooses b then juror 3 will not choose any movement resulting in a, since she can choose y and make c winning the competition). This contradicts that Γ naturally implements the SOCR.

Claim 3. $\gamma(c,a,x)\neq a$ for some $x\in N$ and $\gamma(c,b,y)\neq a$ for some $y\in N$.

Suppose by contradiction that either $\gamma(c,a,x)=a$ for every $x\in N$, or $\gamma(c,b,x)=a$ for every $x\in N$. Note that there exists a preference function $R_1\in\mathcal{R}(I_1,x_1^f,x_1^e)$ such that a is the unique best alternative when $w_d=b$. Similarly, there exists a preference function $R_2\in\mathcal{R}(I_2,x_2^f,x_2^e)$ such that a is the unique best alternative when $w_d=a$. Let $(R,b)\in S(I,x^f,x^e)$ be an admissible state where the preference functions of jurors 1 and 2 are as just described. Then, there is no $\theta\in E(\Gamma,R,b)$ such that jurors 1, 2, and 3 choose b at the first, second, and third stage, respectively. The reason is that, if juror 1 chooses c at the first stage, then the movements of jurors 2 and 3 at the second and third stages will result in a, which is the most preferred alternative for 1 (if juror 1 chooses c, then juror 2 will chose either a, if $\gamma(c,a,x)=a$ for every $x\in N$, or b, if $\gamma(c,b,x)=a$ for every $x\in N$, and then the winning contestant will be a, no matter what juror 3 chooses). This contradicts that Γ naturally implements the SOCR.

Claim 3 contradicts Claim 2, which completes the proof. ■

In order to naturally implement the SOCR in subgame perfect equilibrium, the planner needs to be sure that the jury configuration is minimally impartial. However, if this is the only information that the planner has about the jury configuration, then the SOCR cannot be naturally implemented in subgame perfect equilibrium.

Sometimes the planner knows that some jurors have friends or enemies. Knowing that a juror has a friend or an enemy may facilitate the natural implementation of the SOCR, as it reduces the set of admissible states. For example, if all the information the planner has about juror 3 is that she is impartial with respect to the pair ab, then there are admissible states where contestant c is a friend of 3, admissible states where c is an enemy of 3, etc. The more information the planner has about the jurors' preference functions, the smaller is the set of admissible states and the more likely is that the SOCR is naturally implementable.

It turns out that, knowing that one juror has a friend or an enemy (in addition to the requirement of minimal impartiality) is sufficient to guarantee the natural implementation in subgame perfect equilibrium of the SOCR, at least in the three contestants case. To prove these results, we make the following simplifying assumption:

Assumption 1
(i) $N = \{a, b, c\}$ *(there are three contestants),*
(ii) $J = \{1, 2, 3\}$ *(there are three jurors), and*
(iii) every jury configuration (I, x^f, x^e) *is such that* $I_1 = \{bc\}$, $I_2 = \{ac\}$, *and* $I_3 = \{ab\}$ *(and therefore* (I, x^f, x^e) *is minimally impartial).*[2]

First, we show that, under Assumption 1, knowing that one of the jurors has a friend is a sufficient condition for the natural implementation of the SOCR in subgame perfect equilibrium. Suppose, for example, that the planner knows that c is a friend of juror 3. Then mechanism Γ_3^f represented in

[2]Of course, having only three contestants is a simplification and this case should be considered as a first approach to the problem. This assumption give us a symmetric model in which each contestant can be identified with a juror who could be her friend or her enemy. If there are more than three contestants, there would be many different ways to fulfill the necessary condition of minimal impartiality, and many of them would not be symmetrical. The reason why we assume that there are three jurors is that, as we have seen in Proposition 2, minimal impartiality is a sufficient condition for subgame perfect implementation when there are at least three jurors. We could have more than three jurors, but this would constitute an unnecessary complication of the model.

Figure 2 naturally implements the SOCR in subgame perfect equilibrium. In this mechanism, the first juror to speak is the juror with the known friend. If 3 does not announce her friend (contestant c) then the contestant announced by 3 is chosen. If 3 announces her friend c, then we proceed as follows: (i) if the second juror to speak (say juror 1) does not agree with 3, then b (the contestant that may be a friend or an enemy of the last juror to speak) is chosen; (ii) if the second juror to speak also announces c but the last juror to speak does not agree with her two predecessors, then a (the contestant that may be a friend or an enemy of the second juror to speak) is chosen.

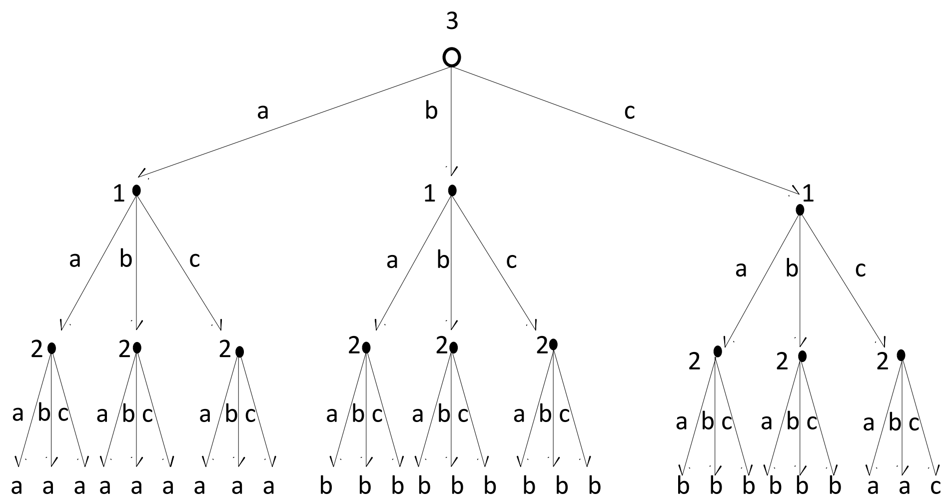

Figure 2 Mechanism Γ_3^f in Proposition 4.

Proposition 4 *Suppose that Assumption 1 holds. Suppose that the jury configuration (I, x^f, x^e) is such that at least one juror (say juror 3) has a known friend. Then, mechanism Γ_3^f naturally implements the SOCR in subgame perfect equilibrium when the jury configuration is (I, x^f, x^e).*

Proof. Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying the conditions of the statement. Then $I_1 = \{bc\}$, $I_2 = \{ac\}$, $I_3 = \{ab\}$, and $x_3^f = c$. Suppose, without loss of generality, that $x_1^f = x_1^e = \emptyset$, and $x_2^f = x_2^e = \emptyset$.[3] Table 4 shows the preference functions that are admissible in this case.

[3]Note that, of all jury configurations satisfying $bc \in I_1$, $ac \in I_2$, $ab \in I_3$, and $x_3^f = c$, this is the one with the largest set of admissible states, and therefore this is the case where implementation is more difficult. In other words, if Γ_3^f works with this jury configuration, it also works with any other jury configuration satisfying $bc \in I_1$, $ac \in I_2$, $ab \in I_3$, and $x_3^f = c$.

R_1			R_2			R_3		
a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	c	c	c
$\vdots$	b	c	a	$\vdots$	c	a	b	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	b	a	$\vdots$
$\vdots$	c	b	c	$\vdots$	a			
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$			

Table 4 Admissible preference functions when only juror 3 has a known friend.

For every $x \in N$, consider any admissible state where the deserving winner is x, $(R, x) \in S(I, x^f, x^e)$, and any profile of subgame perfect equilibrium strategies of mechanism Γ_3^f at state (R, x), $\theta \in E(\Gamma, R, x)$. Note that θ is such that:

(1) in nodes $\Gamma(a, a, .)$, $\Gamma(a, b, .)$, and $\Gamma(a, c, .)$ juror 2 chooses either a, or b, or c, since all of them result in a,

(2) in nodes $\Gamma(b, a, .)$, $\Gamma(b, b, .)$, and $\Gamma(b, c, .)$ juror 2 chooses either a, or b, or c, since all of them result in b, and

(3) in nodes $\Gamma(c, a, .)$, and $\Gamma(c, b, .)$ juror 2 chooses either a, or b, or c, since all of them result in b.

From (1), θ is such that:

(4) in node $\Gamma(a, .)$ juror 1 chooses either a, or b, or c, since all of them result in a, and

From (2), θ is such that:

(5) in node $\Gamma(b, .)$ juror 1 chooses either a, or b, or c, since all of them result in b.

Case 1. $w_d = a$.

Consider any state where $w_d = a$, $(R, a) \in S(I, x^f, x^e)$, and any profile of subgame perfect equilibrium strategies of mechanism Γ^f at state (R, a), $\theta \in E(\Gamma, R, a)$. Any preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ is such that a $P_2(a)$ c. Then, θ must be such that:

(1.1) in node $\Gamma(c, c, .)$ juror 2 chooses either a or b, which results in a.

The fact that $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ does not impose any restriction over the preference relation $R_1(a)$. Therefore, from (3) and (1.1), θ is such that:

(1.2) in node $\Gamma(c, .)$ juror 1 may choose a or b (which results in b), or c (which results in a).

Any preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that $c\ P_3(a)\ a\ P_3(a)\ b$. Therefore, from (4), (5), and (1.2),, θ must be such that:

(1.3) if 1 chooses a or b in $\Gamma(c,.)$, then 3 chooses a in $\Gamma(.)$, which results in a, and

(1.4) if 1 chooses c in $\Gamma(c,.)$, then 3 may choose a or c in $\Gamma(.)$, which results in a.

Therefore, for each admissible state where the deserving winner is a, $(R, a) \in S(I, x^f, x^e)$: (i) the only subgame perfect equilibrium outcome is a, and (ii) there exists $\theta \in E(\Gamma, R, a)$ such that the jurors moving at nodes $\Gamma(.)$, $\Gamma(a,.)$, and $\Gamma(a,a,.)$ choose a.

Case 2. $w_d = b$.

Consider any state where $w_d = b$, $(R, b) \in S(I, x^f, x^e)$, and any profile of subgame perfect equilibrium strategies of mechanism Γ^f at state (R, b), $\theta \in E(\Gamma, R, b)$. The fact that $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ does not impose any restriction over the preference relation $R_2(b)$. Therefore, θ is such that:

(2.1) in node $\Gamma(c,c,.)$, juror 2 may choose a or b (which results in a), or c (which results in c).

The fact that $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ does not impose any restriction on the relative ranking between a and c in $R_1(b)$. Therefore, from (3) and (2.1):

(2.2) in node $\Gamma(c,.)$, juror 1 may choose a or b (which results in b), or c (which results in a).

Any preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that $c\ P_3(b)\ b\ P_3(b)\ a$. Therefore, from (4), (5), and (2.2), θ is such that:

(2.3) if 1 chooses a or b in node $\Gamma(c,.)$, then 3 may choose b or c in node $\Gamma(.)$, which results in b, and

(2.4) if 1 chooses c in node $\Gamma(c,.)$, then 3 chooses b in node $\Gamma(c,.)$, which results in b.

Therefore, for each admissible state where the deserving winner is b, $(R, b) \in S(I, x^f, x^e)$: (i) the only subgame perfect equilibrium outcome is b, and (ii) there exists $\theta \in E(\Gamma, R, b)$ such that the jurors moving at nodes $\Gamma(.)$, $\Gamma(b,.)$, and $\Gamma(b,b,.)$ choose b.

Case 3. $w_d = c$.

Consider any state where $w_d = c$, $(R, c) \in S(I, x^f, x^e)$, and any profile of subgame perfect equilibrium strategies of mechanism Γ^f at state (R, c), $\theta \in E(\Gamma, R, c)$. Any preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ is such that c $P_2(c)\ a$. Then, θ must be such that:

(3.1) in node $\Gamma(c,c,.)$ juror 2 chooses c, which results in c.

Any preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ is such that $c\ P_1(c)\ b$. Therefore, from (3) and (3.1):

(3.2) in node $\Gamma(c,.)$ juror 1 chooses c, which results in c.

Any preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that $c\ P_3(c)\ a$ and $c\ P_3(c)\ b$. Therefore, from (4), (5), and (3.2), θ must be such that:

(3.3) 3 chooses c at node $\Gamma(.)$.

Therefore, for each admissible state where the deserving winner is c, $(R, c) \in S(I, x^f, x^e)$: (i) the only subgame perfect equilibrium outcome is c, and (ii) there exists $\theta \in E(\Gamma, R, c)$ such that the jurors moving at nodes $\Gamma(.)$, $\Gamma(c,.)$, and $\Gamma(c,c,.)$ choose c. ■

In mechanism Γ_3^f the first juror to speak is the juror with a known friend. This is not by chance: if only one juror has a known friend, any straightforward mechanism naturally implementing the SOCR in subgame perfect equilibrium is such that this juror moves at the first stage. The proof of this result is in the Appendix.

Remark 1 *Suppose that Assumption 1 is satisfied. Let (I, x^f, x^e) be a jury configuration such that only one juror has a known friend (and no juror has a known enemy). Then, any straightforward mechanism naturally implementing the SOCR in subgame perfect equilibrium when the jury configuration is (I, x^f, x^e) is such that the juror with the known friend moves at the first stage.*

Next, we study the case in which the jurors have enemies. As in the friends case, under Assumption 1, knowing that one of the jurors has an enemy is sufficient to guarantee that the SOCR can be implemented in subgame perfect equilibrium. Suppose, without loss of generality, that c is a known enemy of juror 3. Then, mechanism Γ_3^e represented in Figure 3 implements the SOCR. The first juror to speak is the juror with the known enemy, and the mechanism consists of three simple rules: (1) if all jurors announce the same contestant, then that contestant is chosen; (2) if all jurors but one announce x and there is one dissident announcing $y \neq x$, then y is chosen only if the dissident is impartial with respect the pair xy, while x is chosen otherwise; (3) if more than two jurors disagree on their announcements then the known enemy of juror 3 (contestant c) is chosen.

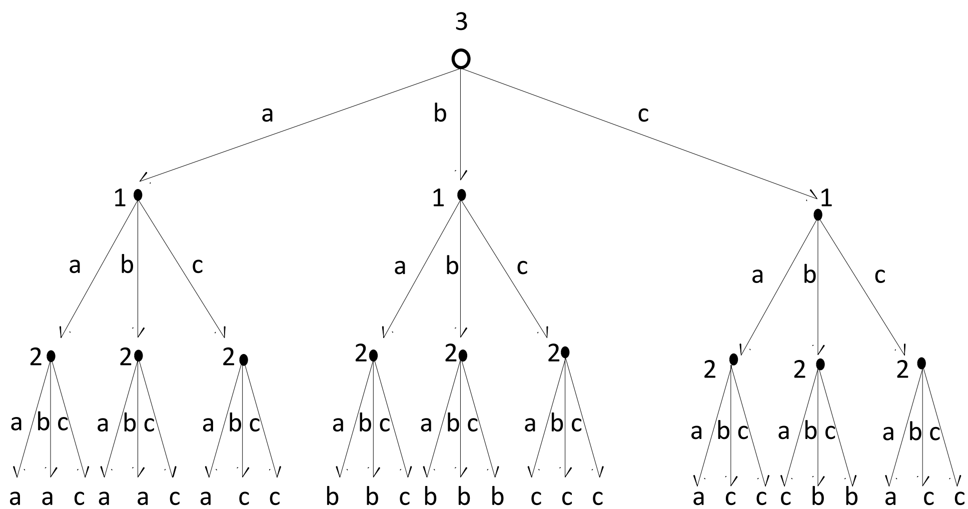

Figure 3 Mechanism Γ_3^e in Proposition 5.

Proposition 5 *Suppose that Assumption 1 holds. Suppose that the jury configuration (I, x^f, x^e) is such that at least one juror (say juror 3) has a known enemy. Then, mechanism Γ_3^e naturally implements the SOCR in subgame perfect equilibrium when the jury configuration is (I, x^f, x^e).*

Proof. Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying the conditions of the statement. Then $I_1 = \{bc\}$, $I_2 = \{ac\}$, $I_3 = \{ab\}$, and $x_3^e = c$. Suppose, without loss of generality, that $x_1^f = x_1^e = \emptyset$, and $x_2^f = x_2^e = \emptyset$.[4] Table 5 shows the preference functions that are admissible in this case.

R_1			R_2			R_3		
a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
⋮	⋮	⋮	⋮	⋮	⋮	a	b	⋮
⋮	b	c	a	⋮	c	b	a	⋮
⋮	⋮	⋮	⋮	⋮	⋮	c	c	c
⋮	c	b	c	⋮	a			
⋮	⋮	⋮	⋮	⋮	⋮			

Table 5 Admissible preference functions when only juror 3 has a known enemy.

[4] Note that, if Γ_3^e works with this jury configuration, it also works with any other jury configuration satisfying $bc \in I_1$, $ac \in I_2$, $ab \in I_3$, and $x_3^e = c$.

Case 1. $w_d = a$.

Consider any state where $w_d = a$, $(R, a) \in S(I, x^f, x^e)$, and any profile of subgame perfect equilibrium strategies of mechanism Γ^f at state (R, a), $\theta \in E(\Gamma, R, a)$. Any preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ is such that a $P_2(a)$ c. However, the fact that $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ does not impose any restriction on the relative ranking between b and c in $R_2(a)$. Then, θ is such that:

(1.1) in node $\Gamma(a, a, .)$ juror 2 chooses either a or b, which results in a,

(1.2) in node $\Gamma(a, b, .)$ juror 2 chooses either a or b, which results in a,

(1.3) in node $\Gamma(a, c, .)$ juror 2 chooses a, which results in a,

(1.4) in node $\Gamma(b, a, .)$ juror 2 may choose a or b (which results in b), or c (which results in c),

(1.5) in node $\Gamma(b, b, .)$ juror 2 chooses either a, or b, or c, since all of them result in b,

(1.6) in node $\Gamma(b, c, .)$ juror 2 chooses either a, or b, or c, since all of them result in c,

(1.7) in node $\Gamma(c, a, .)$ juror 2 chooses a, which results in a,

(1.8) in node $\Gamma(c, b, .)$ juror 2 may choose a (which results in c), or b or c (which results in b), and

(1.9) in node $\Gamma(c, c, .)$ juror 2 chooses a, which results in a.

The fact that a preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ does not impose any restriction on the preference relation $R_1(a)$. Then, from (1.1)-(1.9), θ is such that:

(1.10) in node $\Gamma(a, .)$ juror 1 may choose a, b, or c, which results in a,

(1.11) in node $\Gamma(b, .)$ juror 1 may choose a (which may result in b or c), b (which results in b), or c (which results in c), and

(1.12) in node $\Gamma(c, .)$ juror 1 may choose a (which results in a), b (which may result in c or b), or c (which results in a).

Any preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that a $P_3(a)$ b $P_3(a)$ c. Then, from (1.1)-(1.12), θ must be such that, either (i) in the initial node $\Gamma(.)$ juror 3 chooses a (which results in a), or (ii) in the initial node $\Gamma(.)$ juror 3 chooses c and in node $\Gamma(c, .)$ juror 1 chooses a or c (which results in a). Therefore, for each admissible state where the deserving winner is a, $(R, a) \in S(I, x^f, x^e)$: (i) the only subgame perfect equilibrium outcome is a, and (ii) there exists $\theta \in E(\Gamma, R, a)$ such that the jurors moving at nodes $\Gamma(.)$, $\Gamma(a, .)$, and $\Gamma(a, a, .)$ choose a.

Case 2. $w_d = b$.

Consider any state where $w_d = b$, $(R, b) \in S(I, x^f, x^e)$, and any profile of subgame perfect equilibrium strategies of mechanism Γ^f at state (R, b), $\theta \in E(\Gamma, R, b)$. The fact that $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ does not impose any restriction over the preference relation $R_2(b)$. Therefore, θ must be such that:

(2.1) in node $\Gamma(a, a, .)$ juror 2 may choose a or b (which results in a), or c (which results in c),

(2.2) in node $\Gamma(a, b, .)$ juror 2 may choose a or b (which results in a), or c (which results in c),

(2.3) in node $\Gamma(a, c, .)$ juror 2 may choose b or c (which results in c), or a (which results in a),

(2.4) in node $\Gamma(b, a, .)$ juror 2 may choose a or b (which results in b), or c (which results in c),

(2.5) in node $\Gamma(b, b, .)$ juror 2 may choose a, b, or c, since all of them result in b,

(2.6) in node $\Gamma(b, c, .)$ juror 2 may choose a, b, or c, since all of them result in c,

(2.7) in node $\Gamma(c, a, .)$ juror 2 may choose b or c (which results in c), or a (which results in a),

(2.8) in node $\Gamma(c, b, .)$ juror 2 may choose b or c (which results in b), or a (which results in c), and

(2.9) in node $\Gamma(c, c, .)$ juror 2 may choose b or c (which results in c), or a (which results in a).

Any preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ is such that $b \ P_1(b) \ c$. However, the fact that $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ does not impose any restriction on the relative ranking between a and c in $R_1(b)$. Then, from (2.1)-(2.9), θ is such that:

(2.10) in node $\Gamma(a, .)$ juror 1 may choose a (which may result in a or c), b (which may result in a or c), or c (which may result in a or c),

(2.11) in node $\Gamma(b, .)$ juror 1 may choose a (if 2 chooses a or b in $\Gamma(b, a, .)$, which results in b), or b (which results in b), and

(2.12) in node $\Gamma(c, .)$ juror 1 may choose a (which may result in a or c), b (which may result in c or b), or c (which may result in a or c).

Any preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that $b \ P_3(b) \ a \ P_3(b) \ c$. Then, from (2.1)-(2.12), θ must be such that, either (i) in the initial node $\Gamma(.)$ juror 3 chooses b (which results in b), or (ii) in the initial node $\Gamma(.)$ juror 3 chooses c, in node $\Gamma(c, .)$ juror 1 chooses b, and in node $\Gamma(c, b, .)$ juror 2 chooses b or c (which results in b). Therefore, for each admissible state where the deserving winner is b, $(R, b) \in S(I, x^f, x^e)$: (i) the only subgame perfect equilibrium outcome is b, and (ii) there exists $\theta \in E(\Gamma, R, b)$ such that the jurors moving at nodes $\Gamma(.)$, $\Gamma(b,.)$, and $\Gamma(b,b,.)$ choose b.

Case 3. $w_d = c$.

Consider any state where $w_d = c$, $(R,c) \in S(I, x^f, x^e)$, and any profile of subgame perfect equilibrium strategies of mechanism Γ^f at state (R,c), $\theta \in E(\Gamma, R, c)$. Any preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ is such that c $P_2(c)$ a. However, the fact that $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ does not impose any restriction on the relative ranking between b and c in $R_2(c)$. Then, θ is such that:

(3.1) in node $\Gamma(a,a,.)$ juror 2 chooses c, which results in c,

(3.2) in node $\Gamma(a,b,.)$ juror 2 chooses c, which results in c,

(3.3) in node $\Gamma(a,c,.)$ juror 2 chooses either b or c, which results in c,

(3.4) in node $\Gamma(b,a,.)$ juror 2 may choose a or b (which results in b), or c (which results in c),

(3.5) in node $\Gamma(b,b,.)$ juror 2 chooses either a, or b, or c, since all of them result in b,

(3.6) in node $\Gamma(b,c,.)$ juror 2 chooses either a, or b, or c, since all of them result in c,

(3.7) in node $\Gamma(c,a,.)$ juror 2 chooses either b or c, which results in c,

(3.8) in node $\Gamma(c,b,.)$ juror 2 may choose a (which results in c), or b or c (which results in b), and

(3.9) in node $\Gamma(c,c,.)$ juror 2 chooses either b or c, which results in c.

Any preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ is such that c $P_1(c)$ b. However, the fact that $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ does not impose any restriction on the relative ranking between a and c in $R_1(c)$. Then, from (3.1)-(3.9), θ is such that:

(3.10) in node $\Gamma(a,.)$ juror 1 may choose a (which results in c), b (which results in c), or c (which results in c),

(3.11) in node $\Gamma(b,.)$ juror 1 may choose a (if 2 chooses c in $\Gamma(b,a,.)$, which results in c), or c (which results in c), and

(3.12) in node $\Gamma(c,.)$ juror 1 may choose a (which results in c), b (if 2 chooses a in $\Gamma(c,b,.)$, which results in c), or c (which results in c).

From (3.1)-(3.12), θ is such that in the initial node $\Gamma(.)$ juror 3 may choose a (which results in c), b (which results in c), or c (which results in c). Note that, for each admissible state where the deserving winner is c, $(R,c) \in S(I, x^f, x^e)$: (i) the only subgame perfect equilibrium outcome is c, and (ii) there exists $\theta \in E(\Gamma, R, c)$ such that the jurors moving at nodes $\Gamma(.)$, $\Gamma(c,.)$, and $\Gamma(c,c,.)$ choose c. ■

Similarly to what happens in the friends case, in mechanism Γ_3^e, the first juror to speak is the juror with a known enemy. If only one juror has a known enemy, there is no straightforward mechanism naturally implementing the SOCR in subgame perfect equilibrium where the only juror with a known enemy does not move at the first stage. The proof of this remark is in the Appendix.

Remark 2 *Suppose that Assumption 1 is satisfied. Let (I, x^f, x^e) be a jury configuration such that only one juror has a known enemy (and no juror has a known friend). Then, any straightforward mechanism naturally implementing the SOCR in subgame perfect equilibrium when the jury configuration is (I, x^f, x^e) is such that the juror with the known enemy moves at the first stage.*

4 Conclusion

We have analyzed the problem of designing extensive form mechanisms that give the jurors the right incentives to always choose the deserving winner of a contest when the solution concept is subgame perfect equilibrium. We have restricted ourselves to what can be considered "natural" mechanisms in this setting. These mechanisms are those which satisfy two conditions: (1) the jurors take turns to announce the contestant they think should win the competition and (2) telling the truth (announcing the deserving winner) is always part of a profile equilibrium strategies. The implementation of the deserving winner through this type of mechanisms is called natural implementation. A necessary condition for the implementation of the deserving winner is that, for each possible pair of contestants, there is at least one juror who is impartial with respect to them. This condition (that we call minimal impartiality) is also sufficient for the implementation of the deserving winner in subgame perfect equilibrium if we do not impose any restriction on the mechanisms. Minimal impartiality of the jury, however, does not guarantee natural implementation in this equilibrium concept. In order to naturally implement the deserving winner in subgame perfect equilibrium, the planner needs to know if some jurors have friends or enemies among the contestants. The reason is that, knowing that a juror wants to favor or harm a contestant reduces the size of the set of admissible preferences for that juror, which facilitates implementation. We have shown that, in the three contestants case, it is sufficient that the planner knows that one of the jurors has a friend or an enemy to naturally implement the deserving winner in subgame perfect equilibrium. To prove these results we have proposed two straightforward mechanisms, one for the case in which there is one juror with a friend and another for the case in which there is one juror with an enemy.

Appendix

PROOF OF PROPOSITION 1:

Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$. Suppose that the SOCR is implementable in subgame perfect equilibrium when the jury configuration is (I, x^f, x^e) by means of a mechanism $\Gamma = (\Theta, \gamma)$. Suppose by contradiction and without loss of generality that for some pair $xy \in 2_2^N$ and for every $j \in J$, $xy \notin I_j$. It can be shown that then there exists a profile of preference functions $R = (R_j)_{j \in J}$ such that, for each $j \in J$, (i) $R_j \in \mathcal{R}(I_j, x_j^f, x_j^e)$ and (ii) $R_j(x) = R_j(y)$. Note that, by (i) $(R, x), (R, y) \in S(I, x^f, x^e)$, and by (ii) $E(\Gamma, R, x) = E(\Gamma, R, y)$. Since Γ implements the SOCR in subgame perfect equilibrium, there is $\theta \in E(\Gamma, R, x)$ such that $\gamma(\theta) = x$. But then, $\theta \in E(\Gamma, R, y)$ and $\gamma(m) \neq y$, which contradicts that Γ implements the SOCR in subgame perfect equilibrium. ■

PROOF OF PROPOSITION 2:

We first define essential monotonicity, a sufficient condition for Nash (and subgame perfect) implementation when there are at least three agents (see Danilov, 1992).

Definition 5 *The SOCR is **essentially monotonic** when the jury configuration is (I, x^f, x^e) if, for all $(R, w_d), (\hat{R}, \hat{w}_d) \in S(I, x^f, x^e)$, if $w_d \neq \hat{w}_d$, then there exist $j \in J$ and $\bar{w} \in N$ such that:*

(i) $w_d \ R_j(w_d) \ \bar{w}$ and $\bar{w} \ \hat{P}_j(\hat{w}_d) \ w_d$, and

(ii) there exist $(\tilde{R}, \tilde{w}) \in S(I, x^f, x^e)$ such that, for all $w \in N$, if $w \ P_j(w_d) \ w_d$ then $w \ \tilde{P}_j(\tilde{w}) \ \bar{w}$.

Next we show that, if (I, x^f, x^e) is minimally impartial then the SOCR is essentially monotonic. Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be such that, for every $xy \in 2_2^N$, there is some $j \in J$ such that $xy \in I_j$. Let $(R, w_d), (\hat{R}, \hat{w}_d) \in S(I, x^f, x^e)$ be such that $w_d \neq \hat{w}_d$. Let $j \in J$ be such that $w_d \hat{w}_d \in I_j$. Then, $w_d \ P_j(w_d) \ \hat{w}_d$ and $\hat{w}_d \ \hat{P}_j(\hat{w}_d) \ w_d$, and therefore point (i) of the definition of essential monotonicity is fulfilled for $\bar{w} = \hat{w}_d$.

Let $\tilde{R}_j \in \mathcal{R}(I_j, x_j^f, x_j^e)$ be such that, for all $w \in N$, if $w \ P_j(w_d) \ w_d$ then $w \ \tilde{P}_j(w_d) \ \hat{w}_d$. To see that such a preference function exists, note that (1) $w_d \neq \hat{w}_d$, (2) $\hat{w}_d \neq x_j^f$ (since $w_d \ P_j(w_d) \ \hat{w}_d$ and $R_j \in \mathcal{R}(I_j, x_j^f, x_j^e)$), and (3) if $w \ P_j(w_d) \ w_d$, then (3.1) $w \neq w_d$, (3.2) $w \neq \hat{w}_d$ (since $w_d \ P_j(w_d) \ \hat{w}_d$), and (3.3) $w \neq x_j^e$ (since $R_j \in \mathcal{R}(I_j, x_j^f, x_j^e)$). Then, point (ii) of the definition of essential monotonicity is fulfilled for that $\tilde{R}_j$, any $(\tilde{R}_k)_{k\in J\setminus\{j\}} \in \times_{k\in J\setminus\{j\}}\mathcal{R}(I_k, x_k^f, x_k^e)$, and $\tilde{w} = w_d$. ■

PROOF OF REMARK 1:

Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying the conditions of the statement and suppose without loss of generality that the juror with the known friend is juror 3. Then $I_1 = \{bc\}$, $I_2 = \{ac\}$, $I_3 = \{ab\}$, $x_1^f = x_1^e = \emptyset$, $x_2^f = x_2^e = \emptyset$, and $x_3^f = c$. Table 4 shows the preference functions that are admissible in this case. Suppose, by contradiction, that there is a straightforward-extensive-form mechanism $\Gamma \equiv (\Theta, \gamma)$ that naturally implements the SOCR in subgame perfect equilibrium when the jury configuration is (I, x^f, x^e) where 3 does not move at the first stage. We distinguish two cases.

Case 1. Γ is such that the juror with the known friend (juror 3) moves at the last stage.

Suppose without loss of generality that 1 moves at the first stage and 2 moves at the second stage.

Claim 1.1. Either $\gamma(c, a, x) = c$ for some $x \in N$ or $\gamma(c, a, x) = a$ for all $x \in N$.

Suppose by contradiction that $\gamma(c, a, x) \in \{a, b\}$ for every $x \in N$ and $\gamma(c, a, y) = b$ for some $y \in N$. Note that there exists a preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ such that $b\ P_3(c)\ a$. Similarly, there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that b is the unique best alternative when $w_d = c$. Let $(R, c) \in S(I, x^f, x^e)$ be an admissible state where the preference functions of jurors 3 and 2 are as just described. Then, there is no $\theta \in E(\Gamma, R, c)$ such that jurors 1, 2, and 3 choose c at the first, second, and third stage, respectively. The reason is that, if juror 1 chooses c at the first stage, then the movements of jurors 2 and 3 at the second and third stages will result in b (if juror 2 chooses a then juror 3 will choose some y so that $\gamma(c, a, y) = b$, and 2 likes b more than any other alternative). This contradicts that Γ naturally implements the SOCR.

Claim 1.2. Either $\gamma(c, b, x) = c$ for some $x \in N$ or $\gamma(c, b, x) = a$ for all $x \in N$.

The argument to prove this claim is identical to that in Claim 1.1.

Claim 1.3. Either $\gamma(c, a, x) = a$ for all $x \in N$ or $\gamma(c, b, x) = a$ for all $x \in N$.

Suppose by contradiction that $\gamma(c, a, x) \neq a$ for some $x \in N$ and $\gamma(c, b, y) \neq a$ for some $y \in N$. Then, by Claims 1.1 and 1.2, $\gamma(c, a, x) = c$ for some $x \in N$

and $\gamma(c, b, y) = c$ for some $y \in N$. Moreover, from the definition of natural implementation, we have $\gamma(c, c, c) = c$. Note that any preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that c is the unique best alternative when $w_d = a$. Then, any $\theta \in E(\Gamma, R, a)$ is such that, if 1 chooses c, the candidate finally chosen is c, no matter what candidate 2 chooses at the second stage. Note also that there exists a preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ such that c $P_1(a)$ a. Let $(R, a) \in S(I, x^f, x^e)$ be an admissible state where the preference function of juror 1 is as just described. Then, there is no $\theta \in E(\Gamma, R, a)$ such that jurors 1, 2, and 3 choose a at the first, second, and third stage, respectively (if juror 1 chooses c at the first stage, then the movements of jurors 2 and 3 at the second and third stages will result in c, which is more preferred than a for 1). This contradicts that Γ naturally implements the SOCR.

Claim 1.4. $\gamma(c, a, x) \neq a$ for some $x \in N$ and $\gamma(c, b, y) \neq a$ for some $y \in N$.

Suppose by contradiction and without loss of generality that $\gamma(c, a, x) = a$ for all $x \in N$. Note that there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that a is the unique best alternative when $w_d = b$. Similarly, there exists a preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ such that a is the unique best alternative when $w_d = b$. Let $(R, b) \in S(I, x^f, x^e)$ be an admissible state where the preference functions of jurors 2 and 1 are as just described. Then, there is no $\theta \in E(\Gamma, R, b)$ such that jurors 1, 2, and 3 choose b at the first, second, and third stage, respectively. The reason is that, if juror 1 chooses c at the first stage, then the movements of jurors 2 and 3 at the second and third stages will result in a, which she prefers more than b (if juror 2 chooses a then the candidate finally selected is a, no matter what candidate 3 chooses at the third stage, and 2 likes b more than any other alternative). This contradicts that Γ naturally implements the SOCR.

Claim 1.4 contradicts Claim 1.3, and therefore Case 1 is not possible.

Case 2. Γ is such that the juror with the known friend (juror 3) moves at the second stage.

Suppose without loss of generality that 1 moves at the first stage and 2 moves at the third stage.

Claim 2.1. $\gamma(b, b, x) = b$ for all $x \in N$.

Suppose by contradiction that $\gamma(b, b, x) \neq b$ for some $x \in N$. Note that there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that b is the unique worst alternative when $w_d = b$. Then, there is no $\theta \in E(\Gamma, R, b)$ such that jurors 1, 3, and 2 choose b at the first, second, and third stage, respectively.

The reason is that if jurors 1 and 3 choose b at the first and second stage, respectively, then juror 2 prefers to choose x at the third stage, so that b is not finally selected.

Claim 2.2. $\gamma(b, a, x) \neq c$ for all $x \in N$ and $\gamma(b, c, x) \neq c$ for all $x \in N$.

Suppose by contradiction that $\gamma(b, a, x) = c$ for some $x \in N$. Note that any preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that c is the unique best alternative when $w_d = b$. Note also that there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that c is the unique best alternative when $w_d = b$. Let $(R, b) \in S(I, x^f, x^e)$ be an admissible state where the preference function of juror 2 is as just described. Then, there is no $\theta \in E(\Gamma, R, b)$ such that jurors 1, 3, and 2 choose b at the first, second, and third stage, respectively. The reason is that if juror 1 chooses b at the first stage, then 3 prefers to choose a at the second stage, since in this case 2 will choose x, and then c will be selected, which is the most preferred alternative for 3 and 2. This contradicts that Γ naturally implements the SOCR. The proof that $\gamma(b, c, x) \neq c$ for all $x \in N$ is analogous.

Claim 2.3. Either $\gamma(b, a, x) = a$ for all $x \in N$ or $\gamma(b, c, x) = a$ for all $x \in N$.

Suppose by contradiction that $\gamma(b, a, x) \neq a$ for some $x \in N$ and $\gamma(b, c, y) \neq a$ for some $y \in N$. Then, by Claim 2.2, $\gamma(b, a, x) = b$ for some $x \in N$ and $\gamma(b, c, y) = b$ for some $y \in N$. Moreover, by Claim 2.1, $\gamma(b, b, x) = b$ for all $x \in N$. Note that there exists a preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ such that a is the unique worst alternative when $w_d = a$. Note also that every preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that $c\ P_3(a)\ a\ P_3(a)\ b$. Finally, note that there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that b is the unique best alternative when $w_d = a$. Let $(R, a) \in S(I, x^f, x^e)$ be an admissible state where the preference functions of jurors 1, 2, and 3 are as just described. Then, there is no $\theta \in E(\Gamma, R, a)$ such that jurors 1, 3, and 2 choose a at the first, second, and third stage, respectively. The reason is that, if juror 1 chooses b at the first stage, then the movements of jurors 3 and 2 at the second and third stages will result in b, which she prefers more than a (no matter what juror 3 chooses at the second stage, juror 2 can choose an alternative at the third stage that results in b). This contradicts that Γ naturally implements the SOCR.

Claim 2.4. $\gamma(b, a, x) \neq a$ for all $x \in N$ and $\gamma(b, c, x) \neq a$ for all $x \in N$.

Suppose by contradiction and without loss of generality that $\gamma(b, a, x) = a$ for some $x \in N$. Note that there exists a preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ such that a is the unique best alternative when $w_d = c$. Similarly, there exists a preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ such that $a\ P_3(c)\ b$. Finally, there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that $a\ P_2(c)\ b$. Let $(R, c) \in S(I, x^f, x^e)$ be an admissible state where the preference functions of jurors 1, 3, and 2 are as just described. Then, there is no $\theta \in E(\Gamma, R, c)$ such that jurors 1, 3, and 2 choose c at the first, second, and third stage, respectively. The reason is that, if juror 1 chooses b at the first stage, then the movements of jurors 3 and 2 at the second and third stages will result in a, which she prefers more than c (from Claims 2.1 and 2.2, if juror 1 chooses b at the first stage, the candidate finally selected cannot be c, no matter what are the movements of jurors 3 and 2 at the second and third stages; then, since a is the second most preferred alternative for jurors 3 and 2, juror 3 prefers to announce a at the second stage, in which case juror 2 will chose x at the third stage and c will be selected).

Claim 2.4 contradicts Claim 2.3, and therefore Case 2 is not possible. ■

PROOF OF REMARK 2:

Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying the conditions of the statement and suppose without loss of generality that the juror with the known enemy is juror 3. Then $I_1 = \{bc\}$, $I_2 = \{ac\}$, $I_3 = \{ab\}$, $x_1^f = x_1^e = \emptyset$, $x_2^f = x_2^e = \emptyset$, and $x_3^e = c$. Table 5 shows the preference functions that are admissible in this case. Suppose, by contradiction, that there is a straightforward-extensive-form mechanism $\Gamma \equiv (\Theta, \gamma)$ that naturally implements the SOCR in subgame perfect equilibrium when the jury configuration is (I, x^f, x^e) where 3 does not move at the first stage. We distinguish two cases.

Case 1. Γ is such that the juror with the known enemy (juror 3) moves at the last stage.

Suppose without loss of generality that 1 moves at the first stage and 2 moves at the second stage.

Claim 1.1. $\gamma(c, c, x) = c$ for all $x \in N$.

From the definition of natural implementation, we have $\gamma(c, c, c) = c$. Suppose by contradiction that $\gamma(c, c, x) \neq c$ for some $x \in N\backslash\{c\}$. Note that any preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that $\gamma(c, c, x)\ P_3(c)\ c$. Let $(R, c) \in S(I, x^f, x^e)$ be an admissible state. Then, there is no $\theta \in E(\Gamma, R, c)$ such that jurors 1, 2, and 3 choose c at the first, second, and third stage, respectively (if jurors 1 and 2 choose c at the first and second stages, then juror 3 prefers to choose x at the third stage). This contradicts that Γ naturally implements the SOCR.

Claim 1.2. $\gamma(c, a, x) \neq b$ for all $x \in N$ and $\gamma(c, b, x) \neq b$ for all $x \in N$.

Suppose by contradiction that $\gamma(c, a, x) = b$ for some $x \in N$. Note that there exists a preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ such that b is the unique best alternative when $w_d = c$. Similarly, there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that b is the unique best alternative when $w_d = c$. Let $(R, c) \in S(I, x^f, x^e)$ be an admissible state where the preference functions of jurors 3 and 2 are as just described. Then, there is no $\theta \in E(\Gamma, R, c)$ such that jurors 1, 2, and 3 choose c at the first, second, and third stage, respectively (if juror 1 chooses c at the first stage, then juror 2 prefers to choose a at the second stage, since in this case juror 3 will chose x at the third stage, since this results in b, the most preferred alternative for 2 and 3). This contradicts that Γ naturally implements the SOCR. The proof that $\gamma(c, b, x) \neq b$ for all $x \in N$ is analogous.

Claim 1.3. Either $\gamma(c, a, x) = a$ for some $x \in N$ or $\gamma(c, b, x) = a$ for some $x \in N$.

Suppose by contradiction that $\gamma(c, a, x) \neq a$ and $\gamma(c, b, x) \neq a$ for all $x \in N$. Then, by Claims 1.1 and 1.2, $\gamma(c, x, y) = c$ for all $x, y \in N$. Note that there exists a preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ such that $c \; P_1(a) \; a$. Let $(R, a) \in S(I, x^f, x^e)$ be an admissible state where the preference function of juror 1 is as just described. Then, there is no $\theta \in E(\Gamma, R, a)$ such that jurors 1, 2, and 3 choose a at the first, second, and third stage, respectively. The reason is that if juror 1 chooses c at the first stage, then the movements of jurors 2 and 3 at the second and third stages will result in c, which 1 prefers more than a. This contradicts that Γ naturally implements the SOCR.

Claim 1.4. $\gamma(c, a, x) \neq a$ and $\gamma(c, b, x) \neq a$ for all $x \in N$.

Suppose by contradiction and without loss of generality that $\gamma(c, a, x) = a$ for some $x \in N$. By Claims 1.1 and 1.2, $\gamma(c, x, y) \in \{a, c\}$ for all $x, y \in N$. Note that there exists a preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ such that $a \; P_1(b) \; b$. Similarly, there exists a preference function for juror 2 admissible at (I_2, x_2^f, x_2^e), $R_2 \in \mathcal{R}$, such that $a \; P_2(b) \; c$. Finally, every preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ is such that $a \; P_3(b) \; c$. Let $(R, b) \in S(I, x^f, x^e)$ be an admissible state where the preference functions of juror 1, 2, and 3 are as just described. Then, there is no $\theta \in E(\Gamma, R, b)$ such that jurors 1, 2, and 3 choose b at the first, second, and third stage, respectively. The reason is that if juror 1 chooses c at the first stage, then the movements of jurors 2 and 3 at the second and third stages will result in a, which 1 prefers more than b (once juror 1 chooses c, the only possible result is a or c, and both, 2 and 3, prefer a rather than c; juror 2 can guarantee that a is the contestant finally selected by choosing a in $\Gamma(c,.)$, since in this case juror 3 is going to choose x). This contradicts that Γ naturally implements the SOCR.

Claim 1.4 contradicts Claim 1.3, and therefore Case 1 is not possible.

Case 2. Γ is such that the juror with the known friend (juror 3) moves at the second stage.

Suppose without loss of generality that 1 moves at the first stage and 2 moves at the third stage.

Claim 2.1. $\gamma(c,c,x) \neq b$ for all $x \in N$.

Suppose by contradiction that $\gamma(c,c,x) = b$ for some $x \in N$. Note that there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that $b\ P_2(c)\ c$. Then, there is no $\theta \in E(\Gamma, R, c)$ such that jurors 1, 3, and 2 choose c at the first, second, and third stage, respectively. The reason is that if jurors 1 and 3 choose c at the first and second stage, respectively, then juror 2 prefers to choose x at the third stage, so that b is finally selected.

Claim 2.2. $\gamma(c,a,x) \neq b$ for all $x \in N$ and $\gamma(c,b,x) \neq b$ for all $x \in N$.

Suppose by contradiction that $\gamma(c,a,x) = b$ for some $x \in N$. Note that there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that b is the unique best alternative when $w_d = c$. Similarly, there exists a preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$ such that b is the unique best alternative when $w_d = c$. Let $(R, c) \in S(I, x^f, x^e)$ be an admissible state where the preference functions of jurors 2 and 3 are as just described. Then, there is no $\theta \in E(\Gamma, R, c)$ such that jurors 1, 3, and 2 choose c at the first, second, and third stage, respectively. The reason is that if juror 1 chooses c at the first stage, then 3 prefers to choose a at the second stage, since in this case 2 will choose x, and then b will be selected, which is the most preferred alternative for 3 and 2. This contradicts that Γ naturally implements the SOCR. The proof that $\gamma(c,b,x) \neq b$ for all $x \in N$ is analogous.

Claim 2.3. $\gamma(c,x,y) = a$ for some $x, y \in N$.

Suppose by contradiction that $\gamma(c,x,y) \neq a$ for every $x, y \in N$. Note that there exists a preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ such that a is the unique worst alternative when $w_d = a$. Let $(R, a) \in S(I, x^f, x^e)$ be an admissible state where the preference function of juror 1 is as just described. Then, there is no $\theta \in E(\Gamma, R, a)$ such that jurors 1, 3, and 2 choose a at the first, second, and third stage, respectively. The reason is that if juror 1 chooses c at the first stage, then a cannot be the contestant finally selected, and then $\gamma(c,x,y)\ P_1(a)\ a$ for every $x, y \in N$. This contradicts that Γ naturally implements the SOCR.

Claim 2.4. $\gamma(c,x,y) \neq a$ for every $x, y \in N$.

Suppose by contradiction that $\gamma(c, x, y) = a$ for some $x, y \in N$. Note that there exists a preference function $R_1 \in \mathcal{R}(I_1, x_1^f, x_1^e)$ such that a is the unique best alternative when $w_d = b$. Similarly, there exists a preference function $R_2 \in \mathcal{R}(I_2, x_2^f, x_2^e)$ such that a is the unique best alternative when $w_d = b$. Finally, any preference function $R_3 \in \mathcal{R}(I_3, x_3^f, x_3^e)$, $R_3 \in \mathcal{R}$, such that $a \ P_3(c) \ c$. Let $(R, b) \in S(I, x^f, x^e)$ be an admissible state where the preference functions of jurors 1, 2, and 3 are as just described. Then, there is no $\theta \in E(\Gamma, R, b)$ such that jurors 1, 3, and 2 choose b at the first, second, and third stage, respectively. The reason is that if juror 1 chooses c at the first stage, (and since, by Claims 2.1 and 2.2, b cannot be the contestant finally selected), then the movements of jurors 3 and 2 will result in a (juror 3 plays x and juror 2 plays y, which results in a), which is his/her most preferred alternative at state (R, b). This contradicts that Γ naturally implements the SOCR.

Claim 2.4 contradicts Claim 2.3, and therefore Case 2 is not possible. ■

References

Adachi, T. (2014): A natural mechanism for eliciting ranking when jurors have favorites, Games Econ. Behav., 87, 508-518.

Amorós, P. (2015): Conditions on the jury for the natural implementation of the deserving winner of a competition, mimeo, Universidad de Málaga..

Amorós, P. (2013): Picking the winners, Int. J Game Theory, 42, 845-865.

Amorós, P. (2011): A natural mechanism to choose the deserving winner when the jury is made up of all contestants, Econ. Lett., 110, 241-244.

Amorós, P. (2009): Eliciting socially optimal rankings from unfair jurors, J. Econ. Theory, 144, 1211-1226.

Amorós, P., L. C. Corchón, and B. Moreno (2002): The scholarship assignment problem, Games Econ. Behav., 38, 1-18.

Danilov V (1992): Implementation via Nash equilibria, Econometrica, 60, 43-56.

Dutta, B. and A. Sen (2012): Nash implementation with partially honest individuals, Games Econ. Behav., 74, 154-169.

Jackson, M. O. (1992): Implementation in undominated strategies: A look at bounded mechanisms, Rev. Econ. Stud., 59, 757-75.

Maskin, E. (1999): Nash equilibrium and welfare optimality, Rev. Econ. Stud., 66, 23-38.

Matsushima, H. (2008): Role of honesty in full implementation, J. Econ. Theory, 139, 353-359.

Moskalenko, A. (2013): A mechanism to pick the deserving winner, mimeo, Universitat Rovira i Virgili.

Ng, Y. K., and G. Z. Sun (2003): Exclusion of self evaluations in peer ratings: An impossibility and some proposals, Soc. Choice Welfare, 20, 443-456.

Serrano, R. (2004): The theory of implementation of social choice rules, SIAM Review, 46, 377-414.