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Conditions on the jury for the natural implementation of the deserving winner of a contest*

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Abstract

A jury has to choose the winner of a contest. There exists a deserving winner, whose identity is common knowledge among the jurors, but not known by the planner. Jurors may be biased in favor (friend) or against (enemy) some contestants. We study conditions on the configuration of the jury so that it is possible to implement the deserving winner in Nash equilibrium when we restrict ourselves to mechanisms satisfying two conditions: (1) each juror only has to announce a contestant, and (2) announcing the deserving winner is an equilibrium. We call this notion natural implementation. We show that, in order to naturally implement the deserving winner, the planner needs to know a number of jurors with friends or a number of jurors with enemies. Specifically, the number of jurors with friends that the planner needs to know to naturally implement the deserving winner is less than the number of jurors with enemies that the planner would need to know for it.

Key Words: Mechanism design; contests; jury; Nash equilibrium.

J.E.L. Classification Numbers: C72, D71, D78.

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1 Introduction

Consider the hiring process in a department. The members of the department must choose whom to hire. Suppose that all members of the department know who the best candidate is (there is no private information). The optimal objective is to hire the best candidate available. Some professors, however, may be biased in favor/against some candidates: some professors may not want to hire candidates who are better at their jobs than they are, some others may want to hire candidates who work in the same topics than them, even though they are not the best candidates, etc.

There are many problems with a similar structure to the one just described that can be summarized as follows. A group on contestants are involved in a competition. A group of jurors has to choose one winner among the contestants. All jurors know who the best contestant is: the "deserving winner". The socially optimal choice rule (SOCR) is to select the deserving winner (whoever he is). Jurors, however, may be biased in favor/against some contestants. Biased jurors may try to favor/harm one contestant over another, regardless of who is the best one.

The fact that the jurors are biased does not necessarily imply that their decisions will be unfair. Sometimes, it is possible to design mechanisms that neutralize the particular interests of partial jurors. It depends on the specific bias of the jury. Amorós (2013) provides a necessary condition for the implementability of the SOCR in any equilibrium concept: for each pair of contestants, there must be at least one juror who is impartial with respect to them (a juror is impartial with respect to a pair of contestants if, whenever one of the two contestants is the deserving winner, the juror prefers that contestant to the other one). The previous condition is also sufficient for Nash implementation: if the condition is satisfied, then the canonical mechanism for Nash implementation implements the SOCR.

However, the canonical mechanism have received criticism for being unnatural, having too complex message spaces, and making use of extraneous devices such as "integer games" or "modulo games" (see Jackson, 1992). The reason that these type of mechanisms are abstract and unnatural is that they are designed to characterize what can be implemented and, therefore, they have to handle a large number of problems (see Serrano, 2004).

In this paper, we are interested in implementing the deserving winner through "simple" and "natural" mechanisms that can be used in the real world. Although, in general, it is not easy to define what simple and natural means, we can be more specific when talking about mechanisms used to elicit the deserving winner. Regarding simplicity, we want our mechanisms to have straightforward message spaces. In this sense, the simplest mechanisms are those in which each juror only has to announce the contestant who he thinks should win the competition. We call these mechanisms "straightforward" mechanisms. With respect to naturalness, and since we are asking each juror to tell us who is the deserving winner, we want the mechanism to satisfy unanimity: if all jurors announce the same contestant, then that contestant should be chosen as the winner of the competition. Moreover, in this context it is natural to require that telling the truth (announcing the deserving winner) should always be an equilibrium. We say that the SOCR is naturally implementable if there exists a straightforward mechanism implementing the SOCR in which telling the truth is always an equilibrium.

Natural implementation entails a restriction on the mechanisms that can be used. Not surprisingly, this restriction on the mechanisms hinders implementation. We show that the condition stated by Amorós (2013) is no longer sufficient for the natural implementation of the SOCR in Nash equilibrium (Proposition 3). Imposing that, for each pair of contestants, there is at least one juror who is impartial with respect to them, implies a reduction on the set of admissible states of the world. While this reduction is large enough to allow the implementation of the SOCR, it is not sufficient to permit its natural implementation. Then, a natural question arises: what additional conditions must satisfy the jury in order to be sure that we can design a simple and natural mechanism that induce the jurors to always choose the deserving winner?

Sometimes the planner has more information about the jury and knows that some jurors would like to favor/harm some contestants. We say that a contestant is a known friend (enemy) of a juror if this contestant is always his most (less) preferred alternative, regardless of who is deserving winner. Knowing that jurors have friends or enemies may facilitate the natural implementation, as it reduces the set of admissible states of the world. We analyze the effect of having friends or enemies in the natural implementability of the SOCR. For that, we study a simple model with only three jurors and three contestants, so that we can identify each contestant with a different juror.[1]

We first focus on the case in which jurors have friends. We show that, if

[1] The condition on the jury stated by Amorós (2013) is sufficient for Nash implementation if there are at least three jurors.

all jurors have friends (which, for example, may occur if the jury is made up of the contestants themselves), then the SOCR is naturally implementable in Nash equilibrium (Proposition 4). To prove this result, we propose a straightforward mechanism with three simple rules: (i) if all jurors announce the same contestant then that contestant is chosen, (ii) if there is only one dissident then we chose the contestant announced by the dissident only if he is not his friend, and (iii) if more than two jurors disagree on their announcements then an arbitrary contestant known by all jurors is chosen.

On the one hand, from Proposition 3 we have that, if the planner does not know whether any of the jurors has some friend (or enemy), then the SOCR is not naturally implementable. On the other hand, from Proposition 4 we have that, if the planner knows that all jurors have friends, then the SOCR is naturally implementable. Again, a natural question arises: how many (jurors with) friends would need the planner to know at least for the SOCR to be naturally implementable? The answer is simple: just one (Proposition 5). To prove this result we propose a variation of the mechanism used when all jurors have friends.

We also study the case in which jurors have enemies. Similarly to what happens in the friends case, in Proposition 6 we show that if all jurors have enemies then the SOCR is naturally implementable in Nash equilibrium. The straightforward mechanism proposed to prove this result also consists of three rules, but it is more complicated than the one for the friends case, especially in the case in which there is one dissident. Again, from Proposition 3 we have that if no juror has a known enemy (or friend) then natural implementation of the SOCR is not possible, while from Proposition 6 we have that if all jurors have enemies then natural implementation of the SOCR is possible. What is then the minimum number of jurors with enemies that the planner need to know in order to naturally implement the SOCR?

Unlike what happens in the friends case, having one juror with a known enemy is not sufficient for the natural implementability of the SOCR (Proposition 7). This is surprising in that the reduction in the size of set of admissible states caused by the existence of a juror with an enemy is equal to that produced for having one juror with a friend. However, while in the latter case this reduction is sufficient to allow the natural implementation of the SOCR, in the former case it is not. This does not mean we need to have all jurors with known enemies to naturally implement the SOCR: in Proposition 8 we show that it is sufficient to have two jurors of this type in the jury.

We conclude therefore that, in order to naturally implement the SOCR, we need to reduce the size of the set of admissible states, which is equivalent to increasing the information available to the planner. Knowing with certainty that a juror has a friend or an enemy is a possible way to do this. Our results suggest that, if we can choose between these two possibilities, we better select jurors with friends, since in this case less information about the jury is required: while in the case of enemies at least two members of the jury with known enemies are required, in the case of friends only one member of the jury with a known friend is necessary.

Related literature

Amorós (2013) provides necessary and sufficient conditions on the configuration of jury for the implementability of the SOCR. This paper, however, does not study natural implementation (in particular, the mechanism proposed makes use of modulo games). Amorós (2011) studies the particular case where each contestant has one friend and proposes a natural extensive form mechanism that implements the SOCR in subgame perfect equilibrium. Moskalenko (2013) proposes an alternative to the previous mechanism where each juror can veto a contestant.

Amorós (2009) analyzes a model where the jurors have to choose a full ranking of the contestants instead of selecting one winner. This paper provides necessary and sufficient conditions on the jury for the Nash implementability of the rule that always select the socially optimal ranking, but it does not study implementation by natural mechanisms (it proposes mechanisms à la Maskin, 1999). Amorós et al. (2002) study the same model and analyze implementation in dominant strategies and Nash equilibrium when each juror has one friend and is impartial with respect to the rest of contestants. Adachi (2014) also examines the problem of choosing the socially optimal ranking when jurors may have friends and proposes a natural mechanism which implements in subgame perfect equilibrium. Ng and Sun (2003) investigate the problem of excluding the self-awarded marks in the calculation of the ranking when each contestant is biased in favor of itself.

Doğan (2013) studies a closely related model, where a set of tasks is to be allocated among a set of agents whose preferences over allocations may or may not be "responsive" to the optimal allocation (the notion of an agent being responsible in this paper is similar to that of a juror being impartial in our model). This work shows that the optimal allocation can be implemented in Nash equilibrium if there are at least three responsible agents. Our paper is also related to the literature on the effects of having honest agents on the general implementation problem (Matsushima, 2008; Dutta and Sen; 2011).

The rest of the paper is organized as follows. Section 2 lays out our basic model, states the necessary and sufficient conditions on the jury for Nash implementation, and introduces the concept of natural implementation. Section 3 presents the results on natural implementation when jurors have friends and when jurors have enemies. Section 4 concludes. The Appendix provides the proofs of some of the results.

2 Preliminaries

2.1 The model

Let $N = \{a, b, ...\}$ be a set of contestants in a competition. A group $J = \{1, 2, ...\}$ of jurors have to choose one winner from the contestants. All jurors know who the best contestant is. We call this contestant the **deserving winner**, $w_d \in N$. The socially optimal outcome is that the deserving winner wins. General elements of N are denoted by x, y, etc., and general elements of J are denoted by j, k, etc.

Let $\Re$ be the class of preference relations defined over N. Each juror $j \in J$ has a **preference function** $R_j : N \longrightarrow \Re$ which associates with each possible deserving winner, $w_d \in N$, a preference relation $R_j(w_d) \in \Re$. Let $P_j(w_d)$ denote the strict part of $R_j(w_d)$. Let $\mathcal{R}$ denote the class of all possible preference functions.

Example 1 *Table 1 shows an example of preference function when $N = \{a, b, c\}$ (higher contestants in the table are preferred to lower contestants). In this case, (i) juror j always prefers contestant c to win (regardless of who is the deserving winner) and (ii) if c cannot win and c is not the deserving winner, juror j prefers the deserving winner (a or b) to win (if c is the deserving winner, j is indifferent between a and b).*

R_j

$w_d =$	**a**	**b**	**c**
	c	c	c
Preferences	a	b	ab
	b	a	

Table 1 Preference function in Example 1.

Let 2^N_2 denote the set of all possible pairs of contestants. If $N = \{a, b, c\}$ then $2^N_2 = \{bc, ac, ab\}$. We say that a preference function is impartial with respect to a pair of contestants if, whenever one of the two contestants is the deserving winner, that contestant is preferred to the other one.

Definition 1 *A preference function $R_j \in \mathcal{R}$ is **impartial** with respect to a pair of contestants $xy \in 2^N_2$ if, whenever $x = w_d$, then $x\ P_j(w_d)\ y$.*

Example 2 *The preference function R_j defined in Table 1 is impartial with respect to the pair ab, since $a\ P_j(a)\ b$ and $b\ P_j(b)\ a$. This preference function, however, is not impartial with respect to the pair ac, since $c\ P_j(a)\ a$, or the pair bc, since $c\ P_j(b)\ b$.*

We say that a preference function favors a contestant if this contestant is always the most preferred alternative, regardless of who is the deserving winner.

Definition 2 *A preference function $R_j \in \mathcal{R}$ **favors** a contestant $x \in N$ if, for each $w_d \in N$ and each $y \in N \backslash \{x\}$, $x\ P_j(w_d)\ y$.*

Similarly, we say that a preference function harms a contestant if this contestant is always the less preferred alternative, regardless of who is the deserving winner.

Definition 3 *A preference function $R_j \in \mathcal{R}$ **harms** a contestant $x \in N$ if, for each $w_d \in N$ and each $y \in N \backslash \{x\}$, $y\ P_j(w_d)\ x$.*

Example 3 *The preference function R_j in Table 1 favors c, since, for each $w_d \in N$ and each $y \in N \backslash \{c\}$, $c\ P_j(w_d)\ y$. The preference function $\bar{R}_j$ in Table 2 harms a, since, for each $w_d \in N$ and each $y \in N \backslash \{a\}$, $y\ \bar{P}_j(w_d)\ a$. The preference function $\hat{R}_j$ in Table 2 does not favor or harm any contestant.*

$\bar{R}_j$

a	**b**	**c**
bc	b	c
a	c	b
	a	a

$\hat{R}_j$

a	**b**	**c**
a	b	cb
bc	c	a
	a	

Table 2 Preference functions in Example 3.

Each juror $j \in J$ is characterized by a triple (I_j, x_j^f, x_j^e) where $I_j \subset 2_2^N \cup \{\emptyset\}$, $x_j^f \in N \cup \{\emptyset\}$, and $x_j^e \in N \cup \{\emptyset\}$. We say that a **preference function** $R_j \in \mathcal{R}$ is **admissible for juror** j **at** (I_j, x_j^f, x_j^e) if:

(i) R_j is impartial with respect to every $xy \in I_j$,

(ii) R_j favors x_j^f, and

(iii) R_j harms x_j^e.

The case $x_j^f = \emptyset$ corresponds with a situation where the planner does not know if juror j wants to favor some contestant or not. In this case, an admissible preference function can favor a contestant, but need not to do so (*i.e.*, there are admissible preference functions for j that favor no contestant, admissible preference functions that favor a, admissible preference functions that favor b, and admissible preference functions that favor c). Similarly, $x_j^f = \{\emptyset\}$ represents the situation where the planner does not know if j wants to harm some contestant or not. Finally, $I_j = \{\emptyset\}$ represents the situation where the planner does not know if j is impartial with respect to some pair of contestants. In the extreme case where $I_j = \{\emptyset\}$, $x_j^f = \{\emptyset\}$, and $x_j^e = \{\emptyset\}$, every possible preference function $R_j \in \mathcal{R}$ is admissible for j.

Example 4 *Let $N = \{a, b, c\}$. Let $I_j = \{ab\}$, $x_j^f = \{\emptyset\}$, and $x_j^e = \{\emptyset\}$. A preference function $R_j \in \mathcal{R}$ is admissible for j at (I_j, x_j^f, x_j^e) if it satisfies $a\ P_j(a)\ b$ and $b\ P_j(b)\ a$. For instance, the preference function R_j in Table 1 and $\hat{R}_j$ in Table 2 are admissible for j at (I_j, x_j^f, x_j^e), while the preference function $\bar{R}_j$ in Table 2 is not. Suppose now that $\hat{I}_j = \{ab\}$, $\hat{x}_j^f = \{c\}$, and $\hat{x}_j^e = \{\emptyset\}$. A preference function $R_j \in \mathcal{R}$ is admissible for j at $(\hat{I}_j, \hat{x}_j^f, \hat{x}_j^e)$ if $a\ P_j(a)\ b$, $b\ P_j(b)\ a$, and $c\ P_j(x)\ y$ for all $x \in N$ and all $y \in N \backslash \{c\}$. For instance, the preference function defined R_j in Table 1 is admissible for j at $(\hat{I}_j, \hat{x}_j^f, \hat{x}_j^e)$ while the preference functions defined in Table 2 are not.*

Let $\mathcal{E} \equiv 2_2^N \cup \{\emptyset\} \times N \cup \{\emptyset\} \times N \cup \{\emptyset\}$. For each $(I_j, x_j^f, x_j^e) \in \mathcal{E}$, let $\mathcal{R}(I_j, x_j^f, x_j^e)$ be the class of all preference functions that are admissible for j at (I_j, x_j^f, x_j^e). A **jury configuration** is a profile $(I, x^f, x^e) \equiv (I_j, x_j^f, x_j^e)_{i \in J} \in \mathcal{E}^{|J|}$.

Definition 4 *Given a jury configuration $(I, x^f, x^e) \in \mathcal{E}^{|J|}$, and a juror $j \in J$:*

(i) if $xy \in I_j$ we say that juror j is impartial with respect to the pair xy,

*(i) if $x_j^f \neq \emptyset$, we say that contestant x_j^f is a known **friend** of juror j, and*

*(ii) if $x_j^e \neq \emptyset$, we say that contestant x_j^e is a known **enemy** of juror j.*

A **state** (of the world) is a tuple $(R, w_d) \in \mathcal{R}^{|J|} \times N$, where $R \equiv (R_j)_{j \in J}$ is a profile of preference functions (one for each juror). Given a jury configuration $(I, x^f, x^e) \in \mathcal{E}^{|J|}$, a state (R, w_d) is **admissible** when the jury configuration is (I, x^f, x^e) if $R_j \in \mathcal{R}(I_j, x_j^f, x_j^e)$ for every $j \in J$. Let $S(I, x^f, x^e)$ be the set of all states that are admissible when the jury configuration is (I, x^f, x^e).

Given a jury configuration $(I, x^f, x^e) \in \mathcal{E}^{|J|}$, the **socially optimal choice rule** (SOCR) is the function $\varphi : S(I, x^f, x^e) \rightarrow N$ such that, for each $(R, w_d) \in S(I, x^f, x^e)$, $\varphi(R, w_d) = w_d$; *i.e.*, for each state admissible at (I, x^f, x^e), φ selects the deserving winner.

A **mechanism** is a pair $\Gamma \equiv (M, g)$, where $M = \times_{j \in J} M_j$, M_j is a message space for juror j, $g : M \rightarrow N$ is an outcome function. Given a jury configuration $(I, x^f, x^e) \in \mathcal{E}^{|J|}$, a profile of messages $m \in M$ is a **Nash equilibrium** of mechanism $\Gamma = (M, g)$ at state $(R, w_d) \in S(I, x^f, x^e)$ if for each $j \in J$ and each $\hat{m}_j \in M_j$, $g(m)\ R_j(w_d)\ g(\hat{m}_j, m_{-j})$. Let $N(\Gamma, R, w_d)$ be the set of profiles of messages that are a Nash equilibrium of mechanism Γ at state (R, w_d). A mechanism implements the SOCR in Nash equilibrium if, in every admissible state, the deserving winner is selected in Nash equilibrium.

Definition 5 *A mechanism $\Gamma = (M, g)$ **implements** the SOCR in Nash equilibrium when the jury configuration is $(I, x^f, x^e) \in \mathcal{E}^{|J|}$, if, for each state $(R, w_d) \in S(I, x^f, x^e)$:*
(i) $N(\Gamma, R, w_d) \neq \emptyset$, and
(ii) $m \in N(\Gamma, R, w_d)$ if and only if $g(m) = w_d$.

The SOCR is **implementable** when the jury configuration is $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ if there exists a mechanism that implements it. We are interested in studying what conditions must satisfy the configuration of the jury to ensure that the SOCR is implementable.

2.2 Necessary and sufficient conditions for implementation

A necessary condition for the implementability of the SOCR in Nash equilibrium when the jury configuration is $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ is that, for each pair of contestants, there is at least one juror who is impartial with respect to them. This result can be deduced as a corollary from Amorós (2013, Proposition 1). We include the proof for completeness.

Proposition 1 *If the SOCR is implementable in Nash equilibrium when the jury configuration is* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ *then, for each pair of contestants, there is at least one juror who is impartial with respect to them.*

Proof. Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$. Suppose that the SOCR is implementable in Nash equilibrium when the jury configuration is (I, x^f, x^e) by means of a mechanism $\Gamma = (M, g)$. Suppose by contradiction and without loss of generality that, for each $j \in J$, $ab \notin I_j$. In this case, there exists a profile of preference functions $R = (R_j)_{j \in J}$ such that, for each $j \in J$, (i) $R_j \in \mathcal{R}(I_j, x_j^f, x_j^e)$ and (ii) $R_j(a) = R_j(b)$ (see Example 5 below). Note that then, by (i) $(R, a), (R, b) \in S(I, x^f, x^e)$, and by (ii) $N(\Gamma, R, a) = N(\Gamma, R, b)$. Since Γ implements the SOCR in Nash equilibrium, there is $m \in N(\Gamma, R, a)$ such that $g(m) = a$. But then, $m \in N(\Gamma, R, b)$ and $g(m) \neq b$, which contradicts that Γ implements the SOCR in Nash equilibrium. ■[2]

Example 5 *Suppose that* $J = \{1, 2, 3\}$*. Let* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ *be such that* $(I_1, x_1^f, x_1^e) = (bc, a, \emptyset)$*,* $(I_2, x_2^f, x_2^e) = (ac, \emptyset, b)$*,* $(I_3, x_3^f, x_3^e) = (bc, \emptyset, \emptyset)$*. Note that, for every* $j \in J$*,* $ab \notin I_j$*. Table 3 shows an example of the profile of preference functions* $R = (R_j)_{j \in J}$ *defined in the proof of Proposition 1 for this case (for each* $j \in J$*,* $R_j \in \mathcal{R}(I_j, x_j^f, x_j^e)$ *and* $R_j(a) = R_j(b)$*).*

R_1			R_2			R_3		
a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
a	a	a	a	a	c	b	b	c
b	b	c	c	c	a	c	c	b
c	c	b	b	b	b	a	a	a

Table 3 Profile $R = (R_j)_{j \in J}$ in Example 5.

The necessary condition for the implementability of the SOCR in Nash equilibrium stated in Proposition 1 is also sufficient when there are at least three jurors. The reason is that, under this condition, the SOCR satisfies "essential monotonicity", a sufficient condition for Nash implementation when there are at least three agents (see Danilov, 1992).[3] This result follows as

[2] In fact, the same proof is valid for every equilibrium concept.

[3] The reason that we focus on "essential monotonicity" rather than "monotonicity" is that, under the necessary condition for the implementability of the SOCR stated in Proposition 1, the SOCR may not satisfy "no veto power" (monotonicity is a sufficient condition for Nash implementation when combined with no veto power; see Maskin, 1999). Essential monotonicity is a sufficient property for Nash implementation by itself.

a corollary from Amorós (2013, Proposition 2) and we include the proof for completeness. First, we define essential monotonicity.

Definition 6 *The SOCR is* ***essentially monotonic*** *when the jury configuration is* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ *if, for all* $(R, w_d), (\hat{R}, \hat{w}_d) \in S(I, x^f, x^e)$*, if* $w_d \neq \hat{w}_d$*, then there exist* $j \in J$ *and* $\bar{w} \in N$ *such that:*

(i) $w_d \ R_j(w_d) \ \bar{w}$ *and* $\bar{w} \ \hat{P}_j(\hat{w}_d) \ w_d$*, and*

(ii) there exist $(\tilde{R}, \tilde{w}) \in S(I, x^f, x^e)$ *such that, for all* $w \in N$*, if* $w \ P_j(w_d) \ w_d$ *then* $w \ \tilde{P}_j(\tilde{w}) \ \bar{w}$*.*

Proposition 2 *Suppose that there are at least three jurors. Let* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ *be such that, for each pair of contestants, there is at least one juror who is impartial with respect to them. Then, the SOCR is implementable in Nash equilibrium when the jury configuration is* (I, x^f, x^e)*.*

Proof. Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be such that, for every $xy \in 2^N_2$, there is some $j \in J$ such that $xy \in I_j$. We show that then the SOCR is essentially monotonic. Let $(R, w_d), (\hat{R}, \hat{w}_d) \in S(I, x^f, x^e)$ be such that $w_d \neq \hat{w}_d$. Let $j \in J$ be such that $w_d\hat{w}_d \in I_j$. Then, $w_d \ P_j(w_d) \ \hat{w}_d$ and $\hat{w}_d \ \hat{P}_j(\hat{w}_d) \ w_d$, and therefore point (i) of the definition of essential monotonicity is fulfilled for $\bar{w} = \hat{w}_d$.

Let $\tilde{R}_j \in \mathcal{R}(I_j, x^f_j, x^e_j)$ be such that, for all $w \in N$, if $w \ P_j(w_d) \ w_d$ then $w \ \tilde{P}_j(w_d) \ \hat{w}_d$. To see that such a preference function exists, note that (1) $w_d \neq \hat{w}_d$, (2) $\hat{w}_d \neq x^f_j$ (since $w_d \ P_j(w_d) \ \hat{w}_d$ and $R_j \in \mathcal{R}(I_j, x^f_j, x^e_j)$), and (3) if $w \ P_j(w_d) \ w_d$, then (3.1) $w \neq w_d$, (3.2) $w \neq \hat{w}_d$ (since $w_d \ P_j(w_d) \ \hat{w}_d$), and (3.3) $w \neq x^e_j$ (since $R_j \in \mathcal{R}(I_j, x^f_j, x^e_j)$). Then, point (ii) of the definition of essential monotonicity is fulfilled for that $\tilde{R}_j$, any $(\tilde{R}_k)_{k \in J \setminus \{j\}} \in \times_{k \in J \setminus \{j\}} \mathcal{R}(I_k, x^f_k, x^e_k)$, and $\tilde{w} = w_d$. ■

2.3 Natural implementation

From now on, we assume that there are three jurors and that each of them is impartial with respect to a different pair of contestants. The reason why we assume that there are three jurors is that essential monotonicity is a sufficient condition for Nash implementation when there are at least three agents (see Danilov, 1992). We could have more than three jurors, but this would constitute an unnecessary complication of the model. We also make the simplifying assumption that there are only three contestants. The reason for that assumption is that we are interested in analyzing the effect of having friends or enemies in the implementability of the SOCR via "natural" mechanisms. Having only three contestants allows us to identify each contestant with a different juror who could be his friend or his enemy.[4]

Assumption 1
(i) $J = \{1, 2, 3\}$ *(there are three jurors),*
(ii) $N = \{a, b, c\}$ *(there are three contestants), and*
(iii) $bc \in I_1$, $ac \in I_2$, and $ab \in I_3$ *(for each pair of contestants, there is at least one different juror who is impartial with respect to them).*

Hereafter we suppose that Assumption 1 is fulfilled. Under this assumption, the SOCR is implementable in Nash equilibrium since it satisfies essential monotonicity (Proposition 2). The mechanisms used to prove that essential monotonicity is a sufficient condition for Nash implementation are mechanisms à la Maskin (Maskin, 1999). This type of mechanisms have received criticism for being unnatural, having too complex message spaces, and making use of extraneous devices such as "integer games" or "modulo games" (see Jackson, 1992).

In this paper, however, we are interested in implementing the SOCR through "simple" and "natural" mechanisms that can be used in the real world. Although, in general, it is not easy to define what simple and natural means, we can be more specific when talking about mechanisms used to elicit the deserving winner.

Regarding simplicity, we want our mechanisms to have straightforward message spaces. In our setting, the most straightforward mechanisms are those in which each juror only has to announce which contestant he thinks should win the competition.

Definition 7 *A mechanism $\Gamma = (M, g)$ implementing the SOCR in Nash equilibrium is **straightforward** if, for each $j \in J$, $M_j = N$.*

[4]Of course, this is a simplification and one could consider a more general model in which there are more than three contestants. In this case, however, there would be many different possibilities to fulfill the condition that for each pair of contestants there is at least one juror who is impartial with respect to them (and many of these possibilities would not be symmetrical). Our goal in this paper is to study whether having jurors with friends is better than having jurors with enemies, or the other way around, when it comes to implement the SOCR via "natural" mechanisms. For this comparison to be clear, we prefer to use a simplified and symmetric model in which each contestant can be identified with a juror who could be his friend or enemy.

We also want the outcome function of the mechanisms to be as natural as possible. Since, in a straightforward mechanism, we are asking each juror to tell us who is the deserving winner, it is natural to require that telling the truth (announcing the deserving winner) is always an equilibrium.

Definition 8 *We say that a straightforward mechanism $\Gamma = (M, g)$* ***naturally implements*** *the SOCR in Nash equilibrium when the jury configuration is $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ if:*

(i) Γ implements the SOCR in Nash equilibrium when the jury configuration is $(I, x^f, x^e) \in \mathcal{E}^{|J|}$, and

(ii) for each $(R, w_d) \in S(I, x^f, x^e)$, we have $(w_d, ..., w_d) \in N(\Gamma, R, w_d)$.[5]

The SOCR is **naturally implementable** in Nash equilibrium when the jury configuration is (I, x^f, x^e) if there exists a straightforward mechanism that naturally implements it.

As we have seen, Assumption 1 is a sufficient condition for the SOCR to be implementable in Nash equilibrium. However, as we show in the next section, this condition is not strong enough to guarantee that the SOCR is naturally implementable. Our aim is then to study what additional conditions must be imposed on the configuration of the jury to ensure that the SOCR is naturally implementable.

3 Conditions for natural implementation in Nash equilibrium

Assumption 1 guarantees that the planner has enough information to design a mechanism à la Maskin that implements the SOCR in Nash equilibrium. However, as we have argued in the previous section, this type of mechanisms have received much criticism for being unnatural. The reason that these mechanisms are unnatural is that they are designed to characterize what can be implemented and therefore, they have to handle a large number of problems. This is precisely what happens in our setting. If all the information that the planner has is that juror 1 is impartial with respect to bc, juror 2 is impartial with respect to ac, and juror 3 is impartial with respect to ab, then

[5] Conditions (i) and (ii) of the definition of natural implementation imply that the outcome function of the mechanism satisfies unanimity; *i.e,* for each $(R, w_d) \in S(I, x^f, x^h)$, $g(w_d, ..., w_d) = w_d$.

the same mechanism must work in many different situations: (i) when a is a friend of juror 1, b is a friend of juror 2, and c is a friend of juror 3; (ii) when a is a friend of juror 1, b is an enemy of juror 2, and c is an enemy of juror 3; (iii) when a is a friend of juror 1, b is an enemy of juror 2, and c is a friend of juror 3; etc. Our next proposition shows that there is no straightforward mechanism that naturally implements the SOCR in all these situations.

Proposition 3 *Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration such that no juror has a known friend or enemy. Then, the SOCR is not naturally implementable in Nash equilibrium when the jury configuration is (I, x^f, x^e).*

Proof. Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying the conditions of the statement. Then, for each $j \in J$, $x_j^f = \emptyset$ and $x_j^e = \emptyset$. Therefore, any profile of preference functions $R \in \mathcal{R}^{|J|}$ satisfying $b\ P_1(b)\ c$, $c\ P_1(c)\ b$, $a\ P_2(a)\ c$, $c\ P_2(c)\ a$, $a\ P_3(a)\ b$, and $b\ P_3(b)\ a$ is admissible when the jury configuration is (I, x^f, x^e) (see Table 4). Suppose, by contradiction, that there is a straightforward mechanism $\Gamma = (M, g)$ that naturally implements the SOCR in Nash equilibrium when the jury configuration is (I, x^f, x^e).

Step 1. $g(a, a, a) = a$, $g(b, b, b) = b$, and $g(c, c, c) = c$.

It follows from conditions (i) and (ii) of the definition of natural implementation.

Step 2. $g(m_1, a, a) = a$ for all $m_1 \in N$, $g(b, m_2, b) = b$ for all $m_2 \in N$, and $g(c, c, m_3) = c$ for all $m_3 \in N$.

There exists an admissible state of the world $(R, a) \in S(I, x^f, x^e)$ such that $x\ P_1(a)\ a$ for every $x \in N \backslash \{a\}$. Then, since $g(a, a, a) = a$, $(a, a, a) \in N(\Gamma, R, a)$, and a is the worst alternative for juror 1 according to the preference relation $R_1(a)$, it must be that $g(m_1, a, a) = a$ for all $m_1 \in N$. The cases $g(b, m_2, b) = b$ for all $m_2 \in N$ and $g(c, c, m_3) = c$ for all $m_3 \in N$ are analogous.

Step 3. $g(a, a, m_3) \neq c$ for all $m_3 \in N$, $g(m_1, b, b) \neq a$ for all $m_1 \in N$, and $g(c, m_2, c) \neq b$ for all $m_2 \in N$.

There exists an admissible state of the world $(R, a) \in S(I, x^f, x^e)$ such that $c\ P_3(a)\ a$. Then, since $g(a, a, a) = a$ and $(a, a, a) \in N(\Gamma, R, a)$, it must be that $g(a, a, m_3) \neq c$ for all $m_3 \in N$. The cases $g(m_1, b, b) \neq a$ for all $m_1 \in N$ and $g(c, m_2, c) \neq b$ for all $m_2 \in N$ are analogous.

Step 4. $g(c, a, b) = b$.

There exists an admissible state of the world $(R, b) \in S(I, x^f, x^e)$ such that $a\ R_2(b)\ x$ for every $x \in N \backslash \{a\}$ and $a\ R_3(b)\ c$. Since, by Step 2,

$g(c, a, a) = a$, then $(c, a, a) \notin N(\Gamma, R, b)$. Note that, when the state of the world is (R, b), neither juror 2 nor Juror 1 (by Step 2) have incentives to deviate unilaterally from (c, a, a). Moreover, juror 3 only has incentives to deviate unilaterally from (c, a, a) if, by doing so, b is the winner. Since, by Step 3, $g(c, a, c) \neq b$, then it must be that $g(c, a, b) = b$.

Step 5. There exists an admissible state of the world $(R, c) \in S(I, x^f, x^e)$ such that $(b, a, b) \in N(\Gamma, R, c)$.

There exists an admissible state of the world $(R, c) \in S(I, x^f, x^e)$ such that $b\ R_1(c)\ a$ and $b\ P_3(c)\ x$ for every $x \in N \backslash \{b\}$. Then, since $g(b, a, b) = b$ (by Step 2), $g(a, a, b) \neq c$ (by Step 3), and $g(c, a, b) = b$ (by Step 4), we have that juror 1 does not have incentives to deviate unilaterally from (b, a, b) when the state of the world is (R, c). Since $g(b, m_2, b) = b$ for all $m_2 \in N$ (by Step 2), then juror 2 does not have incentives to deviate unilaterally from (b, a, b) when the state of the world is (R, c). Finally, since b is the best alternative according to the preference relation $R_3(c)$, juror 3 does not have incentives to deviate unilaterally from (b, a, b) when the state of the world is (R, c).

Since (by Step 2) $g(b, a, b) = b$, Step 5 contradicts that Γ implements the SOCR in Nash equilibrium. ■

R_1			R_2			R_3		
a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	b	c	a	⋮	c	a	b	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	c	b	c	⋮	a	b	a	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 4 Admissible preference functions in Proposition 3.

Sometimes the planner has more information about the jury configuration than that stated in Assumption 1. In particular, in some situations the planner knows that some jurors would like to favor/harm some contestants. Contrary to what one could think at first sight, knowing that a juror wants to favor/harm some contestant may facilitate the natural implementation of the SOCR, as it reduces the set of admissible states of the world (if the planner does not know if a juror wants to favor or harm a contestant, there will be admissible states of the world in which the juror wants to favor the contestant, admissible states of the world in which the juror wants to harm the contestant, and admissible states of the world in which the juror does not want to favor or to harm the contestant).

3.1 Natural implementation in Nash equilibrium when jurors have friends

Suppose, that the jury is made up of the contestants themselves, so that the planner knows that each juror wants to win the competition and is impartial with respect to the rest. In this case, we can identify juror 1 with contestant a, juror 2 with contestant b, and juror 3 with contestant c, and we have $x_1^f = a$, $x_2^f = b$, and $x_3^f = c$. Table 5 shows the preference functions that are admissible in this case. Under this jury configuration, the SOCR can be naturally implemented in Nash equilibrium with the following straightforward mechanism.

Mechanism Γ^f:

Let $\Gamma^f = (M, g)$ be such that, for each $j \in J$, $M_j = N$, and for each $m = (m_j)_{j \in J} \in M$, $g(m)$ is defined by the following three rules:
Rule 1. If, for each $j \in J$, $m_j = x$, then $g(m) = x$ (if all jurors announce the same contestant x, then x is chosen).
Rule 2. If there is $k \in J$ and $x \in N$ such that, for each $j \neq k$, $m_j = x$, but $m_k \neq x$, then

$$g(m) = \begin{cases} m_k; \text{ if } m_k \neq x_k^f \\ x; \text{ otherwise} \end{cases}$$

(if there is only one dissident $k \in J$ announcing $m_k \neq x$, then m_k is chosen only if m_k is not the friend of k).
Rule 3. In all other cases $g(m) = a$ (if more than two jurors disagree on their messages then an arbitrary contestant known by all jurors, say a, is chosen).

Proposition 4 *Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration such that each juror has a known friend. Then, mechanism Γ^f naturally implements the SOCR in Nash equilibrium when the jury configuration is (I, x^f, x^e).*

The proof of this result is in the Appendix.

R_1			R_2			R_3		
a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
a	a	a	b	b	b	c	c	c
$\vdots$	b	c	a	$\vdots$	c	a	b	$\vdots$
$\vdots$	c	b	c	$\vdots$	a	b	a	$\vdots$

Table 5 Admissible preference functions in Proposition 4.

From our previous analysis, we know that (i) if no juror has a known friend (or enemy), then natural implementation of the SOCR is not possible (Proposition 3), and (ii) if all jurors have known friends, then natural implementation of the SOCR is possible (Proposition 4). Then, a natural question arises: how many jurors with known friends are needed at least for the SOCR to be naturally implementable?

The answer is simple: just one. Suppose, without loss of generality, that all the information the planner has is that c is a friend of juror 3; *i.e.*, the jury configuration $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ is such that $x_1^f = \emptyset$, $x_2^f = \emptyset$, $x_3^f = c$, $x_1^e = \emptyset$, $x_2^e = \emptyset$, $x_3^e = \emptyset$. Table 6 shows the preference functions that are admissible in this case. The following straightforward mechanism naturally implements the SOCR in Nash equilibrium under this jury configuration.

Mechanism Γ_3^f:

$\Gamma_3^f = (M, g)$ is such that, for each $j \in J$, $M_j = N$. For each $m = (m_j)_{j \in J} \in M$, $g(m)$ is defined by the following three rules:

Rule 1. If, for each $j \in J$, $m_j = x$, then $g(m) = x$; *i.e.*, if all jurors announce the same contestant x, then x is chosen.

Rule 2. If there is $k \in J$ and $x \in N$ such that, for each $j \neq k$, $m_j = x$, but $m_k \neq x$, then

$$g(m) = \begin{cases} m_k; \text{ if } k \neq 3 \text{ and } xm_k \in I_k \\ x; \text{ if either (i) } k = 1 \text{ and } xm_1 \notin I_1, \\ \quad \text{or (ii) } k = 2,\ xm_2 \notin I_2, \text{ and } x \neq x_3^f \\ \quad \text{or (iii) } k = 3 \text{ and } xm_3 \in I_3 \\ y \notin \{x, m_k\}; \text{ otherwise} \end{cases}$$

i.e.; if there is only one dissident k announcing $m_k \neq x$ then, given an arbitrary juror different from 3 (the juror with the known friend), say juror 2, we proceed as follows: (1) if the dissident is impartial with respect to the pair xm_k, then m_k is chosen, unless the deviator is 3 (the juror with the

known friend), in which case x is chosen; (2) if the dissident is not impartial with respect to the pair xm_k, then x is chosen, unless (2.1) the dissident is 3 (the juror with the known friend) or (2.2) the dissident is 2 (the arbitrary juror different from 3) and the rest of jurors are announcing the friend of 3, in which case a contestant who is neither x nor m_k is chosen.
Rule 3. If, for each $j, k \in N$, $m_j \neq m_k$, then

$$g(m) = \begin{cases} a; \text{ if, for each } j \in J,\ m_j \notin I_j \\ x_3^f; \text{ otherwise} \end{cases}$$

i.e., if more than two jurors disagree on their messages then, given the arbitrary juror defined in Rule 2, say juror 2, we proceed as follows: the friend of juror 3 is chosen unless, for each juror j, $m_j \notin I_j$, in which case a contestant in I_2 different from x_3^f is chosen.

Proposition 5 *Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration such that only one juror (say juror 3) has a known friend. Then, mechanism Γ_3^f naturally implements the SOCR in Nash equilibrium when the jury configuration is (I, x^f, x^e).*

The proof of this proposition is in the Appendix.

R_1			R_2			R_3		
a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
⋮	⋮	⋮	⋮	⋮	⋮	c	c	c
⋮	b	c	a	⋮	c	a	b	⋮
⋮	⋮	⋮	⋮	⋮	⋮	b	a	⋮
⋮	c	b	c	⋮	a			
⋮	⋮	⋮	⋮	⋮	⋮			

Table 6 Admissible preference functions in Proposition 5.

3.2 Natural implementation in Nash equilibrium when jurors have enemies

Similarly to what happens in the case where all jurors have friends, if the planner knows that each juror has an enemy, then the SOCR can be naturally implemented in Nash equilibrium. Suppose that $x^e_1 = a$, $x^e_2 = b$, and $x^e_3 = c$. Table 7 shows the preference functions that are admissible in this case. Under this jury configuration, the SOCR can be naturally implemented in Nash equilibrium with the following straightforward mechanism.

Mechanism Γ^e:

$\Gamma^e = (M, g)$ is such that, for each $j \in J$, $M_j = N$. For each $m = (m_j)_{j\in J} \in M$, $g(m)$ is defined by the following three rules:
Rule 1. If, for each $j \in J$, $m_j = x$, then $g(m) = x$; *i.e.*, if all jurors announce the same contestant x, then x is chosen.
Rule 2. If there is $k \in J$ and $x \in N$ such that, for each $j \neq k$, $m_j = x$, but $m_k \neq x$, then

$$g(m) = \begin{cases} x\text{; if either (i) } x = x^e_k \text{ or (ii) } k = 1 \text{ and } m_1 = x^e_1 \\ y \notin \{x, m_k\}\text{; if } k \neq 1 \text{ and } x \neq x^e_k \\ m_1\text{; if } k = 1,\ x \neq x^e_1, \text{ and } m_1 \neq x^e_1 \end{cases}$$

i.e.; if there is only one dissident k announcing $m_k \neq x$ then, given an arbitrary juror known by all jurors, say juror 1, we proceed as follows: (1) if all jurors but the dissident are announcing the enemy of the dissident, then that contestant is chosen; (2) if all jurors but the dissident are not announcing the enemy of the dissident, then (2.1) if the dissident is not juror 1, then a contestant who is neither x nor m_k is chosen, and (2.2) if the dissident is juror 1, then (2.2.1) if 1 is not announcing his own enemy, then m_1 is chosen, and (2.2.2) if 1 is announcing his own enemy, then the contestant announced by the rest of jurors is chosen.
Rule 3. If, for each $j, k \in N$, $m_j \neq m_k$, then

$$g(m) = \begin{cases} m_k\text{; if there is } k \in J \text{ such that } m_k = x^e_k \text{ and, for each } j \neq k,\ m_j \neq x^e_j \\ x^e_1\text{; otherwise} \end{cases}$$

i.e., if more than two jurors disagree on their messages then, given the arbitrary juror defined in Rule 2, say juror 1, we proceed as follows: if only one of the jurors is announcing his enemy, then that contestant is chosen; otherwise, the enemy of juror 1 is chosen.

Proposition 6 *Let* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ *be a jury configuration such that each juror has a known enemy. Then, mechanism* Γ^e *naturally implements the SOCR in Nash equilibrium when the jury configuration is* (I, x^f, x^e)*.*

The proof is in the Appendix.

R_1				R_2				R_3		
a	**b**	**c**		**a**	**b**	**c**		**a**	**b**	**c**
$\vdots$	b	c		a	$\vdots$	c		a	b	$\vdots$
$\vdots$	c	b		c	$\vdots$	a		b	a	$\vdots$
a	a	a		b	b	b		c	c	c

Table 7 Admissible preference functions in Proposition 6.

As in the friends case, we know that (i) if no juror has a known enemy (or friend), then natural implementation of the SOCR is not possible (Proposition 3), and (ii) if all jurors have known enemies, then natural implementation of the SOCR is possible (Proposition 6). Therefore, the same question arises here: how many jurors with known enemies are needed at least for the SOCR to be naturally implementable?

The answer to this question is different to that in the "friends case". The next proposition shows that having one juror with a known enemy is not sufficient for the SOCR to be natural implementable in Nash equilibrium. This is surprising in that the size of the set of admissible states of the world when only one juror has a friend is equal to the size of this set when only one juror has an enemy.

Proposition 7 *Let* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ *be a jury configuration such that only one juror (say juror* 3*) has a known enemy (and no juror has a known friend). Then, the SOCR is not naturally implementable in Nash equilibrium when the jury configuration is* (I, x^f, x^e)*.*

Proof. Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying the conditions of the statement. Then any profile of preference functions $R \in \mathcal{R}^{|J|}$ satisfying $b\ P_1(b)\ c$, $c\ P_1(c)\ b$, $a\ P_2(a)\ c$, $c\ P_2(c)\ a$, $a\ P_3(a)\ b\ P_3(a)\ c$, $b\ P_3(b)\ a\ P_3(b)\ c$, $a\ P_3(c)\ c$, and $b\ P_3(c)\ c$ is admissible when the jury configuration is (I, x^f, x^e) (see Table 8). Suppose, by contradiction, that there is a straightforward mechanism $\Gamma = (M, g)$ that naturally implements the SOCR in Nash equilibrium when the jury configuration is (I, x^f, x^e).

Step 1. $g(a,a,a) = a$, $g(b,b,b) = b$, and $g(c,c,c) = c$.

It is analogous to Step 1 in Proposition 3.

Step 2. $g(m_1,a,a) = a$ for all $m_1 \in N$, $g(b,m_2,b) = b$ for all $m_2 \in N$, and $g(c,c,m_3) = c$ for all $m_3 \in N$.

It is analogous to Step 1 in Proposition 3.

Step 3. $g(a,m_2,a) \neq b$ for all $m_2 \in N$, and $g(m_1,c,c) \neq a$ for all $m_1 \in N$.

There exists an admissible state of the world $(R,a) \in S(I,x^f,x^e)$ such that $b\ P_2(a)\ a$. Then, since by Step 1 $g(a,a,a) = a$ and $(a,a,a) \in N(\Gamma,R,a)$, it must be that $g(a,m_2,a) \neq b$ for all $m_2 \in N$. The case $g(m_1,c,c) \neq a$ for all $m_1 \in N$ is analogous.

Step 4. $g(b,c,a) = b$.

Note that there exists an admissible state of the world $(R,b) \in S(I,x^f,x^e)$ such that (i) $c\ P_2(b)\ x$ for every $x \in N \backslash \{c\}$ and (ii) $c\ P_1(b)\ a$. Moreover, by Step 2, $g(c,c,m_3) = c$ for all $m_3 \in N$. Since $g(c,c,a) = c$, $(c,c,a) \notin N(\Gamma,R,b)$, and therefore juror 1 must have incentives to unilaterally deviate from (c,c,a). Hence, it must be that $g(m_1,c,a) = b$ for some $m_1 \in N$. Since, by Step 3, $g(a,c,a) \neq b$, we have $g(b,c,a) = b$.

Step 5. Either $g(b,c,c) = a$ or $g(b,c,a) = a$.

Note that there exists an admissible state of the world $(R,a) \in S(I,x^f,x^e)$ such that $b\ P_1(a)\ x$ for every $x \in N \backslash \{a\}$. Moreover, by Step 2, $g(b,m_2,b) = b$ for all $m_2 \in N$. Since $g(b,c,b) = b$, $(b,c,b) \notin N(\Gamma,R,a)$, and therefore juror 3 must have incentives to unilaterally deviate from (b,c,b). Hence, it must be that $g(b,c,c) = a$ or $g(b,c,a) = a$.

Since, by Steps 3 and 4, $g(b,c,c) \neq a$ and $g(b,c,a) \neq a$, Step 5 contradicts that Γ implements the SOCR in Nash equilibrium. ■

R_1			R_2			R_3		
a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
⋮	⋮	⋮	⋮	⋮	⋮	a	b	⋮
⋮	b	c	a	⋮	c	b	a	⋮
⋮	⋮	⋮	⋮	⋮	⋮	c	c	c
⋮	c	b	c	⋮	a			
⋮	⋮	⋮	⋮	⋮	⋮			

Table 8 Admissible preference functions in Proposition 7.

Despite the above result, we do not need to have all jurors with known enemies to naturally implement the SOCR in Nash equilibrium: Proposition 8 shows that it suffices to have two of these jurors in the jury.

Suppose, without loss of generality, that all the information the planner has is that b is an enemy of juror 2 and c is an enemy of juror 3; *i.e.*, the jury configuration $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ is such that $x_1^f = \emptyset$, $x_1^e = \emptyset$, $x_2^e = b$, $x_3^e = c$. Table 9 shows the preference functions that are admissible in this case. The following straightforward mechanism naturally implements the SOCR in Nash equilibrium under this jury configuration.

Mechanism $\Gamma^e_{2,3}$:

$\Gamma^e_{2,3} = (M, g)$ is such that, for each $j \in J$, $M_j = N$. For each $m = (m_j)_{j \in J} \in M$, $g(m)$ is defined by the following three rules:
Rule 1. If, for each $j \in J$, $m_j = x$, then $g(m) = x$; *i.e.*, if all jurors announce the same contestant x, then x is chosen.
Rule 2. If there is $k \in J$ and $x \in N$ such that, for each $j \neq k$, $m_j = x$, but $m_k \neq x$, then

$$g(m) = \begin{cases} m_k; \text{ if } k = 1 \text{ and } xm_1 \in I_1 \\ x; \text{ if either (i) } k = 1 \text{ and } xm_1 \notin I_1, \\ \quad \text{or (ii) } k \in \{2, 3\} \text{ and } x = x_k^e \\ y \notin \{x, m_k\}; \text{ otherwise} \end{cases}$$

i.e.; if there is only one dissident k announcing $m_k \neq x$, then we proceed as follows: (1) if the dissident is juror 1 (the juror without known enemies or friends) then (1.1) m_1 is chosen if 1 is impartial with respect to the pair xm_1, (1.2) otherwise, x is chosen; (2) if the dissident is one of the jurors with a known enemy (2 or 3) then (2.1) if the contestant announced by the rest of jurors, x, is the enemy of the dissident, then that contestant is chosen, (2.2) otherwise, a contestant who is neither x nor m_k is chosen.
Rule 3. If, for each $j, k \in N$, $m_j \neq m_k$, then

$$g(m) = \begin{cases} a; \text{ if, for each } k \in \{2, 3\}, m_k \neq x_k^e \\ x_2^e; \text{ if } m_2 = x_2^e \\ x_3^e; \text{ otherwise} \end{cases}$$

i.e., if more than two jurors disagree on their messages then, given an arbitrary juror between those with a known enemy (say juror 2) we proceed as follows: (1) if none of the jurors with a known enemy is announcing his enemy, then the contestant who is not the enemy of any juror, a, is chosen; (2) if juror 2 is announcing his enemy, then that contestant is chosen; (3) if juror 2 is not announcing his enemy but juror 3 is announcing his enemy, then the enemy of 3 is chosen.

Proposition 8 *Let* $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ *be a jury configuration such that only two jurors (say jurors* 2 *and* 3*) have known enemies (and no juror has a known friend). Then, mechanism* $\Gamma^e_{2,3}$ *naturally implements the SOCR in Nash equilibrium when the jury configuration is* (I, x^f, x^e)*.*

The proof of this proposition is in the Appendix.

R_1			R_2			R_3		
a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
⋮	⋮	⋮	a	⋮	c	a	b	⋮
⋮	b	c	c	⋮	a	b	a	⋮
⋮	⋮	⋮	b	b	b	c	c	c
⋮	c	b						
⋮	⋮	⋮						

Table 9 Admissible preference functions in Proposition 8.

4 Conclusion

We consider the problem of a jury choosing the winner of a contest. There is a deserving winner which is common knowledge among the jurors, but not known by the planner. Moreover, some of the jurors may be partial and want to favor/harm some contestants over others. We study conditions on the configuration of the jury so that it is possible to induce the jurors to always choose the deserving winner, whoever he is, when we restrict ourselves to mechanisms that must satisfy two conditions: (1) each juror only has to announce which contestant he thinks should win the competition and (2) announcing the deserving winner must be an equilibrium. We call this notion natural implementation. Amorós (2013) provides a necessary condition for Nash implementation of the deserving winner: for each pair of contestants, the planner must know at least one juror who is impartial with respect to them. This condition, however, is not sufficient for natural implementation in Nash equilibrium. In addition, the planner must know if some of the other contestants are friends or enemies of the jurors (this information reduces the size of the set of admissible states). We show that, given the choice, we better select jurors with friends than jurors with enemies. The reason is that the number of jurors with friends that the planner needs to know to implement the deserving winner with simple and natural mechanisms is less than the number of jurors with enemies that the planner would need to know for it.

Appendix

PROOF OF PROPOSITION 4:

Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying conditions of the statement. Then $I_1 = \{bc\}$, $I_2 = \{ac\}$, $I_3 = \{ab\}$, $x_1^f = a$, $x_2^f = b$, and $x_3^f = c$. First, note that, for each $(R, w_d) \in S(I, x^f, x^e)$, $(w_d, ..., w_d) \in N(\Gamma^f, R, w_d)$. To see this, note that, on the one hand, if $x_j^f = w_d$, juror j has no incentives to deviate from $(w_d, ..., w_d)$ since he is getting his best alternative. On the other hand, if $x_j^f \neq w_d$, then x_j^f is the most preferred alternative for juror j and w_d is his second most preferred alternative. In this case, juror j does not want to deviate from $(w_d, ..., w_d)$ since, by Rule 2 of mechanism Γ^f, he can only obtain an alternative that is less preferred than w_d for him.

Next, we show that, for each $(R, w_d) \in S(I, x^f, x^e)$ and $m \in N(\Gamma^f, R, w_d)$, $g(m) = w_d$ (every Nash equilibrium results in the deserving winner). Suppose, on the contrary, that $g(m) = x \neq w_d$. Next we show that any such profile m is such that some juror can benefit by deviating unilaterally, which contradicts that $m \in N(\Gamma^f, R, w_d)$. Let j be the juror such that $xw_d \in I_j$. Then $w_d \ P_j(w_d) \ x$ and $x_j^f \notin \{w_d, x\}$. We distinguish three cases:

Case 1. Rule 1 of mechanism Γ^f applies to m. Then, all jurors are announcing x in m. Therefore, juror j can improve his welfare if he deviates and announces w_d (in this case Rule 2 applies and, since $w_d \neq x_j^f$, $g(m) = w_d$).

Case 2. Rule 2 of mechanism Γ^f applies to m. We distinguish three subcases.

Subcase 2.1. The dissident in m is j. Then $m_j \neq w_d$ (otherwise, since $x_j^f \neq w_d$, $g(m) = w_d$). Therefore, juror j can improve if he deviates and announces $\hat{m}_j = w_d$ (if all other jurors are announcing w_d then Rule 1 would apply and $g(\hat{m}_j, m_{-j}) = w_d$; if all other jurors are announcing some $y \neq w_d$, then Rule 2 would apply and, since $x_j^f \neq w_d$, $g(\hat{m}_j, m_{-j}) = w_d$).

Subcase 2.2. The dissident in m is $k \neq j$ and $m_k = x$. Then, since $g(m) = x$, by Rule 2 we have $x_k^f \neq x$. Therefore, $w_d \ P_k(w_d) \ x$. If all jurors but k are announcing w_d, then k can improve if he announces $\hat{m}_k = w_d$ (in this case, Rule 1 would apply and $g(\hat{m}_k, m_{-k}) = w_d$). If all jurors but k are announcing $y \notin \{x, w_d\}$, then j can improve if he announces $\hat{m}_j = x$ (since $x_j^f \notin \{w_d, x\}$, then $x_j^f = y$; therefore, for juror $l \notin \{j, k\}$, we have $x_l^f \neq y$; then, by Rule 2, $g(\hat{m}_j, m_{-j}) = y$).

Subcase 2.3. The dissident in m is $k \neq j$ and $m_k \neq x$. Then, all other jurors are announcing x. Since $g(m) = x$, by Rule 2 we have $m_k = x_k^f$. If $m_k = w_d$, then j can improve if he announces $\hat{m}_j = w_d$ (since $x \notin \{x_j^f, x_k^f\}$, then, for juror $l \notin \{j, k\}$, we have $x_l^f = x$; then, by Rule 2, $g(\hat{m}_j, m_{-j}) = w_d$). If $m_k \neq w_d$, then k can improve if he announces $\hat{m}_k = w_d$ (since $x_k^f = m_k \notin \{x, w_d\}$ and all other agents are announcing x then, by Rule 2, $g(\hat{m}_k, m_{-k}) = w_d$; moreover, since $x_k^f \neq x$, $w_d \ P_k(w_d) \ x$).

Case 3. Rule 3 of mechanism Γ^f applies to m. We distinguish three subcases.

Subcase 3.1. Juror j is announcing $m_j \notin \{w_d, x\}$. In this case $m_j = x_j^f$. Let k be the juror such that $m_k = w_d$ and let l be the juror such that $m_l = x$. If $w_d = x_k^f$, then $x = x_l^f$ and j can improve if he announces $\hat{m}_j = w_d$ (in this case Rule 2 would apply and $g(\hat{m}_j, m_{-j}) = w_d$). If $w_d \neq x_k^f$, then $x = x_k^f$, $w_d = x_l^f$, and l can improve if he announces $\hat{m}_l = w_d$ (in this case Rule 2 would apply and $g(\hat{m}_l, m_{-l}) = w_d$).

Subcase 3.2 Juror j is announcing $m_j = x$. In this case $m_j \neq x_j^f$. Let k be the juror such that $m_k = w_d$ and let l be the juror such that $m_l \notin \{x, w_d\}$. Then $m_l = x_j^f \neq x_l^f$. Therefore, juror j can improve if he announces $\hat{m}_j = w_d$ (in this case Rule 2 would apply and $g(\hat{m}_j, m_{-j}) = x_j^f$).

Subcase 3.3. Juror j is announcing $m_j = w_d$. Then $m_j \neq x_j^f$. Let k be the juror such that $m_k = x$ and let l be the juror such that $m_l \notin \{x, w_d\}$. Then $m_l = x_j^f \neq x_l^f$. Therefore, juror j can improve if he announces $\hat{m}_j = x$ (in this case Rule 2 would apply and $g(\hat{m}_j, m_{-j}) = x_j^f$). ■

PROOF OF PROPOSITION 5:

Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying conditions of the statement (*i.e.*, $I_1 = \{bc\}$, $I_2 = \{ac\}$, $I_3 = \{ab\}$, $x_3^e = c$, and, for each $j \neq 3$, $x_j^f = \emptyset$ and $x_j^e = \emptyset$.). First, we show that for each $(R, w_d) \in S(I, x^f, x^e)$, $(w_d, ..., w_d) \in N(\Gamma_3^f, R, \ w_d)$. Juror 3 would have incentives to deviate from $(w_d, ..., w_d)$ only if $w_d \neq c$ and, by doing so, c is chosen. However, by Rule 2 of Γ_3^f, if $w_d \neq c$, no unilateral deviation from $(w_d, ..., w_d)$ of juror 3 results in c. By Rule 2, if $w_d = a$, juror 1 cannot avoid that w_d is chosen by unilaterally deviating from $(w_d, ..., w_d)$, while any unilateral deviation of juror 2 results in a or c (and therefore neither 1 nor 2 have incentives to unilaterally deviate). Similarly, if $w_d = b$, juror 2 cannot avoid that w_d is chosen by unilaterally deviating from $(w_d, ..., w_d)$, while any unilateral deviation of juror 1 results in b or c (and therefore neither 1 nor 2 have incentives to unilaterally deviate). Finally, if $w_d = c$, any unilateral deviation from $(w_d, ..., w_d)$ of juror 1 results in b or c, while any unilateral deviation of juror 2 results in a or c (and therefore neither 1 nor 2 have incentives to unilaterally deviate).

Now, we show that, for each $(R, w_d) \in S(I, x^f, x^e)$ and $m \in N(\Gamma_3^f, R, w_d)$, $g(m) = w_d$. Suppose on the contrary that $g(m) = x^* \neq w_d$. Next we show that any such profile m is such that some juror can benefit by deviating unilaterally, which contradicts that $m \in N(\Gamma_{2,3}^e, R, w_d)$. Let $j^* \in J$ be such that $x^* w_d \in I_{j^*}$. Then $w_d \; P_{j^*}(w_d) \; x^*$. We distinguish three cases.

Case 1. Rule 1 of mechanism Γ_3^f applies to m; *i.e.*, for each $j \in J$, $m_j = x^*$.

If $j^* \neq 3$, then j^* can improve his welfare if he deviates and announces $\hat{m}_{j^*} = w_d$ (in this case, Rule 2 applies and, since $x^* w_d \in I_{j^*}$, w_d is chosen). If $j^* = 3$, then juror 3 can improve his welfare if he deviates and announces $\hat{m}_3 \notin \{x^*, w_d\}$ (in this case, Rule 2 applies and, since the dissident is juror 3 and $x^* \hat{m}_3 \notin I_3$, then $w_d \notin \{x^*, \hat{m}_3\}$ is chosen).

Case 2. Rule 2 of mechanism Γ_3^f applies to m; *i.e.*,there is $k \in J$ and $x \in N$ such that, for each $j \neq k$, $m_j = x$, but $m_k \neq x$.

We distinguish three subcases.

Subcase 2.1. Each juror $j \neq k$ announces $m_j = x^*$ and the dissident k announces $m_k \neq x^*$.

By Rule 2, since $g(m) = x^*$, there are three possible subsubcases.

Subsubcase 2.1.1. $k = 1$ and $x^* m_1 \notin I_1$.

By Rule 2, $m = (m_1, m_2, m_3) \in \{(a, b, b), (b, a, a), (a, c, c), \; (c, a, a)\}$. If $m = (a, b, b)$, then $g(m) = b$ and, since we are assuming $g(m) \neq w_d$, we have two possibilities: either (i) $w_d = a$, in which case juror 3 can benefit by announcing $\hat{m}_3 = a$ (a is chosen), or (ii) $w_d = c$, in which case juror 1 can benefit by announcing $\hat{m}_1 = c$ (c is chosen). If $m = (b, a, a)$, then $g(m) = a$ and juror 3 can benefit by announcing $\hat{m}_3 = c$ (c is chosen). If $m = (a, c, c)$, then $g(m) = c$ and we have two possibilities: either (i) $w_d = a$, in which case juror 2 can benefit by announcing $\hat{m}_2 = b$ (a is chosen), or (ii) $w_d = b$, in which case juror 1 can benefit by announcing $\hat{m}_1 = b$ (b is chosen). If $m = (c, a, a)$, then $g(m) = a$ and juror 3 can benefit by announcing $\hat{m}_3 = b$ (c is chosen).

Subsubcase 2.1.2. $k = 2$, $x^* m_2 \notin I_2$, and $x^* \neq c$.

In this case, by Rule 2, $m = (m_1, m_2, m_3) \in \{(a, b, a), (b, a, b), (b, c, b)\}$. If $m = (a, b, a)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_3 = b$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen).

If $m = (b, a, b)$, then $g(m) = b$ and juror 3 can benefit by announcing $\hat{m}_3 = c$ (c is chosen). If $m = (b, c, b)$, then $g(m) = b$ and juror 3 can benefit by announcing $\hat{m}_3 = a$ (c is chosen).

Subsubcase 2.1.3. $k = 3$ and $x^* m_3 \in I_3$.

In this case, by Rule 2, $m = (m_1, m_2, m_3) \in \{(a, a, b), (b, b, a)\}$. If $m = (a, a, b)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_3 = c$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen). If $m = (b, b, a)$, then $g(m) = b$ and we have two possibilities: either (i) $w_d = a$, in which case juror 3 can benefit by announcing $\hat{m}_3 = c$ (a is chosen), or (ii) $w_d = c$, in which case juror 1 can benefit by announcing $\hat{m}_1 = c$ (c is chosen).

Subcase 2.2. Each juror $j \neq k$ announces $x \neq x^*$ and the dissident k announces $m_k = x^*$.

By Rule 2, $m = (m_1, m_2, m_3) \in \{(b, c, c), (c, b, b), (c, a, c), (a, c, a)\}$. If $m = (b, c, c)$, then $g(m) = b$ and juror 3 can benefit by announcing $\hat{m}_3 = a$ (c is chosen). If $m = (c, b, b)$, then $g(m) = c$ and we have two possibilities: either (i) $w_d = a$, in which case juror 2 can benefit by announcing $\hat{m}_2 = c$ (a is chosen), or (ii) $w_d = b$, in which case juror 1 can benefit by announcing $\hat{m}_1 = b$ (b is chosen). If $m = (c, a, c)$, then $g(m) = a$ and juror 3 can benefit by announcing $\hat{m}_3 = b$ (c is chosen). If $m = (a, c, a)$, then $g(m) = c$ and we have two possibilities: either (i) $w_d = a$, in which case juror 2 can benefit by announcing $\hat{m}_2 = a$ (a is chosen), or (ii) $w_d = b$, in which case juror 1 can benefit by announcing $\hat{m}_1 = c$ (b is chosen).

Subcase 2.3. Each juror $j \neq k$ announces $x \neq x^*$ and the dissident k announces $m_k \neq x^*$.

By Rule 2, there are two possible subsubcases.

Subsubcase 2.3.1. $k = 2$, $x = c$ and $cm_2 \notin I_2$.

In this case, $m = (m_1, m_2, m_3) = (c, b, c)$, $g(m) = a$, and juror 3 can benefit by announcing $\hat{m}_3 = a$ (c is chosen).

Subsubcase 2.3.2. $k = 3$ and $xm_k \notin I_3$.

In this case, $m = (m_1, m_2, m_3) \in \{(a, a, c), (c, c, a), (b, b, c), (c, c, b)\}$.

If $m = (a, a, c)$, then $g(m) = b$ and we have two possibilities: either (i) $w_d = a$, in which case juror 3 can benefit by announcing $\hat{m}_3 = a$ (a is chosen), or (ii) $w_d = c$, in which case juror 1 can benefit by announcing $\hat{m}_1 = b$ (c is chosen). If $m = (c, c, a)$, then $g(m) = b$ and juror 3 can benefit by announcing $\hat{m}_3 = c$ (c is chosen). If $m = (b, b, c)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_2 = b$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = a$ (c is chosen). If $m = (c, c, b)$, then $g(m) = a$ and juror 3 can benefit by announcing $\hat{m}_3 = c$ (c is chosen).

Case 3. Rule 3 of mechanism Γ_3^f applies to m; *i.e.*, for each $j, k \in N$, $m_j \neq m_k$.

We distinguish two subcases.

Subcase 3.1. For each $j \in J$, $m_j \notin I_j$.

In this case, $m = (m_1, m_2, m_3) = (a, b, c)$ and $g(m) = a$. If $w_d = b$ then juror 3 can benefit by announcing $\hat{m}_3 = b$ (b is chosen), and if $w_d = c$ then juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen).

Subcase 3.2. There is some $j \in J$ such that $m_j \in I_j$.

Then $m = (m_1, m_2, m_3) \in \{(c, b, a), (a, c, b), (b, a, c), (c, a, b), (b, c, a)\}$ and $g(m) = c$. If $m \in \{(c, b, a), (a, c, b)\}$, we have two possibilities: either (i) $w_d = a$, in which case juror 2 can benefit by announcing $\hat{m}_2 = a$ (a is chosen), or (ii) $w_d = b$, in which case juror 1 can benefit by announcing $\hat{m}_1 = b$ (b is chosen). If $m = (b, a, c)$, we have two possibilities: either (i) $w_d = a$, in which case juror 2 can benefit by announcing $\hat{m}_2 = b$ (a is chosen), or (ii) $w_d = b$, in which case juror 1 can benefit by announcing $\hat{m}_1 = a$ (b is chosen). If $m = (c, a, b)$, we have two possibilities: either (i) $w_d = a$, in which case juror 2 can benefit by announcing $\hat{m}_2 = c$ (a is chosen), or (ii) $w_d = b$, in which case juror 1 can benefit by announcing $\hat{m}_1 = b$ (b is chosen). If $m = (b, c, a)$, we have two possibilities: either (i) $w_d = a$, in which case juror 2 can benefit by announcing $\hat{m}_2 = a$ (a is chosen), or (ii) $w_d = b$, in which case juror 1 can benefit by announcing $\hat{m}_1 = c$ (b is chosen). ■

PROOF OF PROPOSITION 6:

Proof. Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying conditions of the statement (*i.e.*, $I_1 = \{bc\}$, $I_2 = \{ac\}$, $I_3 = \{ab\}$, $x_1^e = a$, $x_2^e = b$, and $x_3^e = c$). We first show that, for each $(R, w_d) \in S(I, x^f, x^e)$, $(w_d, ..., w_d) \in N(\Gamma^e, R, w_d)$. If $x_j^e = w_d$, by Rule 2 of mechanism Γ^e, juror j cannot avoid that w_d is chosen by unilaterally deviating from $(w_d, ..., w_d)$. If $x_j^e \neq w_d$, then juror j does not want to deviate from $(w_d, ..., w_d)$ since w_d is his most preferred alternative at state (R, w_d).

Next, we show that, for each $(R, w_d) \in S(I, x^f, x^e)$ and $m \in N(\Gamma^e, R, w_d)$, $g(m) = w_d$. Suppose by contradiction that $g(m) = x^* \neq w_d$. We show that any such profile m is such that some juror can benefit by deviating unilaterally, which contradicts that $m \in N(\Gamma^e, R, w_d)$. Let $j^* \in J$ be such that $x^* w_d \in I_{j^*}$. Then $w_d \ P_{j^*}(w_d) \ x^*$ and $x^e_{j^*} \notin \{w_d, x^*\}$. We distinguish three cases.

Case 1. Rule 1 of mechanism Γ^e applies to m; *i.e.*, for each $j \in J$, $m_j = x^*$.

If $j^* \neq 1$, then j^* can improve his welfare if he deviates and announces $\hat{m}_{j^*} \notin \{w_d, x^*\}$ (in this case, Rule 2 applies and, since $x^* \neq x^e_{j^*}$, $j^* \neq 1$, and $w_d \notin \{x^*, \hat{m}_{j^*}\}$, then w_d is chosen). If $j^* = 1$, then j^* can improve his welfare if he deviates and announces $\hat{m}_{j^*} = w_d$ (in this case, Rule 2 applies and, since $x^* \neq x^e_{j^*}$, $j^* = 1$, and $\hat{m}_{j^*} = w_d \neq x^e_{j^*}$, then w_d is chosen).

Case 2. Rule 2 of mechanism Γ^e applies to m; *i.e.*,there is $k \in J$ and $x \in N$ such that, for each $j \neq k$, $m_j = x$, but $m_k \neq x$.

We distinguish three subcases.

Subcase 2.1. Each juror $j \neq k$ announces $m_j = x^*$ and the dissident k announces $m_k \neq x^*$.

By Rule 2, since $g(m) = x^*$, there are two possible subsubcases:

Subsubcase 2.1.1. $x^* = x^e_k$.

Note that then $m = (m_1, m_2, m_3) \in \{(b, a, a), (c, a, a), (b, a, b), (b, c, b), (c, c, a), (c, c, b)\}$. For any such profile m, juror j^* can benefit by deviating unilaterally. Thus, if $m = (b, a, a)$, then $g(m) = a$, and since we are assuming $g(m) \neq w_d$, we have two possibilities: either (i) $w_d = b$, in which case $j^* = 3$ and he can benefit by announcing $\hat{m}_3 = b$ (b is chosen), or (ii) $w_d = c$, in which case $j^* = 2$ and he can benefit by announcing $\hat{m}_2 = b$ (c is chosen). If $m = (c, a, a)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case $j^* = 3$ and he can benefit by announcing $\hat{m}_3 = c$ (b is chosen), or (ii) $w_d = c$, in which case $j^* = 2$ and he can benefit by announcing $\hat{m}_2 = c$ (c is chosen). If $m = (b, a, b)$, then $g(m) = b$ and we have two possibilities: either (i) $w_d = a$, in which case $j^* = 3$ and he can benefit by announcing $\hat{m}_3 = a$ (a is chosen), or (ii) $w_d = c$, in which case $j^* = 1$ and he can benefit by announcing $\hat{m}_1 = a$ (c is chosen). If $m = (b, c, b)$, then $g(m) = b$ and we have two possibilities: either (i) $w_d = a$, in which case $j^* = 3$ and he can benefit by announcing $\hat{m}_3 = a$ (a is chosen), or (ii) $w_d = c$, in which case $j^* = 1$ and he can benefit by announcing $\hat{m}_1 = c$ (c is chosen). If $m = (c, c, a)$, then $g(m) = c$ and we have two possibilities: either (i) $w_d = a$, in which case $j^* = 2$ and he can benefit by announcing $\hat{m}_2 = a$ (a is chosen), or (ii) $w_d = b$, in which case $j^* = 1$ and he can benefit by announcing $\hat{m}_1 = a$ (b is chosen). If $m = (c, c, b)$, then $g(m) = c$ and we have two possibilities: either (i) $w_d = a$, in which case $j^* = 2$ and he can benefit by announcing $\hat{m}_2 = a$ (a is chosen), or (ii) $w_d = b$, in which case $j^* = 1$ and he can benefit by announcing $\hat{m}_1 = b$ (b is chosen).

Subsubcase 2.1.2. $k = 1$ and $m_1 = a$.

Note that in tis case $m = (m_1, m_2, m_3) \in \{(a, b, b), (a, c, c)\}$. For any such profile m, juror j^* can benefit by deviating unilaterally. If $m = (a, b, b)$, then $g(m) = b$, and since $g(m) \neq w_d$, we have two possibilities: either (i) $w_d = a$, in which case $j^* = 3$ and he can benefit by announcing $\hat{m}_3 = c$ (a is chosen), or (ii) $w_d = c$, in which case $j^* = 1$ and he can benefit by announcing $\hat{m}_1 = c$ (c is chosen). If $m = (a, c, c)$, then $g(m) = c$ and we have two possibilities: either (i) $w_d = a$, in which case $j^* = 2$ and he can benefit by announcing $\hat{m}_2 = b$ (a is chosen), or (ii) $w_d = b$, in which case $j^* = 1$ and he can benefit by announcing $\hat{m}_1 = b$ (b is chosen).

Subcase 2.2. Each juror $j \neq k$ announces $x \neq x^*$ and the dissident k announces $m_k = x^*$.

Since $g(m) = x^*$, then by Rule 2 we have $m = (m_1, m_2, m_3) \in \{(c, b, b), (b, c, c)\}$. Again, for any such profile m, juror j^* can benefit by deviating unilaterally. Thus, if $m = (c, b, b)$, then $g(m) = c$ and we have two possibilities: either (i) $w_d = a$, in which case $j^* = 2$ and he can benefit by announcing $\hat{m}_2 = a$ (a is chosen), or (ii) $w_d = b$, in which case $j^* = 1$ and he can benefit by announcing $\hat{m}_1 = b$ (b is chosen). If $m = (b, c, c)$, then $g(m) = b$ and we have two possibilities: either (i) $w_d = a$, in which case $j^* = 3$ and he can benefit by announcing $\hat{m}_3 = a$ (a is chosen), or (ii) $w_d = c$, in which case $j^* = 1$ and he can benefit by announcing $\hat{m}_1 = c$ (c is chosen).

Subcase 2.3. Each juror $j \neq k$ announces $x \neq x^*$ and the dissident k announces $m_k \neq x^*$.

By Rule 2, $k \neq 1$ and $x \neq x_k^e$, and therefore $m = (m_1, m_2, m_3) \in \{(a, b, a), (a, c, a), (c, a, c), (c, b, c), (a, a, b), (a, a, c), (b, b, a), (b, b, c)\}$. Any such profile m is such that some juror can benefit by deviating unilaterally. Thus, if $m = (a, b, a)$, $g(m) = c$ and then juror 3 can benefit by announcing $\hat{m}_3 = c$ (a is chosen), whether $w_d = a$ or $w_d = b$. If $m = (a, c, a)$, $g(m) = b$ and then juror 2 can benefit by announcing $\hat{m}_2 = a$ (a is chosen), whether $w_d = a$ or $w_d = c$. If $m = (c, a, c)$, $g(m) = b$ and then juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen), whether $w_d = a$ or $w_d = c$. If $m = (c, b, c)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case $j^* = 3$ and he can benefit by announcing $\hat{m}_3 = a$ (b is chosen), or (ii) $w_d = c$, in which case $j^* = 2$ and he can benefit by announcing $\hat{m}_2 = c$ (c is chosen). If $m = (a, a, b)$, $g(m) = c$ and then juror 3 can benefit by announcing $\hat{m}_3 = a$ (a is chosen), whether $w_d = a$ or $w_d = b$. If $m = (a, a, c)$, $g(m) = b$ and then juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen), whether

$w_d = a$ or $w_d = c$. If $m = (b, b, a)$, $g(m) = c$ and then juror 3 can benefit by announcing $\hat{m}_3 = b$ (b is chosen), whether $w_d = a$ or $w_d = b$. If $m = (b, b, c)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case $j^* = 3$ and he can benefit by announcing $\hat{m}_3 = b$ (b is chosen), or (ii) $w_d = c$, in which case $j^* = 2$ and he can benefit by announcing $\hat{m}_2 = a$ (c is chosen).

Case 3. Rule 3 of mechanism Γ^e applies to m; *i.e.*, for each $j, k \in N$, $m_j \neq m_k$.

We distinguish two subcases.

Subcase 3.1. There is $k \in J$ such that $m_k = x_k^e$ and, for each $j \neq k$, $m_j \neq x_j^e$.

Then $m = (m_1, m_2, m_3) \in \{(a, c, b), (c, b, a), (b, a, c)\}$. By Rule 3 we have $g(m) = x_k^e$. Then, juror k can benefit by deviating unilaterally to any $\hat{m}_k \neq x_k^e$, since in this case Rule 2 applies and $\hat{m}_k$ is chosen.

Subcase 3.2. Either (i) for each $j \in J$, $m_j \neq x_j^e$, or (ii) for each $j \in J$, $m_j = x_j^e$.

Then $m = (m_1, m_2, m_3) \in \{(b, c, a), (c, a, b), (a, b, c)\}$. By Rule 3 we have $g(m) = x_1^e$. Any such profile m is such that some juror can benefit by deviating unilaterally. If, for each $j \in J$, $m_j \neq x_j^e$, then, juror 1 can benefit by deviating unilaterally to $\hat{m}_1 = x_1^e$, since in this case Rule 2 applies and some $y \neq x_1^e$ is chosen. If, for each $j \in J$, $m_j = x_j^e$, we have two possibilities: either (i) $w_d = b$, in which case $j^* = 3$ and he can benefit by announcing $\hat{m}_3 = b$ (b is chosen), or (ii) $w_d = c$, in which case $j^* = 2$ and he can benefit by announcing $\hat{m}_2 = c$ (c is chosen). ■

PROOF OF PROPOSITION 8:

Let $(I, x^f, x^e) \in \mathcal{E}^{|J|}$ be a jury configuration satisfying conditions of the statement (*i.e.*, $x_1^f = \emptyset$, $x_1^e = \emptyset$, $x_2^e = b$, $x_3^e = c$). First, we show that, for each $(R, w_d) \in S(I, x^f, x^e)$, $(w_d, ..., w_d) \in N(\Gamma_{2,3}^e, R, w_d)$. On the one hand, Juror 1 does not want to deviate from $(w_d, ..., w_d)$ since, by Rule 2 of mechanism $\Gamma_{2,3}^e$, an unilateral deviation by 1 may result in an alternative x different from w_d only if $w_d x \in I_1$ (and, in this case, $w_d \; P_1(w_d) \; x$). On the other hand, for each juror $j \in \{2, 3\}$, (i) if $w_d \neq x_j^e$, then w_d is the most preferred contestant for j and therefore he does not want to deviate from $(w_d, ..., w_d)$, and (ii) if $w_d = x_j^e$ then, by Rule 2, juror j cannot prevent w_d from being chosen by unilaterally deviating from $(w_d, ..., w_d)$.

Now, we show that, for each $(R, w_d) \in S(I, x^f, x^e)$ and $m \in N(\Gamma_{2,3}^e, R, w_d)$, $g(m) = w_d$. Suppose on the contrary that $g(m) = x^* \neq w_d$. Next we show that any such profile m is such that some juror can benefit by deviating unilaterally, which contradicts that $m \in N(\Gamma^e_{2,3}, R, w_d)$. Let $j^* \in J$ be such that $x^* w_d \in I_{j^*}$. Then $w_d \ P_{j^*}(w_d) \ x^*$. We distinguish three cases.

Case 1. Rule 1 of mechanism $\Gamma^e_{2,3}$ applies to m; *i.e.*, for each $j \in J$, $m_j = x^*$.

If $j^* = 1$, then juror 1 can improve his welfare if he deviates and announces $\hat{m}_1 = w_d$ (in this case, Rule 2 applies and, since $x^* w_d \in I_1$, then w_d is chosen). If $j^* \in \{2, 3\}$, then $x^* \neq x^e_{j^*}$ and therefore j^* can improve his welfare if he deviates and announces $\hat{m}_{j^*} \notin \{x^*, w_d\}$ (in this case, Rule 2 applies and, since $x^* \neq x^e_{j^*}$, $w_d \notin \{x^*, \hat{m}_{j^*}\}$ is chosen). This contradicts that $m \in N(\Gamma^e_{2,3}, R, w_d)$.

Case 2. Rule 2 of mechanism $\Gamma^e_{2,3}$ applies to m; *i.e.*,there is $k \in J$ and $x \in N$ such that, for each $j \neq k$, $m_j = x$, but $m_k \neq x$.

We distinguish four subcases.

Subcase 2.1. $k = 1$ and $x m_1 \in I_1$.

By Rule 2, since $g(m) = x^*$, we have $m_1 = x^*$. Therefore, since $x^* x \in I_1$, we have $m = (m_1, m_2, m_3) \in \{(b, c, c), (c, b, b)\}$. If $m = (b, c, c)$, then $g(m) = b$ and juror 2 can benefit by announcing $\hat{m}_2 = b$ (a is chosen). If $m = (c, b, b)$, then $g(m) = c$ and juror 3 can benefit by announcing $\hat{m}_3 = c$ (a is chosen).

Subcase 2.2. $k = 1$ and $x m_1 \notin I_1$.

Then $g(m) = x \neq w_d$ and, since $x m_1 \notin I_1$, $m = (m_1, m_2, m_3) \in \{(a, b, b), (b, a, a), (a, c, c), (c, a, a)\}$. If $m = (a, b, b)$, then $g(m) = b$ juror 2 can benefit by announcing $\hat{m}_2 = a$ (c is chosen). If $m = (b, a, a)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_3 = b$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = b$ (c is chosen). If $m = (a, c, c)$, then $g(m) = c$ and juror 3 can benefit by announcing $\hat{m}_3 = a$ (b is chosen). If $m = (c, a, a)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_3 = c$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen).

Subcase 2.3. $k \in \{2, 3\}$ and $x = x^e_k$.

In this case $m = (m_1, m_2, m_3) \in \{(b, a, b), (b, c, b), (c, c, a), (c, c, b)\}$. By Rule 2 of mechanism $\Gamma^e_{2,3}$, $g(m) = x^e_k$. If $m = (b, a, b)$, then $g(m) = b$ and we have two possibilities: either (i) $w_d = a$, in which case juror 3 can benefit by announcing $\hat{m}_3 = a$ (a is chosen), or (ii) $w_d = c$, in which case juror 1 can benefit by announcing $\hat{m}_1 = a$ (c is chosen). If $m = (b, c, b)$, then $g(m) = b$ and we have two possibilities: either (i) $w_d = a$, in which case juror 3 can benefit by announcing $\hat{m}_3 = a$ (a is chosen), or (ii) $w_d = c$, in which case juror 1 can benefit by announcing $\hat{m}_1 = c$ (c is chosen). If $m = (c, c, a)$, then

$g(m) = c$ and we have two possibilities: either (i) $w_d = a$, in which case juror 2 can benefit by announcing $\hat{m}_2 = a$ (a is chosen), or (ii) $w_d = b$, in which case juror 1 can benefit by announcing $\hat{m}_1 = a$ (b is chosen). If $m = (c, c, b)$, then $g(m) = c$ and we have two possibilities: either (i) $w_d = a$, in which case juror 2 can benefit by announcing $\hat{m}_2 = a$ (a is chosen), or (ii) $w_d = b$, in which case juror 1 can benefit by announcing $\hat{m}_1 = b$ (b is chosen).

Subcase 2.4. $k \in \{2, 3\}$ and $x \neq x_k^e$.

In this case $m = (m_1, m_2, m_3) \in \{(a, b, a), (a, c, a), (c, b, c), (c, a, c), (a, a, c), (a, a, b), (b, b, c), (b, b, a)\}$. If $m = (a, b, a)$, then $g(m) = c$ juror 3 can benefit by announcing $\hat{m}_3 = b$ (b is chosen). If $m = (a, c, a)$, then $g(m) = b$ juror 2 can benefit by announcing $\hat{m}_2 = a$ (a is chosen). If $m = (b, c, b)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_3 = a$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen). If $m = (c, a, c)$, then $g(m) = b$ juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen). If $m = (a, a, c)$, then $g(m) = b$ juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen). If $m = (a, a, b)$, then $g(m) = c$ juror 3 can benefit by announcing $\hat{m}_3 = a$ (a is chosen). If $m = (b, b, c)$, then $g(m) = a$ and we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_3 = b$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = a$ (c is chosen). If $m = (b, b, a)$, then $g(m) = c$ juror 3 can benefit by announcing $\hat{m}_3 = b$ (b is chosen).

Case 3. Rule 3 of mechanism $\Gamma_{2,3}^e$ applies to m; *i.e.*, for each $j, k \in N$, $m_j \neq m_k$.

We distinguish three subcases.

Subcase 3.1. For each $k \in \{2, 3\}$, $m_k \neq x_k^e$.

In this case $m = (m_1, m_2, m_3) \in \{(c, a, b), (b, c, a), (a, c, b)\}$. By Rule 2 of mechanism $\Gamma_{2,3}^e$, $g(m) = a$. If $m = (c, a, b)$, we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_3 = c$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen). If $m = (b, c, a)$, we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_3 = b$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = b$ (c is chosen). If $m = (a, c, b)$, we have two possibilities: either (i) $w_d = b$, in which case juror 3 can benefit by announcing $\hat{m}_3 = a$ (b is chosen), or (ii) $w_d = c$, in which case juror 2 can benefit by announcing $\hat{m}_2 = a$ (c is chosen).

Subcase 3.2. $m_2 = x_2^e$.

In this case $m = (m_1, m_2, m_3) \in \{(a, b, c), (c, b, a)\}$ and, by Rule 2 of mechanism $\Gamma^e_{2,3}$, $g(m) = b$. Given any such profile m, juror 2 can benefit by announcing $\hat{m}_2 = c$ (c is chosen).

Subcase 3.3. $m_2 \neq x^e_2$ and $m_3 = x^e_3$.

In this case $m = (m_1, m_2, m_3) = (b, a, c)$ and $g(m) = c$. Then, juror 3 can benefit by announcing $\hat{m}_3 = a$ (a is chosen). ■

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