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MANIPULABILITY IN RESTRICTED SEPARABLE DOMAINS*

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Abstract

We study a problem where a group of voters must decide which candidates are elected from a set of alternatives. The voters' preferences on the combinations of elected candidates are represented by orderings. We propose a family of restrictions of the domain of separable preferences. These subdomains are generated from a partition that identifies, for each voter, her friends, her enemies and the unbiased candidates. We characterize the family of rules that satisfy strategy-proofness and tops-onliness on each of those subdomains.

Keywords: separable preferences, aggregation, strategy-prooness, tops-onliness.

JEL Classification: D63, D70

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1 Introduction.

Quite often we find situations where societies have to choose one or several alternatives among the available possibilities. That is the case of a club that have to decide which of the candidates will become new members. Each candidate can be accepted or rejected separately. Another example is when in a parliament a collection of bills is submitted for consideration to the legislators. Each single bill can be passed or not. We find similar problems in the allocation of public projects, or the settlement of facilities in some districts of a city.

A set of *voters* N has to choose one or several alternatives among a set of *candidates* or objects K. The voters' *preferences* on the combinations of winning candidates are represented by orderings. A *social choice function* (SCF) is mapping that, taking as an input the preferences of all the voters, results in a set of elected candidates. One of the key aspects of the social choice functions is their potential attractiveness, understood as the interesting properties they may satisfy. Those properties (or axioms) usually refer to different behaviors of the SCFs. The second key aspect is the domain of preferences on which the SCFs are defined. In this paper we focus on the latter ingredient, we introduce a family of subdomains, and we characterize the social choice functions that fulfill, on those subdomains, two of the most analyzed axioms in the literature.

Among the several properties a SCF may satisfy, two of them have been intensively studied: *strategy-proofness* and *tops-onliness*. The first one says that none of the voters can be better off by misrepresenting her preferences. The second property sets that all the information the SCF uses to compute the elected candidates is contained in the tops of the preferences. Gibbard (1973) and Satterthwaite (1975) showed that, if the domain of the SCFs is the universal one and every ordering is admissible, the only SCF that is strategy-proof and tops-only is the *dictatorial rule*. These results have motivated a wide literature to delimit the set of preferences in order to scape from this impossibility result. Black (1948) and Moulin (1980) provides characterization results in the domain of single-peaked preferences. Border and Jordan (1983) narrow the domain ever more and they restrict to quadratic preferences. Alternatively, Barberà et al. (1991) consider a different type of domain: *separable preferences* (the position of a candidate in the ordering is not affected by the position of other candidates). These authors show that the only SCFs that satisfy strategy-proofness and tops-onliness on separable preferences are the *voting by committees rules*.

In a trial to make a further step in this research line, we propose to restrict the the domain of separable preferences. Actually, we consider a family of restrictions. Each voter $i \in N$ has a *type* V_i, that consists in a partition of the set of candidates into friends, enemies, and unbiased.[1] The agents' type is common knowledge. Associated with each partition V_i we

[1] Although in a different context, Barberà et al. (2001) use the idea of friendship and enmity (without the possibility of being unbiased) in a dynamic model where the current members of a club must choose

have a subdomain of separable preferences, that we call V_i-*domain*. The friends are those candidates that i always wants to be elected, and therefore will be above the empty set in any preference of the V_i-domain. The enemies are those candidates that i never wants to be elected, and therefore will be below the empty set in any preference of the V_i-domain. The unbiased are the remaining candidates, and there is not information on what i thinks about them. They can be above or below the empty set in the preference relation. Notice that different V_i's lead to different V_i-domains, and hence we have a family of subdomains of separable preferences. As we illustrate in Section 2 the structure of these subdomains can be used to accommodate several frameworks. As a very special case, if V_i is such that all candidates are unbiased for i, then the V_i-domain coincides with the whole domain of separable preferences.

Our main goal is to characterize the SCFs that are strategy-proof and tops-only on each possible V_i-domain. Those SCFs are extensions of the voting by committees rules. We also find that, within the SCFs fulfilling the two previous properties, the smaller the V_N-domain, (i) the fewer elected candidates there are for a fixed profile, and (ii) the smaller the range is.

The rest of the paper is structured as follows. In Section 2 we set the basic model, the key axioms, and the elementary results in the literature. We also introduce our family of restricted separable domains and their potential to be applied to different frameworks. In Section 3 we present an extension of the voting by committees rules and our main results. In Section 4 we conclude with some final comments and further research.

2 Basic model

Let $N = \{1, 2, \ldots, n\}$ be the set of voters, whose typical elements are denoted by $i, j, \ldots$ Let $K = \{1, 2, \ldots, k\}$ be the set of candidates, whose typical elements are denoted by $x, y, \ldots$ Subset of N are $M, M' \ldots$, and subsets of K are $L, L', \ldots$ In both cases the empty set is denoted by ϕ.

If 2^K denotes the collection of subsets of K, a **preference** R is is a reflexive, complete, and transitive binary relation defined over 2^K. Given a preference R, the most preferred element of R is called *top* and denoted by $t(R)$:

$$t(R) = \{L \in 2^K : L\,R\,L' \text{ for all } L' \in 2^K\}.$$

For each voter $i \in N$, $\mathcal{D}_i$ is the domain of preferences for i. Note that, in principle, different agents may have different domains. Elements of $\mathcal{D}_i$ are denoted by R_i, R_i',... The cartesian product of the domains of voters in N is $\mathcal{D}_N = \mathcal{D}_1 \times \ldots \times \mathcal{D}_n$. Elements of $\mathcal{D}_N$ are denoted by $R_N, R_N', \ldots$

which candidates are accepted to enter.

A *social choice function* is a way to select one or several candidates as an output of the voters' preferences. More formally,

Social choice function (SCF). It is a mapping $S : \mathcal{D}_N \longrightarrow 2^K$ such that $S(R_N) \in 2^K$ for each $R_N \in \mathcal{D}_N$.

For the rest of the paper, we are mostly interested in SCFs that satisfy two properties. The first one imposes that the SCF must be immune to individual manipulations, and the second one says that the SCF must not request to much information on the agents' preferences to compute the elected candidates.

A SCF satisfies *stratey-proofness* when it is not in the profit of a voter to misrepresent her preference.[2]

Strategy-proofness. For each $i \in N$, each $R_N \in \mathcal{D}_N$, and each $R'_i \in \mathcal{D}_i$, $S(R'_i, R_{-i})\, R'_i\, S(R_i, R_{-i})$.

Tops-onliness requires that the SCF depends only on the tops of the preferences. This property is quite useful from an informational point of view, since we do not need to know every detail of the voters' preferences to compute the outcome of the SCF.

Tops-onliness. For each $R_N, R'_N \in \mathcal{D}_N$ such that $t(R_N) = t(R'_N)$, $S(R_N) = S(R'_N)$.

Which are the SCFs that satisfy strategy-proofness and tops-onliness together? As the existing literature illustrates, the answer to this question strongly depends on the domain of admissible preferences $\mathcal{D}_N$ we consider. Hence, if $\mathcal{D}_i$ is the universal domain of preferences ($\mathcal{D}_i = \mathcal{U}$ for all $i \in N$), and any ordering on 2^K is admissible, Gibbard (1973) and Satterthwaite (1975) showed that the unique strategy-proof and tops-only SCF is the dictatorial rule.

Theorem 1 (Gibbard-Satterwaite theorem)**.** *A social choice function $S : \mathcal{U}_N \longrightarrow 2^K$ whose range contains more than two elements of 2^K is strategy-proof and tops-only if and only if it is the dictatorial rule.*

Several restrictions on the domain of preferences have been proposed to scape from the previous impossibility result. In situations where a set of candidates has to be chosen a natural restriction of the domain of preferences is that of separability. Roughly, a preference is *separable* if the position of a candidate in the ordering is not affected by the position of other candidates.

Separability. A preference relation R is separable if for each $L \subseteq K$ and each $x \notin L$, $L \cup \{x\}\, P\, L$ if and only if $\{x\}\, P\, \phi$.[3] The domain of separable preferences is denoted by $\mathcal{D}^{sep}$.

Barberà et al. (1991) characterize the family of SCFs that satisfy both strategy-proofness

[2]The reader is referred to Barberà (2010) for an extensive exposition of the literature.

[3]We occasionally abuse the nation and write simply x instead of $\{x\}$.

and tops-onliness when $\mathcal{D}_i = \mathcal{D}^{\text{sep}}$ for all $i \in N$. Those SCFs are what they call *voting by committees SCFs*. A *committee* is a collection of coalitions of voters that satisfies a monotonicity condition.

Committee, $\boldsymbol{C}$. It is set of coalitions of N, $C \in 2^{2^N}$, that satisfies that, if $M \in C$ and $M' \supseteq M$, then $M' \in C$.

Let C_x be a committee for candidate x. Let $\Delta = (C_x)_{x \in K}$ be a collection of committees, one for each candidate. Associated to each parameter Δ, a Barberà, Sonnenschein and Zhou voting SCF (BSZ, hereafter) is defined as follows

BSZ rule associated to Δ, $\mathbf{BSZ_\Delta}$. For each $R_N \in \mathcal{D}^{V_N}$,

$$x \in \text{BSZ}_\Delta(R_N) \Leftrightarrow \{i \in N : x \in t(R_i)\} \in C_x$$

Theorem 2 (Barberà et al. (1991)). *A social choice function $S : \mathcal{D}_N^{sep} \longrightarrow 2^K$ is strategy-proof and tops-only if and only if it is a BSZ rule.*

2.1 A family of separable domains.

The restriction we propose in this paper introduces more structure on $\mathcal{D}^{sep}$. For each voter $i \in N$, there exists an exogenous partition of the set of candidates, $K = F_i \cup U_i \cup E_i$. This partition is common knowledge, and it means that, for agent i, some of the candidates are friends (F_i), some others are enemies (E_i), and for the rest she is unbiased (U_i), they may be regarded as friends or enemies depending on the particular preference. We denote by $\mathcal{V}$ the set of all possible partitions of K into three sets like the previous ones. Elements of $\mathcal{V}$ are generally denoted by V, or V_i when it refers to the voter i. We call to the partition V_i the **type** of voter i.

V-compatibility. Given a partition $V = (F, U, E) \in \mathcal{V}$, a preference relation R is V-compatible if the following requirements hold

(i) $R \in \mathcal{D}^{sep}$.

(ii) If $x \in F$ then $xP\phi$.

(iii) If $x \in E$ then ϕPx.

(iv) If $x \in U$ then either $xP\phi$ or ϕPx.

A preference R_i is therefore V_i-compatible if (i) it is separable, (ii) i prefers her friends to be elected rather than not to be, (iii) i prefers her enemies not to be elected rather than to be, and (iv) none of the rest of candidates is indifferent with the empty set.

Given a voter $i \in N$ and a partition into friends, enemies, and unbiased $V_i \in \mathcal{V}$, the domain of V_i-compatible preferences (V_i**-domain**, hereafter) is denoted by $\mathcal{D}^{V_i}$. Since, the partitions V_i and V_j may be different for different voters i and j, the corresponding domains $\mathcal{D}^{V_i}$ and $\mathcal{D}^{V_j}$ may also be different. We denote by $V_N = (V_1, \ldots, V_n) \in \mathcal{V}^N$ a profile of partitions for voters in N. Finally, $\mathcal{D}^{V_N} = \mathcal{D}^{V_1} \times \ldots \times \mathcal{D}^{V_n}$ is the domain of preferences profiles for the type profile V_N.

Next, we present a collection of particular examples of V-domains. We argue that the partition structure is rich enough to accommodate a wide variety of frameworks and real life situations. In what follows we assume that the preferences are V-compatible, and therefore separable.

Unbiased society

In this type of society, all the voters are unbiased towards all the candidates, this is, $F_i = \phi$, $U_i = K$, and $E_i = \phi$ for each $i \in N$. Alternatively, we may say that there is not information at all on the voters' preferences. When this happens, every V_i-domain coincides with the domain of separable preferences, $\mathcal{D}^{V_i} = \mathcal{D}^{sep}\ \forall i \in N$.

Friendly and enemily societies

Any voter in a friendly society considers all the candidates as friends: $F_i = K$, $U_i = \phi$, and $E_i = \phi$ for each $i \in N$. This implies that, no matter the preference, the most preferred option for every voter is that all the candidates are elected; that is, $t(R_i) = K$ for all $R_i \in \mathcal{D}^{V_i}$. Analogously, one may refer to a society where all candidates are considered as enemies, that is $F_i = \phi$, $U_i = \phi$, and $E_i = K$ for each $i \in N$.

Polarized society

Esteban and Ray (1994) conclude that a society achieves the greatest degree of polarization when it is split in two groups of equal size, that are significantly different one from the other on all the attributes, but internally homogeneous. If we assume that there is only one attribute, and this corresponds to the type of the voter, the idea of polarized society can be easily captured by the mean of the V-domains. Consider that the voters in N are partitioned into two groups, $N = M \cup M'$, of equal size ($m = m' = \frac{n}{2}$).[4] The friends and enemies of one group are the enemies and friends of the other, respectively. For example, imagine that there are three bills $K = \{x, y, z\}$ submitted for to the Congress that affect to controversial and key ideological issues. Assume that the leftist legislators M like two of the bills but hate the other ($F_i = \{x, y\}$, $U_i = \phi$, and $E_i = \{z\}$), while for the rightist legislators it is exactly the opposite ($F_i = \{z\}$, $U_i = \phi$, and $E_i = \{x, y\}$). Some leftist legislators will prefer x to y while some others will prefer y to x; however, they all have in common that they do not want the law z to pass.

[4] Obviously, we are assuming here that n is even.

Andina and Meléndez-Jiménez (2009) present a model where the set of voters have to elect some representatives among themselves ($N = K$). The innovative aspect of that paper is that friendship relations among the individuals is captured through a bipartite network whose nodes are the voters.[5] Such a network is exogenous and common knowledge. A framework similar to the one proposed by Andina and Meléndez-Jiménez (2009) can be studied by using the V-domains structure. To do that is enough to consider that N is divided into two groups, $N = M \cup M'$ (not necessarily of equal size), and the partition profile V is such that $F_i \subseteq M'$ for all $i \in M$, $F_j \subseteq M$ for all $j \in M'$, and $i \in F_j \Leftrightarrow j \in F_i$.

Selfish preferences

A usual situation is when the set of voters coincides with the set of candidates $K = N = \{1, \ldots, n\}$. That is the case of the election of the head of the department or a chairman of a Congress Committee. It is not difficult to justify that voter i is a friend of herself ($iP_i\phi$), which implies that $\{i\} \subseteq F_i$. Depending on how opened-minded i is to the other candidates/voters' ideas, the set of friends and unbiased candidates will be bigger or smaller. Hence, in an extreme situation, we may have that $F_i = \{i\}$, $U_i = \phi$, and $E_i = N - \{i\}$ for all $i \in N$. Or if voters are not too biased toward themselves, $F_i = \{i\}$, $U_i = N - \{i\}$, and $E_i = \phi$, for example.

NIMBY situations

Consider a situation where undesirable facilities (dumps, prisons,...) must be located in some districts. If those districts have to collectively decide where to site in the facilities, it is quite reasonable to assume the last place they want the facilities to be located is their own neighborhood. This is known as a NIMBY ("non in my back yard") problem.[6] Again, the election of the locations can be studied by the mean of the V-domains. To do that we only need to impose that the set of voters (or districts) and candidates are the same ($N = K$), and for each $i \in K$ the partition V_i such that $E_i = \{i\}$. In other words, the only enemy of any district is itself.

The V-domains structure is rich enough to easily generalize the previous idea. For instance, imagine that the districts are points of an Euclidean space. For each district there there are two numbers $\overline{d}_i, \underline{d}_i \in \mathbb{R}_+$. If the distance between i and the district j having the facility is greater or equal to $\overline{d}_i$, then the j is a desirable place to locate the facility according to i. Similarly, if the distance between i and the district j having the facility is smaller or equal to $\underline{d}_i$, then the j is an undesirable place to locate the facility according to i. In other words the i's preferences are in a V_i-domain where the partition V_i is such

[5]A network is *bipartite* when the nodes can be divided into two disjoint sets M and M' such that every node in M is connected to one in M'.

[6]The reader is referred to Brion (1991) for a survey on NIMBY problems.

that $F_i = \{j \in N : d(i,j) \geq \underline{d}_i\}$ and $E_i = \{j \in N : d(i,j) \geq \underline{d}_i\}$.

In the next section we study the SCFs that, on the V-domains, satisfy strategy-proofness and tops-onliness.

3 Results

Using the same approach as Barberà et al. (1991), we use the notion of committee to define the V_N*-voting by committee* SCFs, which, as the reader may advance, are extensions of the BSZ rules. Let C_x be a committee for candidate x, and let $\Gamma = (C_x)_{x \in K}$ be a collection of committees, one for each candidate. Associated to each parameter Γ, we define a V_N-voting by committee SCF as follows

V_N**-Voting by committee SCF associated to** Γ**,** $\boldsymbol{VC_\Gamma^{V_N}}$. For each $R_N \in \mathcal{D}^{V_N}$,

$$x \in VC_\Gamma^{V_N}(R_N) \Leftrightarrow \{i \in N : x \in t(R_i) \text{ or } x \in E_i\} \in C_x$$

Several comments are in order,

(a) The V_N-voting by committees SCFs are slightly different from BSZ SCFs introduced in the previous section. To decide whether a candidate x is elected or not, in the V-voting by committees SCFs we have to consider both the voters for whom x in the top and the voters that have x as an enemy. The BSZ SCFs only consider the tops.

(b) The V_N-voting by committees SCFs are domain dependent, if we change the V_N-domain, we change the rule. This implies that a SCF defined on a separable domain differs in general from a SCF defined on another V_N-domain.

(c) When the V_N-domain is such that $E_i = \phi$ for all $i \in N$ ($\mathcal{D}^{sep}$, for example), the V_N-voting by committees SCFs coincide with the BSZ SCFS proposed by Barberà et al. (1991).

Before introducing our main result, we present two properties that will be very useful to prove our characterization theorem. We also show in Propositions 3 and 4 below how they are related to strategy-proofness and tops-onliness.

The first property is a stronger version of tops-onliness. It says that, to decide whether a candidate is elected or not, we only need to know the voters for whom she is at the top of the preference. In contrast with tops-onliness, *tops-onliness by candidate* states that her election is independent, not only of the preference profile below the tops, and also of the rest of the candidates on those tops.

Tops-onliness by candidate. For each $x \in K$ and for each $R_N, R'_N \in \mathcal{D}^{V_N}$ such that $x \in t(R_N) \Leftrightarrow x \in t(R'_N)$, we have that $x \in S(R_N) \Leftrightarrow x \in S(R'_N)$.

Tops monotonicity states that, if a candidate $x \in K$ is elected and the preference profile changes in a way such that x increases her support among the voters' tops, then x should still be elected.

Tops monotonicity. For each $x \in K$ and for each $R_N, R'_N \in \mathcal{D}^{V_N}$, if $x \in S(R_N)$ and $\{i \in N : x \in t(R_i)\} \subseteq \{i \in N : x \in t(R'_i)\}$, then $x \in S(R'_N)$.

Proposition 3. *Strategy-proofness and tops-onliness together imply tops monotonicity.*

Proof. Let $x \in K$ and $R_N \in \mathcal{D}^{V_N}$. Let $R'_i \in \mathcal{D}^{V_i}$. We show that if $x \in S(R_i, R_{-i})$ and $x \in t(R'_i)$, then $x \in S(R'_i, R_{-i})$. We distinguish two cases.

(a) If $x \in t(R_i)$. We can find a preference $R''_i \in \mathcal{D}^{V_i}$ such that $t(R''_i) = t(R'_i)$ and $(M \cup x)\, R''_i\, M'$ for all $M, M' \subseteq K$ that do not contain the candidate x. *Strategy-proofness* implies that $x \in S(R''_i, R_{-i})$. By *top-onlyness* we know that $x \in S(R'_i, R_{-i})$.

(b) If $x \notin t(R_i)$. We can find a preference $R''_i \in \mathcal{D}^{V_i}$ such that $t(R''_i) = t(R_i)$ and $M\, R''_i\, (M' \cup x)$ for all $M, M' \subseteq K$ that do not contain the candidate x. By *top-onlyness* $S(R''_i, R_{-i}) = S(R_i, R_{-i})$. *Strategy-proofness* implies that $x \in S(R'_i, R_{-i})$.

Applying iteratively this reasoning over the set of voters we conclude the statement. □

It is quite obvious that tops-onliness by candidates implies tops-onliness. The converse, in general, is not true. However, the next result states that, in presence of strategy-proofness, both properties are equivalent.

Proposition 4. *Let S be a strategy-proof social choice function, then it satisfies tops monotonicity if and only if it satisfies tops-onliness by candidate.*

Proof. We omit the proof because the argument is similar to the previous proposition. □

Now we are ready to present our main result. It is a characterization of the strategy-proof and tops-only rules on the V-domains.

Theorem 5. *For any partition $V \in \mathcal{V}$, a social choice function $S : \mathcal{D}^{V_N} \longrightarrow 2^K$ is strategy-proof and tops-only if and only if it is a V_N-voting by committees rule.*

Proof. It is not difficult to check that any $VC_\Gamma^{V_N}$ satisfies both tops-onliness and strategy-proofness. Conversely, assume that S is a social choice function that is tops-onliness and strategy-proof. We probe that S is actually a voting by committees rule. We divide the proof in two steps. Firstly we determine the parameter Γ, and secondly we show that S coincides with $VC_\Gamma^{V_N}$ for the Γ previusly obtained.

Step 1: Definition of the committees. For each candidate $x \in K$ we define her committee as follows. Let $R_N \in \mathcal{D}^{V_N}$ be a preference profile such that $x \in S(R_N)$; then

$\{i \in N : x \in t(R_i)$ or $x \in E_i\} \in C_x$. Now, do the same same for any profile that results in the election of candidate x. Therefore, the committee C_x is

$$\begin{aligned} C_x &= \bigcup_{\substack{R_N \in \mathcal{D}^{V_N} \\ x \in S(R)}} \{i \in N : x \in t(R_i) \text{ or } x \in E_i\} \\ &= \bigcup_{\substack{R_N \in \mathcal{D}^{V_N} \\ x \in S(R)}} [F_N(x) \cup E_N(x)] \cup \bigcup_{\substack{R_N \in \mathcal{D}^{V_N} \\ x \in S(R)}} \{i \in N : x \in t(R_i) \text{ and } x \in U_i\} \\ &= \bigcup_{\substack{R \in \mathcal{D}^{V_N} \\ x \in S(R)}} [F_N(x) \cup E_N(x) \cup \{i \in U_N(x) : x \in t(R_i)\}], \end{aligned}$$

where $F_N(x) = \{i \in N : x \in F_i\}$, $E_N(x) = \{i \in N : x \in E_i\}$, and $U_N(x) = \{i \in N : x \in U_i\}$. This implies that any coalition in the committee of x contains all the voters for which x is a friend or an enemy. The rest of the coalition structure depends on the voters that are unbiased towards x and they have it in the their tops, as soon as x is elected.

We need to show that the committee is well-defined and it satisfies the monotonicity condition. Indeed, let $M \in C_X$ and $M' \supseteq M$; we probe that $M' \in C_x$. To do that we have to define a preference profile $R'_N \in \mathcal{D}^{V_N}$ such that $x \in S(R'_N)$ and $M' = F_N(x) \cup E_N(x) \cup \{i \in U_N(x) : x \in t(R'_i)\}$. Since we know that $M \in C_x$, there exists $R_N \in \mathcal{D}^{V_N}$ such that $x \in S(R_N)$ and $M = F_N(x) \cup E_N(x) \cup \{i \in U_N(x) : x \in t(R_i)\}$. Now, define $R'_N \in \mathcal{D}^{V_N}$ as a profile such that

$$t(R'_i) = \begin{cases} F_i \cup U_i & \text{if } i \in M' \setminus M \\ t(R_i) & \text{otherwise} \end{cases}$$

Since $x \in S(R_N)$ and $\{i \in N : x \in t(R_i)\} \subseteq \{i \in N : x \in t(R'_i)\}$, by *tops monotonicity* (implied by *stratetegy-proofness* and *tops-onliness* according to Proposition 3) we have that $x \in S(R'_N)$. Finally, we show that $M' = F_N(x) \cup E_N(x) \cup \{i \in U_N(x) : x \in t(R'_i)\}$. It is quite obvious that $F_N(x) \cup E_N(x) \cup \{i \in U_N(x) : x \in t(R'_i)\} \subseteq M'$. To see the opposite direction, let $i \in M'$. There are two possibilities, either $i \in M$ or $i \notin M$: In the former case, $i \in F_N(x) \cup E_N(x) \cup \{i \in U_N(x) : x \in t(R_i)\} = F_N(x) \cup E_N(x) \cup \{i \in U_N(x) : x \in t(R'_i)\}$ because of the definition of R'_N. And in the latter, since $i \notin M$, $i \in U_N(x)$ and then $x \in t(R'_i) = U_i \cup F_i$.

Therefore, applying this reasoning to each candidate, we define the parameter $\Gamma = (C_x)_{x \in K}$.

Step 2: $\boldsymbol{S = VC_{\Gamma}^{V_N}}$. Now we prove that S is a voting by committees rule with the committees defined in the previous step. That is, we have to show that for each $R_N \in \mathcal{D}^{V_N}$, $x \in S(R_N)$ if and only if $M = \{i \in N : x \in t(R_i)$ or $x \in E_i\} \in C_x$. If $x \in S(R_N)$ the result is immediately obtained by the definition of the committees. Let us prove the converse. Suppose that $M \in C_x$, then there exists a preference profile $R' \in \mathcal{D}^{V_N}$ (maybe different from R_N) such that $x \in S(R'_N)$ and $M = F_N(x) \cup E_N(x) \cup \{i \in U_N(x) : x \in t(R'_i)\} = \{i \in N : x \in t(R'_i)$ or $x \in E_i\}$. Notice that $x \in t(R_i)$ if and only if $x \in t(R'_i)$. Since S satisfies tops-onliness by candidate (implied by strategy-proofness and tops-onliness according to Proposition 4), we conclude that $x \in S(R'_N)$. $\square$

The previous result must be understood in the following way. Since we have a collection of possible partitions V_N, we also have a collection of possible domains $\mathcal{D}^{V_N}$. On each of those domains the unique strategy-proof and tops-only SCFs are the V_N- voting by committees rules, that, by definition, depend on the domain $\mathcal{D}^{V_N}$. Since Theorem 5 holds for any V_N-domain, actually we have a "collection of characterizations".

It is quite usual in the literature that a narrowing on the set of admissible ordering entails the emergence of new rules that fulfill strategy-proofness and tops-onliness. Since we are proposing restrictions on the set of separable preferences through the partitions V_N, one would expect to find more strategy-proof and tops-only rules. However, this is not the case; even if we restrict $\mathcal{D}^{V_N}$ until the limit ($F_i = K$ for all $i \in N$, for example) we do not get more solution satisfying both axioms. Quite the reverse, some of the $VC_\Gamma^{V_N}$ rules may collapse to a single one in some V_N-domains. The reason is that the V_N-voting by committees family of rules can be interpreted as a rearrangement of the BSZ family; this is, to every $VC_\Gamma^{V_N}$ we can associate a BSZ_Δ that is outcome equivalent, and vice versa.

The *range* of a SCF is the collection of subsets of candidates that are selected by the rule for some preference profile. More formally,

$$rg(S) = \{L \in 2^K : \exists R_N \in \mathcal{D}_N \text{ s.t. } L = S(R_N)\}.$$

The next two propositions states that, within the SCFs fulfilling strategy-proofness and tops-onliness, the smaller the V_N-domain, (i) the fewer elected candidates there are for a fixed profile, and (ii) the smaller the range is.

Proposition 6. *If $\mathcal{D}^{V_N}$, $\mathcal{D}^{V'_N}$ are two domains such that $\mathcal{D}^{V_N} \subseteq \mathcal{D}^{V'_N}$,[7] then $VC_\Gamma^{V_N}(R_N) \supseteq VC_\Gamma^{V'_N}(R_N)$ for all $R_N \in \mathcal{D}^{V_N}$*

Proof. Notice first that $\mathcal{D}^{V_N} \subseteq \mathcal{D}^{V'_N}$ if and only if $F_i \supseteq F'_i$, $U_i \subseteq U'_i$, and $E_i \supseteq E'_i$ for all $i \in N$. Given $R_N \in \mathcal{D}^{V_N}$, $x \in VC_\Gamma^{V'_N}(R_N) \Leftrightarrow \{i \in N : x \in t(R_i) \text{ or } x \in E'_i\} \in C_x$. Because of the monotonicity condition of the committee, since $E'_i \subseteq E_i$, $\{i \in N : x \in t(R_i) \text{ or } x \in E_i\} \in C_x$. And then, $x \in VC_\Gamma^{V_N}(R_N)$. $\square$

Proposition 7. *If $\mathcal{D}^{V_N}$, $\mathcal{D}^{V'_N}$ are two domains such that $\mathcal{D}^{V_N} \subseteq \mathcal{D}^{V'_N}$, then $rg(VC_\Gamma^{V_N}) \subseteq rg(VC_\Gamma^{V'_N})$.*

Proof. Let $L \in rg(VC_\Gamma^{V_N})$. Then, there exist a profile $R_N \in \mathcal{D}^{V_N}$ such that $L = VC_\Gamma^{V_N}(R_N)$, which means that $\{i \in N : x \in t(R_i) \text{ or } x \in E_i\} \in C_x$ for all $x \in L$. Now, consider a preference profile R'_N that satisfies that $t(R'_i) = t(R_i) \cup (E'_i \setminus E_i)$ for all

[7]This notation means that $\mathcal{D}^{V_i} \subseteq \mathcal{D}^{V'_i}$ for all $i \in N$

$i \in N$. Then, it is easy to check that $R'_N \in \mathcal{D}^{V'_N}$ and $\{i \in N : x \in t(R'_i) \text{ or } x \in E'_i\} \in C_x$ for all $x \in L$. Therefore, $L \in rg(VC_{\Gamma}^{V'_N})$. □

4 Conclusions.

In this paper we have presented a family of restriction of the domain of separable preferences. Those restrictions are the V_i-domains, and are based on partitioning the set of candidates into friends, enemies, and unbiased for each voter. As we illustrate, the structure of the partitions is rich enough to accommodate different situations. In our main result (Theorem 5) we provide a characterization of the SCFs that are strategy-proof and tops-only on each possible V_N-domain. Those SCFs are the V_N-voting by committees rules, that result to be extensions of the rules presented by Barberà et al. (1991). In addition, we also find that, the larger the V_i domain is, (i) the fewer elected candidates there are, and (ii) the larger the range of the SCFs satisfying strategy-proofness and tops-onliness there are.

Regarding strategy-proofness, it is possible to argue that, in practice, even when we have objective or declared information on who the voters' friends, enemies, and unbiased candidates are, we are not legally allow to restrict their space of actions.[8] In other words, the voters whose true preferences are in $\mathcal{D}^{V_N}$ can submit another preference of a larger domain, like $\mathcal{D}^{\text{sep}}$ or even $\mathcal{U}$. We would need then to reformulate the property of strategy-proofness to accommodate this fact. To do that, consider the axiom *extended strategy-proofness*, that is in the line of strategy-proofness but allows the voters to provide a preference in the universal domain.[9] Our characterization result (Theorem 5) do not change if we replace strategy-proofness by its extended version.

There is one key assumption made in this work: there is no restriction on the number of candidates that can be elected. Of course, this is what happens many real-life problems; however, in other cases only one or few candidates can win the election. This situation is beyond the purpose of this paper and it is left for future research.

[8] Penn et al. (2011) consider environments in which individual may provide a preference in the universal domain and show that in these environments, any SCF that is coalitionally strategy-proof must be dictatorial.

[9] **Extended strategy-proofness**. For each $i \in N$, each $R_N \in \mathcal{U}_N$, and each $R'_i \in \mathcal{D}^{V_i}$, $S(R'_i, R_{-i})\, R'_i\, S(R_i, R_{-i})$.

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