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Noemí Navarro

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Departamento de Teoría e Historia Económica
Facultad de Ciencias Económicas y Empresariales
Universidad de Málaga

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Noemí Navarro[†]
Universidad de Málaga

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Abstract

I analyze incentives for provision of quality in a market for an experience good. There is a single producer who is choosing quality and price taking into account three features. First, consumers do not know the quality of the good before purchasing it but use their acquaintances in order to obtain information about it. Second, consumers assign a common initial willingness-to-pay before information transmission takes place. Third, the social network of acquaintances is known to the producer. I define an equilibrium concept taking the point of view of the producer and characterize the set of resulting equilibria for any possible social network. One implication from this characterization is that, if there is a maximal level of quality (given by technological knowledge) that can be chosen, the producer may choose lower levels of quality as the population of consumers is getting more internally connected. This is due to free-riding of information by consumers when quality levels are low. In addition, I identify necessary and sufficient conditions for a new producer arriving in the market to provide a lower quality level though a higher price than the initial producer.

Keywords: Networks, word-of-mouth information, referral consumption.
JEL Classification: D4, D8, L1.

1 Introduction

This paper analyzes incentives for provision of quality in markets where the quality level is unknown to consumers, but where the network of social relationships in which consumers are

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†Universidad de Málaga, Departamento de Teoría e Historia Económica, Plaza El Ejido s/n, Ap. Ofc. Suc. 4, 29071 Málaga, Spain; navarro.prada@gmail.com.

engaged has an effect on their consumption decision. I study the effects of the particular social network structure on the choice of price and quality by producers.

Examples that fit this type of situation are experience goods whose service quality level is not perceived before purchasing takes place and where consumers share opinions that are to some extent containing useful information. Think of provision of services like doctors, lawyers, car diagnosis, etc. Empirical evidence on the effect that word-of-mouth has on consumption decisions can be found in the marketing literature. For instance, Arndt (1965) found that a positive word-of-mouth about a new food product makes it more likely to be purchased. Engel, Blackwell & Kegerreis (1969) conducted a survey on arriving clients to a new center for car diagnosis. They found that 60% of the respondents who recalled the reason to try the new establishment named word-of-mouth. On another level, Freick & Price (1987) find evidence of the existence of individuals who are key in information transmission through casual conversations with their friends (market mavens.)

The model is as follows. There is one good provided by a single producer choosing quality and price. This quality level is parameterized in terms of willingness-to-pay on the consumers' side and there is a maximum quality level available, given by technological knowledge. Consumers have a common (meaning equal) willingness-to-pay in the absence of information transmission. This willingness-to-pay prior to the word-of-mouth communication can be interpreted as the level of reputation, or as being provided by some public device, available to all consumers. Nowadays, one can think of that public device to include the internet.

The social network and purchasing decisions of other agents in the network have an effect on the willingness-to-pay of each consumer. As a simplifying assumption, the willingness-to-pay for any consumer on the social network is a convex combination of the initial willingness-to-pay and the true value of quality. The weights in such a convex combination depend (i) on the particular structure of the network and the relative position of the consumer in it, and (ii) on the purchasing decision of the other agents. These weights hold the following properties. First, if a consumer does not know (directly or indirectly) of anybody acquiring the good her willingness-to-pay is equal to the prior. Second, as soon as a consumer learns through the network that someone also acquires the good her willingness-to-pay gets closer, in the weak sense, to the true value of quality (therefore information is useful). Third, the willingness-to-pay of a consumer cannot depend on purchasing decisions of other consumers non connected (directly or indirectly) to her. Four, the willingness-to-pay for a consumer gets closer, in the weak sense, to the true value of quality if more people connected in the network acquire the good or if new connections get her closer in the network to the other buyers of the good. By this I mean not only that information trough the network is useful but also that first-hand information is more valuable than second-hand information.

An equilibrium concept is defined where the producer chooses first quality and then price, and, afterwards, consumers choose simultaneously to buy or not. Consumers take their decision given the price and the initial willingness-to-pay, and taking into account word-of-mouth effects as explained above. In this new equilibrium concept, where the key notion is network-consistent beliefs on the consumers' side, consumers are simultaneously taking their purchasing decision, but their beliefs about the choice of all the other consumers to whom they are directly or indirectly connected are correct in equilibrium. Their willingness-to-pay for the quality of the good is consistent with the purchasing decisions of all the other consumers and the word-of-mouth effects through the social structure.

A very intuitive interpretation of the equilibrium concept takes the point of view of the producer. Think for example of a dentist. Higher quality service means that he has to become a better dentist by paying a better university, making higher efforts, etc. This means that he becomes a better dentist at a cost. Once he got his title he has to decide the fee he will charge to his potential clients (or patients). These two decisions take place before his clients come to his office. When taking expectations about how many clients he may have per choice of quality and price, he is aware of the social structure and of the fact that word-of-mouth information has an effect on the decisions of his potential clients, but he does not know in which order they may need his services. The equilibrium concept introduced here is an attempt to write down the possible final (or total) demand that the producer may expect to arise when he takes into account word-of-mouth effects on his potential consumers' decisions.

When comparing the choice of quality for two different social structures or networks, one finds that the producer may choose lower levels of quality for social networks that are internally more connected, or, equivalently, for an improvement in the accuracy of information. The intuition behind this feature follows an argument based on free-riding of costly information.

When quality levels are lower than the reputation, information is costly in the sense that (i) in order to obtain information about the real quality level consumers need to buy the good, and (ii) consumers are paying a price higher than the actual level of quality, therefore losing utility when they buy the good. As in Galeotti (2008) consumers have to decide whether to acquire the information (which in this context it is associated with paying the price of the good) or to rely on the information arriving through the network. An improvement in the word-of-mouth accuracy, for example through a more connected network, makes individuals more prone to rely on the information arriving through the network. In addition, an individual acquiring the good does not take into account the impact her purchasing decision has on the information held by other consumers in the network. A consumer who is key for getting other consumers informed (a market maven) might choose to rely on the network instead of acquiring the information himself after such an improvement in word-of-mouth, therefore providing worse information to consumers who are dependant on her consumption to obtain information (given that second-hand information is in general less valuable than first-hand information.) The producer might benefit from that since those dependant consumers decide to buy the good while before the improvement in word-of-mouth they would not. Therefore, the producer gets higher demand at low quality levels even when the accuracy of word-of-mouth information improves.

If social networks on the internet, i.e. Facebook, are interpreted as an improvement in the accuracy of word-of-information, since agents get informed faster and with an idea closer to the true value of information, then we can conclude that its introduction might work in favor of the producer and against consumer's welfare.

The intuition for this negative effect works in a similar way to the one in Bramoullé & Kranton (2007) and Galeotti (2008). The former find in a context of private provision of a local public good (diffusing through social links) that new links may damage overall welfare by reducing individual's incentives due to free-riding effects. Galeotti (2008) finds, in a context of price search by consumers where these ones share information on prices that are privately found through the social network, that a better word-of-mouth information may crowd out consumer's private search and therefore soften firms' competition.

Finally, I introduce competition by the arrival of a second producer once quality, price and buying decisions have been realized for the initial producer. The main conclusion from this type of competition is that if the initial producer provides high quality level and high prices so that there is at least a set of consumers connected among them, but disconnected to the rest, who did not try the good, a second producer can hope to capture them as clients even with lower quality and higher price than the first producer.[1]

There is a literature studying effects of word-of-mouth communication on consumers behavior. For example, Ellison & Fudenberg (1995), Vettas (1997), Corneo & Jeanne (1999) and Banerjee & Fudenberg (2004). Contrary to what I do here, these papers explore the causes of herding behavior and fashion on a population. In these papers, the information transmission takes place in a different way than it does here. All their settings are dynamic and, with the exception of Corneo & Jeanne (1999), the way information transmission is modeled is not bilateral in spirit. In Corneo & Jeanne (1999), each bilateral meeting implies a transmission of consumption skills, and meetings are random, while here consumers are meeting all people who are already socially related to them. In Ellison & Fudenberg (1995) each consumer hears of the current experiences of a random sample of the other players. In Vettas (1997) the knowledge about the quality is an increasing function of past purchases. Finally, in Banerjee & Fudenberg (2004), as in Ellison & Fudenberg (1995), consumers consult a sample from the rest of the population, with the difference that those consulted people report not only what they themselves have chosen, but they may also send signals that are correlated with the payoffs from the choices (an indication of how satisfied the consulted consumers are with their chosen alternative). The present paper differs from the papers cited above in the following features. First, I introduce a static model where the concept of equilibrium includes the effect of the word-of-mouth communication through the network. Second, I model the social structure specifically as a network of bilateral relations. Finally, as mentioned before, while their aim is to characterize herding behavior by consumers, mine is to study the effect of the specifics of the social structure on the choice of quality by the producer.

This paper proceeds as follows. In Section 2 the model is presented. Section 3 discusses the results of the paper. Section 4 concludes.

[1] If there is no one in this component of the network acquiring the good to the first producer then there are no good news about his quality among the participants of such a component.

2 The Model

There is one homogeneous good produced by a single agent, referred to as the producer. The quality of this good can be parameterized by $\theta \in [0, \theta^{max}]$, where θ^{max} denotes the maximal level of quality given by the present level of technological knowledge. This parametrization corresponds to monetary units, or, in other words, the common willingness-to-pay for consumers. Let $N = \{1, ..., n\}$ denote the finite set of (potential) consumers. Throughout the paper, I will refer to θ as the "quality" when I am dealing with the producer, and as the "willingness-to-pay" when I am dealing with consumers.

The quality parameter θ is known to (and eventually chosen by) the producer, but unknown to consumers. Consumers have a common, initial willingness-to-pay $\bar{\theta}$, based on a public information device about the real value of θ. Note that $\bar{\theta}$ could be equal to 0. There is a network of social relations among consumers in N, and the structure of this network will have an effect on the purchasing decision of a consumer. In what follows, the network of social relations will be referred to as the social structure.

The timing of the process is as follows. The producer chooses quality θ and then price given θ. Then consumers choose simultaneously to consume or not, given the prices and the social structure.

2.1 Production

For the sake of simplicity, the good with quality θ in $[0, \theta^{max}]$ is produced at a marginal cost of $c\theta$, for $0 < c < 1$, with zero fixed costs.

The expected profit to the producer is then given by

$$\pi (\theta, p) = q^d (\theta, p) [p - c\theta], \tag{1}$$

where $q^d (\theta, p)$ is the expected number of consumers buying one unit of the good (and therefore the expected demand) when the producer chooses quality θ and price p. The way this expected value is computed by the producer is formalized in the equilibrium concept.

2.2 Consumers: Utility

The good with quality θ is available for consumers at a market price denoted by p. Consumers are risk neutral, need at most one unit of the good and prefer higher quality and lower price. Thus, the utility function for each consumer i can be written as

$$U_i (\theta) = \begin{cases} \theta - p, & \text{if } i \text{ buys the good,} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Consumers do not know the value of θ. They have two sources of information: (1) as I said before, there exists a common (to all consumers) willingness-to-pay in the absence of word-of-mouth communication, denoted $\bar{\theta}$, and (2) the word-of-mouth communication among consumers takes place through the existing social structure. This will be formalized in the equilibrium concept defined in 2.4 below.

2.3 Consumers: The Social Structure

I proceed to formally describe the social structure since some notions on networks are needed prior to the definition of the equilibrium concept in 2.4. The network of social relations among consumers in N can be represented by an undirected graph g, which is a set of unordered pairs ij, where i, $j \in N$, and $i \neq j$. Throughout the paper, each unordered pair ij will be referred to as a link. A link in the network (or social structure) means that those two consumers have casual conversations containing relevant information about the quality of the good θ. For the rest of this paper, the set N is considered to be fixed. I denote by g^N the complete graph, meaning the graph where all possible unordered pairs are listed, and by g^i the graph where agent i is directly connected to everybody else and everybody else is only connected to agent i. This structure g^i is normally called the star with center in agent i. Figure 1 shows different social structures for $n = 5$.

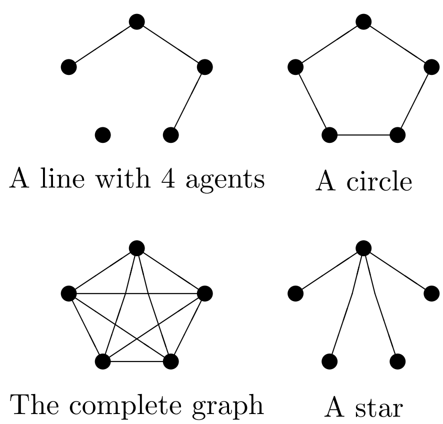

FIGURE 1

Given a graph g, a group of consumers $S \subseteq N$ is called a component of g if: (1) for every two consumers in S, there is a path, that is, a set of consecutive links in g connecting them, and (2) for any consumer i in S and any consumer j not in S, there is no path in g which connects them. Let $\mathcal{C}(g)$ be the set of components of g. Note that $\mathcal{C}(g)$ is a partition of N. Denote by $\varsigma(g)$ the number of elements in $\mathcal{C}(g)$. In Figure 1 above, the line with four agents has two components, one where there is one isolated consumer and the other one with four of them together. The other graphs have only one component.

A graph g is *connected* if $\mathcal{C}(g) = \{N\}$, or, in other words, if $\varsigma(g) = 1$. Therefore, in the examples in Figure 1 above only the line with four agents is not connected.

Two disjoint sets of consumers S and S' are said to be connected if they belong to the same component of g. Note that these sets can be singletons.

2.4 Equilibrium concept

The equilibrium concept is defined backwards, formalizing first the consumers' possible choices from the producer's point of view. The producer's objective will consist of maximizing expected profit.

The social structure allows consumers to extract new information about the quality of the good. I will formalize this effect of the social structure on referral consumption by a family of functions $\lambda_i(S, g) \in [0, \delta]$ for every i in N, where g is the given network structure and S is a subset of N, with $\delta \in \Re$ close to 1. These functions $\lambda_i(S, g)$ have the following properties:

1. $\lambda_i(S, g) = \lambda_i(S \backslash \{i\}, g)$,
2. $\lambda_i(\emptyset, g) = 0$,
3. $\lambda_i(N, g) = \delta$ whenever $g^i \subseteq g$,
4. $\lambda_i(S, g) = \lambda_i(S_i, g)$, where S_i is the maximal subset of S that is connected to i in g,
5. $\lambda_i(S, g) > 0$ if i is connected to a nonempty subset of S in g, and
6. $\lambda_i(S, g) \leq \lambda_i(S \cup \{j\}, g)$, for any $j \notin S$, and $\lambda_i(S, g) \leq \lambda_i(S, g \cup jk)$, for $jk \notin g$.

Assume agent i is considering buying the good, knowing that the given subset S in N is a group of buyers. Property 1 for $\lambda_i(S, g)$ formalizes the information that an agent i has **before** purchasing the good. Properties 2 and 3 state that an agent i has no information available if no one is buying the good, while she has the best information available if she knows all the other agents through the network and they are all purchasing the good in the economy. Property 4 states that an agent i can only extract information from people to whom she is connected through the social network. This together with Property 2 implies that any agent isolated from the set of buyers cannot learn any new information from the network. Property 5 states that one agent buying always generates useful information for the agents directly or indirectly connected to them. Property 6 states monotonicity properties. The information an agent i could extract cannot worsen either when one new agent in the economy buys or when the network structure g gets more connected. Finally, the fact that the maximum amount of information that can be obtained is equal to δ formalizes the part of information in the consuming experience that can be transmitted through the network, while $1 - \delta$ is the part of information that can only be acquired through direct consumption.

The family of functions λ_i are used to define the consumer's part of the equilibrium concept. Let me first introduce the formal definition and an intuitive discussion will follow.

Definition 2.1 *Given any choice of θ and p by the producer, a subset of consumers $B \subseteq N$ is called a buyers configuration after θ and p if (i) for every $i \in B$: $\lambda_i(B,g)\,\theta + (1-\lambda_i(B,g))\,\bar{\theta} \geq p$, and (ii) for every $i \notin B$: $\lambda_i(B,g)\,\theta + (1-\lambda_i(B,g))\,\bar{\theta} < p$.*

Let $Q(\theta,p)$ denote the set of all possible buyers configurations $B \subseteq N$ if the producer chooses quality θ and price p. Since consumers are choosing simultaneously there may be multiplicity of equilibria on the consumers side. This implies that $Q(\theta,p)$ consists of several subsets of consumers in general.

Some comments about the intuition of this definition are in order.

1. I am saying here that consumers update their initial willingness to pay according to the family of functions $\lambda_i(.,.)$. By definition, the closest is $\lambda_i(.,.)$ to 1 the more accurate the information about quality, and therefore the closer the willingness-to-pay to θ, and vice versa.

2. Consumers are price-takers and it looks as if they do not extract information about quality from the observed price. Imagine a market in which higher price is associated to higher quality. Then one could formalize this idea by including another parameter α such that the willingness to pay for a consumer is in part influenced by the network and in part influenced by the price. More explicitly, the willingness-to-pay for a consumer will be $\alpha[\lambda_i(B,g)\,\theta + 1 - \lambda_i(B,g)\,\bar{\theta}] + (1-\alpha)\,p$. Note that this willingness-to-pay is greater or equal to p if and only if $\lambda_i(B\backslash\{i\},g)\,\theta + (1-\lambda_i(B\backslash\{i\},g))\,\bar{\theta} \geq p$, and vice versa, as far as $\alpha > 0$. This definition of buyers configuration is therefore consistent with (i) an updating process of beliefs on quality by consumers that include information on prices, and (ii) the comparison of this updated belief on quality with the observed price. Note that α could be itself a function of the parameters, and still be irrelevant as far as $\alpha(.) > 0$. Hence, separability, and not linearity, in the parameters like p or θ is the key.

3. Recall that from the producer's point of view, the problem is to estimate the number of consumers that could purchase the good for each choice of quality and price. This is made taking into account that there is referral consumption through the social network. For the producer, the dynamic process behind the purchasing decision - i.e., a sequence of individual information updating according to the acquaintances in the social network and purchasing decision consequently afterwards - is a black box that depends on the order consumers need the good and purchase. The definition of a buyers configuration is a method to select subsets of potential consumers that are consistent with referral behavior given the social network and each other's purchasing decision. Furthermore, once consumers are purchasing according to a buyers configuration, no further purchase will take place. What the producer does not know is the dynamical process that has lead the consumers to purchase according to that configuration. It is therefore a consistent and stable candidate for a demand configuration for the producer, but does not specify ad-hoc dynamics behind the purchasing process.

4. Finally, the fact that the updating structure is written as a convex combination with weights keeps the exposition simple without loss of generality. Note that the weights $\lambda_i(.,.)$ can reflect different updating behaviors, depending: on the distance in the network among purchasing consumers (second hand information is less valuable), on the number of purchasing consumers (more purchasing consumers delivers better information) or even Bayesian updating based on probabilities (which generates anyway a willingness to pay that lies between θ and $\bar{\theta}$ and it is monotonic in the sense defined by Property 6).

Next subsection presents the equilibrium on the producer's side.

Definition 2.2 *The producer has network-consistent beliefs if for any choice of θ and p the expected number of consumers $q^d(\theta, p)$ can be written as the convex combination*

$$\sum_{B \in \ Q(\theta,p)} \rho\left(B, Q\left(\theta, p\right)\right) |B|, \tag{3}$$

where $\rho\left(B, Q\left(\theta, p\right)\right) > 0$, for all $B \in Q\left(\theta, p\right)$, and $\sum_{S \in Q(\theta,p)} \rho\left(B, Q\left(\theta, p\right)\right) = 1$.

Network-consistent beliefs for the producer mean that the producer is assigning positive probability only to sets of consumers who are a buyer's configuration in the sense defined above. Note that the probability assigned to each of the possible equilibria B on the consumers side, for given θ and p, denoted $\rho\left(B, Q\left(\theta, p\right)\right)$, depends only on B itself and on the set $Q\left(\theta, p\right)$. This means that if there are two choices (θ, p) and (θ', p') the beliefs for the producer are the same if $Q\left(\theta, p\right) = Q\left(\theta', p'\right)$. In other words, the beliefs do not depend on the particular choices of θ or p, or on the graph g, as far as the resulting structure of consumers' equilibria $Q\left(\theta, p\right)$ is the same. Note that this does not preclude the monopolist to have a "refining" tool or privileged information that will select one configuration out of all possible configurations, as $\rho\left(B, Q\left(\theta, p\right)\right)$ could be equal to 1 for one B in $Q\left(\theta, p\right)$. Being network-consistent means that if the "refining" tool selects B out of $Q\left(\theta, p\right)$, then, B is also selected for θ' and p' whenever $Q\left(\theta', p'\right) = Q\left(\theta, p\right)$.

I proceed to formally introduce the concept of equilibrium.

Definition 2.3 *An equilibrium is a triple $\mathcal{E} = (\theta^*, p^*, Q^*)$, where θ^* and p^* are non-negative real numbers and Q^* consists of subsets of consumers such that*

1. *The producer chooses first quality θ^* and then price p^* maximizing expected profit given that his beliefs about the demand for each possible choice of quality and price are network-consistent (see Definition 2.2).*

2. *$Q^* = Q\left(\theta^*, p^*\right)$.*

It is clearly seen from the definition of the equilibrium that $\mathcal{E}$ is a function of the parameters of the model: the impact of quality on marginal costs c and the system of beliefs ρ, on the producer side, and the prior $\bar{\theta}$, the family of function $\lambda_i(.,.)$ and the social network g, on the consumers side. To avoid abuse of notation I simply write $\mathcal{E}$ instead of $\mathcal{E}\left(c, \rho, \bar{\theta}, \lambda, g\right)$, as these parameters are considered fixed.

3 Results

3.1 The equilibrium

The following theorem characterizes the quality choice by the producer in equilibrium, denoted θ^*. Recall that there is a maximum level of θ, denoted θ^{max}, given by the frontier of technological knowledge. I need the following definitions. If the producer chooses a quality level equal to the common prior $\bar{\theta}$ the best he can do is to charge a price equal to $\bar{\theta}$, getting all consumers acquiring the good, and thus obtaining a profit equal to $n(1-c)\bar{\theta}$. Let c_0 be such that $n(1-c_0)\bar{\theta}$ is equal to the maximum profit the consumer would obtain if he were to choose a quality level equal to 0 given the network and the family of λ's.

For $B \subseteq N$, let $\lambda^{\max}(B) = \max_{i \in B} \lambda_i(B, g)$ and $\lambda^{\min}(B) = \min_{i \in B} \lambda_i(B, g)$ be the maximal and the minimal information measure inside B, respectively, and let $\lambda^{\max}(\neg B) = \max_{i \notin B} \lambda_i(B, g)$ and $\lambda^{\min}(\neg B) = \min_{i \notin B} \lambda_i(B, g)$ be the maximal and minimal information, resp., outside B. By definition, $\lambda^{\max}(\neg\emptyset) = \lambda^{\min}(\neg\emptyset) = 0$ and therefore $\lambda^{\max}(\neg B) = \lambda^{\min}(\neg B) = 0$ if $B \in \mathcal{C}(g)$. Furthermore, even if $B \in \mathcal{C}(g)$, $\lambda^{\min}(B) = \lambda^{\max}(B) = 0$ if B is a singleton. Note that $\lambda^{\max}(N)$ is the maximum amount of information that can be extracted through the network.

Theorem 3.1 *Let c_0, and $\lambda^{\max}(B)$ and $\lambda^{\min}(\neg B)$ be as defined above. Then $p^* = \bar{\lambda}\theta^* + \left[1-\bar{\lambda}\right]\bar{\theta}$, for some $\bar{\lambda} \in [0, \delta]$ and*

1. *Assume $c \geq \lambda^{\max}(N)$. Then $\theta^* = 0$.*

2. *Assume now that $\lambda^{\max}(N) \geq c > \max_{B \subset N : \lambda^{\min}(B) > \lambda^{\max}(\neg B)} \lambda^{\min}(B)$. Then*

$$\theta^* = \begin{cases} 0, & \text{if } c \geq c_0, \\ \bar{\theta}, & \text{if } c \leq c_0. \end{cases}$$

3. *Finally, assume that $\max_{B \subset N : \lambda^{\min}(B) > \lambda^{\max}(\neg B)} \lambda^{\min}(B) \geq c > 0$. Then, there are threshold levels θ_1 and θ_2 both greater than $\bar{\theta}$ such that:*

$$\theta^* = \begin{cases} 0, & \text{if } c \geq c_0 \text{ and } \theta^{\max} \leq \theta_1, \\ \bar{\theta}, & \text{if } c \leq c_0 \text{ and } \theta^{\max} \leq \theta_2, \\ \theta^{\max} & \text{either if } c \geq c_0 \text{ and } \theta^{\max} \geq \theta_1, \text{ or if } c \leq c_0 \text{ and } \theta^{\max} \geq \theta_2. \end{cases}$$

The proof of this theorem is in the appendix.

Theorem 3.1 states the following. If the impact of quality on marginal costs c is bigger than the maximum amount of information that consumers could extract through the network, the producer has no incentives to provide quality. For intermediate values of c, where the limits depend on the amount of information that consumers could extract from the network, the producer is willing to provide at most the reputation level given by $\bar{\theta}$. Finally, for low values of c, or, the other way around, for information functions high enough, the producer chooses $\theta^{\max}$ if $\theta^{\max}$ is above a certain threshold level, while he will choose quality levels equal to $\bar{\theta}$ or 0 depending on which is the most profitable one. Note that a c higher than c_0 means that 0 is more profitable than $\bar{\theta}$, and viceversa. This parameter c_0 depends on the structure of the network, and, as we will see later, it might not be monotonic on the density of the network.[2]

The intuition behind the proof is deeply rooted in the definition of a buyers configuration. First note that when quality levels are low (smaller than $\bar{\theta}$) consumers who acquire the good have less information than consumers who do not acquire the good in any buyers configuration. Therefore, if a set B is a buyers configuration for low quality levels it has to be that $\lambda^{\max}(B) < \lambda^{\min}(\neg B)$. On the contrary, when quality levels are high (greater than $\bar{\theta}$) consumers who acquire the good have more information than the ones who do not acquire it in a buyers configuration. Therefore, if a set B is a buyers configuration for high quality levels it has to be that $\lambda^{\min}(B) > \lambda^{\max}(\neg B)$. Second, the producer faces a trade-off. On one hand, if he provides higher level of quality marginal cost is going to get higher at a proportion equal to c. On the other hand, if he provides higher level of quality he can charge higher prices and expect for a non zero number of consumers given that there is referral consumption through the network. These higher prices that can be charged are intimately related to the λ functions. The highest price that could be charged is (i) $\lambda^{\max}(N)\,\theta + [1-\lambda^{\max}(N)]\bar{\theta}$ when θ is bigger than $\bar{\theta}$, and (ii) $\bar{\lambda}\theta + \left(1-\bar{\lambda}\right)\bar{\theta}$, where $\bar{\lambda} = \min_{\lambda^{\max}(B)<\lambda^{\min}(\neg B)} \lambda^{\max}(B)$, when θ is smaller than $\bar{\theta}$. The marginal cost is $c\theta$. Note that, by monotonicity, $\bar{\lambda} < \lambda^{\max}(N)$ and therefore the marginal benefit of quality θ is always a decreasing function when $c > \lambda^{\max}(N)$. Therefore, the best θ is zero for that case. Only when the marginal benefit of quality could be positive could the producer find it worth it to provide non zero quality levels. Finally, the producer provides the maximum quality level available if (i) marginal costs of quality are small with respect to the information that can be transmitted through the network and (ii) above a certain threshold. Note that below that threshold, the increasing part of the marginal benefit might not hit higher than the level at $\bar{\theta}$ or at 0, therefore, thresholds for θ^{max} are needed.

The following comment on efficiency is in order. Note that the market surplus in any equilibrium outcome is equal to the number of consumers who are actually buying times $(1-c)\,\theta^*$. This implies that, if $c < 1$, the best equilibrium (in terms of maximizing total surplus) is the one in which the producer provides maximum quality level and all consumers buy. In this sense,

[2]By density of the network I mean the following. One network is denser than another if everybody has at least the same number of links in the former than in the latter, and at least one agent has strictly more links.

even when the impact on marginal costs c is small enough, one still needs a threshold level to give the producer the incentives to provide the highest quality level possible. Note that this problem appears even when the network is the complete one.

To conclude this section, the following comment of monotonicity of θ^* is in order. The reader might expect θ^* to be increasing in the connectivity of the network or in the accuracy of the information transmission given by λ (both comparative statics affect the equilibrium through the same way.) *Example 3* in Navarro (2006) shows, for a fixed family of functions λ, that the producer could provide lower levels of quality at networks that are more connected. Formally, the example illustrates two networks g' and g where $g' \subset g$ and such that the quality choice by the producer is higher in g' than in g. The intuition for this result lies on the fact that consumers free-ride information about quality, when quality levels are low. Information in this case is costly as the price that it is paid is typically higher than the real quality level, since the higher reputation level allows the producer to push up the price. If highly connected agents buy the good in the initial network the information about low quality is better transmitted. This configuration is also cheaper as less people buy the bad quality good. Now imagine some highly connected agent gets directly connected to someone new. This highly connected agent, say i, who at the new network extracts better information than before, might decide not to buy the good (free-ride) while before she would. Agents connected to i receive worse information at the new network given that second hand information is worse than first hand information. Those agents may decide to buy the good at the new network while they did not buy the good in the initial network. In turn, for certain values of the parameters more consumers end up buying bad quality good in the more connected network, and incentives for provision of bad quality by the producer are stronger.

3.2 Introducing competition

Assume now that once the producer has decided θ^* and p^* and a set of consumers $B^* \in Q(\theta^*, p^*)$ have bought the product, another producer can enter the market with the same technology, i.e., producing units of quality θ at a marginal cost equal to c. The potential consumers to this new producer are now only $N \backslash B^*$ since consumers who have already acquired the good only need one unit. Assume further that consumers are short-sighted, in the sense that when deciding whether to buy the good or not to the initial producer they do not compute expectations about other producers that could enter the market. Given that the new producer cannot steal consumers from the incumbent and that consumers are short-sighted, the choice of the initial producer is not affected by the fact that there will be other producers in the market.

Let θ_c^* and p_c^* be the choice of the new producer (challenger) that maximizes expected profit assuming that the new producer has consistent beliefs. The definition of a buyers configuration for the challenger is slightly different than for the initial producer, as there are consumers in the market who have acquired the good from the initial consumer but can still transmit information about the new producer. Furthermore, only individuals who did not buy to the initial producer could buy to the new one.

Definition 3.2 *Let $B^* \in Q(\theta^*, p^*)$ be the buyers configuration that has been realized after θ^* and p^* were chosen by the initial producer and let θ_c and p_c be the choice of quality and price by the new producer. For any agent $i \notin B^*$ let $\bar{\theta}_c^i = \lambda_i(B^*, g)\theta^* + (1 - \lambda_i(B^*, g))\bar{\theta}$. A set of consumers $B \subseteq N \backslash B^*$ is called a buyers configuration for the new producer after θ_c and p_c given θ^*, p^* and B^* if (i) for every $i \in B$: $\lambda_i(B, g)\theta_c + (1 - \lambda_i(B, g))\bar{\theta}_c^i \geq p_c$, and (ii) for every $i \notin B$: either $\lambda_i(B, g)\theta_c + (1 - \lambda_i(B, g))\bar{\theta}_c^i < p_c$ or $i \in B^*$.*

Note that the definition of a buyers configuration implicitly assumes that consumers who do not acquire the good to the first producer build a belief about what to expect from the next producer according to what they have already observed in the network, which is $\bar{\theta}_c^i = \lambda_i(B^*, g)\theta^* + (1 - \lambda_i(B^*, g))\bar{\theta}$ for any agent $i \notin B^*$. Next results compare the quality and price chosen by the new producer as compared to the ones chosen by the initial producer, assuming that $B^* \neq N$.

Theorem 3.3 *Let θ^*, p^*, B^*, θ_c and p_c be defined as before. Let B_c be a buyers configuration for the new producer after θ_c and p_c given θ^*, p^* and B^*. Assume $B_c \neq \emptyset$. Then:*

1. *If $\theta_c < \theta^*$ then $p_c < max\{p^*, \theta^*\}$.*

2. *If $p_c > p^*$, then (i) $|B_c \cap S| > 1$ for any $S \in \mathcal{C}(g)$ such that $|B_c \cap S| \neq \emptyset$, and (ii) $\theta_c > p^*$.*

Proof of Theorem 3.3. If we assume that $B_c \neq \emptyset$ there is at least one $i \in B_c$ for which (i) $\lambda_i(B^*, g)\theta^* + (1 - \lambda_i(B^*, g))\bar{\theta} = \bar{\theta}_c^i < p^*$, since i cannot be in B^*, and (ii) $\lambda_i(B_c, g)\theta_c + (1 - \lambda_i(B_c, g))\bar{\theta}_c^i \geq p_c$. Given that $\bar{\theta}_c^i < p^*$ it has to be true that $p_c < \lambda_i(B_c, g)\theta_c + (1 - \lambda_i(B_c, g))p^*$ for each $i \in B_c$.

- I prove (i) first. Note that, from above, if $\theta_c < \theta^*$ then $p_c < \lambda_i(B_c, g)\theta^* + (1 - \lambda_i(B_c, g))p^*$ for each $i \in B_c$. Since B_c is an non-empty set and $\lambda_i(B_c, g)$ is a number between 0 and 1 for any i, this implies that $p_c < max\{\theta^*, p^*\}$.

- Consider part (ii) now. If $p^* < p_c$ then it has to be that $p^* < \lambda_i(B_c, g)\theta_c + (1 - \lambda_i(B_c, g))p^*$ for each $i \in B_c$. Note, first, that this can only be true if $\lambda_i(B_c, g) > 0$ for each $i \in B_c$. By definition of the family of λ's, this implies that any $i \in B_c$ has to be connected to at least one other buyer in B_c. Formally, for any $S \in \mathcal{C}(g)$ such that $|S \cap B_c| \neq 0$ it has to be that $|S \cap B_c| > 1$ (otherwise, there is one buyer in B_c disconnected from the rest of buyers). Note finally that if $p^* < \lambda_i(B_c, g)\theta_c + (1 - \lambda_i(B_c, g))p^*$ for each $i \in B_c$ then it has to be that $p^* < \theta_c$. This completes the proof of Theorem 3.3. ■

By Theorem 3.3 I can show that if the new producer expects a nonempty set of buyers then (i) providing lower quality implies that price is bounded above, and (ii) announcing a higher price implies that quality is bounded below by the price of the incumbent. Note that if quality by the initial producer is low then $\bar{\theta} \geq p^* > \theta^*$. By Theorem 3.3 we can therefore conclude that lower quality level and lower prices have to be together for the new producer if he expects a nonempty set of buyers. On the other hand, if $\theta^* = \bar{\theta}$ then all consumer actually buy the product to the initial producer, and therefore no market is left for a new producer. The question left consists then of, assuming that quality provided by the initial producer is high, whether the new producer could announce a higher price and provide a lower quality level yet expect a nonempty set of buyers. Sufficient conditions for this are stated in the following theorem.

Theorem 3.4 *Let θ^*, p^* and $B^* \neq N$ be defined as before. Assume that $\theta^* > p^*$ and there is a component $S \in \mathcal{C}(g)$ such that (i) $S \subseteq N \backslash B^*$ and (ii) $S \in Q(\theta^*, p^*)$. Then, there exists a choice of quality and price θ_c and p_c by the new producer such that (i) $p^* \leq p_c$ and $\theta_c \leq \theta^*$, and (ii) at least one buyer's configuration after θ_c and p_c given θ^*, p^* and B^* is nonempty.*

Proof of Theorem 3.4. Note that if $B^* \neq N$ and $\theta^* > p^*$ then $\theta^* > p^* > \bar{\theta}$ by Theorem 3.1. Take a component $S \in \mathcal{C}(g)$ such that (i) $|S| > 1$, (ii) $S \in N \backslash B^*$ and (iii) $S \in Q(\theta^*, p^*)$. Recall that $\lambda^{min}(S)$ is equal to $min_{i \in S} \lambda(S, g)$. By design of S, $\lambda^{min}(S) > 0$.

Fix $p_c = \lambda^{min}(S)\,\theta_c + \left(1 - \lambda^{min}(S)\right)\bar{\theta}$ for any θ_c in the interval $\left[\frac{p^* - \left(1 - \lambda^{min}(S)\right)\bar{\theta}}{\lambda^{min}(S)}, \bar{\theta}\right]$. Note that this interval makes sense given that S is disconnected from $N \backslash S$ so if $S \in Q(\theta^*, p^*)$ it is true that $\lambda^{min}(S)\,\theta^* + \left(1 - \lambda^{min}(S)\right)\bar{\theta} \geq p^*$. By construction, $\theta_c \leq \theta^*$, since $\theta^* > \bar{\theta}$, and $p^* \leq p_c$. Furthermore, given that $\theta^* > p^*$ and $B^* \neq N$ we know that $p^* \geq \bar{\theta}$, and therefore $p_c > \bar{\theta}$. I show that S is a buyers configuration after θ_c and p_c given θ^*, p^* and B^*.

- For any $i \in S$: $p_c = \lambda^{min}(S)\,\theta_c + \left(1 - \lambda^{min}(S)\right)\bar{\theta} \leq \lambda_i(S, g)\,\theta_c + (1 - \lambda_i(S, g))\,\bar{\theta} = \lambda_i(S, g)\,\theta_c + (1 - \lambda_i(S, g))\,\bar{\theta}_i^c$. Recall that, since $S \in \mathcal{C}(g)$ we know that $\bar{\theta}_i^c = \bar{\theta}$ for any $i \in S$.

- For any $i \notin S$, $\lambda_i(S, g) = 0$ given that $S \in \mathcal{C}(g)$. If $i \notin B^*$ it has to be that $\bar{\theta}_i^c < p^*$. Since $p^* \leq p_c$, then for all $i \notin S$ either $i \in B^*$ or $\lambda_i(S, g)\,\theta_c + [1 - \lambda_i(S, g)]\,\bar{\theta}_i^c = \bar{\theta}_i^c < p_c$.

Hence, S is a buyers configuration after θ_c and p_c given θ^*, p^* and B^*. This completes the proof of Theorem 3.4. ■

By Theorem 3.4 I have identified sufficient conditions for the new producer to announce a higher price and provide a lower level of quality yet expecting a nonempty set of buyers. Note that these conditions satisfy themselves the necessary conditions stated by Theorem 3.3. Note that the existence of a component $S \in N|g$ such that (i) $S \subseteq N \backslash B^*$ and (ii) $S \in Q(\theta^*, p^*)$

becomes a necessary condition to have both higher prices and lower quality level at the same time for the new producer if the set of buyers to the initial producer B^* is disconnected in g from $N \backslash B^*$. The careful reader will realize that if an agent i not in B^* is isolated, her willingness-to-pay has not changed, i.e., $\bar{\theta}_i^c = \bar{\theta}$. Therefore, this agent will only buy at prices lower than $\bar{\theta}$. If a new producer wants to charge a higher price than the initial one, p^*, being the later already greater than $\bar{\theta}$, will never convince an isolated consumer to purchase his product either.

4 Conclusion

I have presented a model to study the way information sharing by consumers through word-of-mouth gives incentives for provision of quality in the context of a market with asymmetric information.[3] The main result characterizes the choice of quality by the producer in terms of threshold levels for both the maximum quality level that can be provided and the impact of higher quality levels on marginal costs. This result nicely implies that the choice of quality by the producer is not monotonic on the density of the network, due to free-riding on information on the consumers' side. When sequentially introducing a second producer, I have identified sufficient conditions for this arriving producer to provide lower quality at the same time as imposing a higher price than the initial producer.

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[3] Here, asymmetric information means that one side of the market has more information than the other one, as the producer knows the quality, but consumers do not.

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Appendix

Theorem 3.1 characterizes the equilibrium as a function of the parameters, for different ranges of the latter. As a previous step, Lemma 4.1 characterizes the structure of the buyers configurations for any possible choice of quality and price by the producer. Lemma 4.1 not only helps proving the results stated in Theorem 3.1 but it also helps understanding the equilibrium behavior for consumers.

Lemma 4.1 *Let θ and p be the choices of quality and price made by the producer.*

1. *Assume that $\theta > \bar{\theta}$.*
   - (a) *If $0 \leq p \leq \bar{\theta}$ then $Q(\theta, p) = \{N\}$ and $q^d(\theta, p) = n$.*
   - (b) *If $p > \bar{\theta}$ then, there exist numbers $\{\lambda_k\}_{k=1}^{K}$ with $0 = \lambda_1 \leq \lambda_2 \leq ... \leq \lambda_k \leq ... \leq \lambda_K \leq \delta$ such that*
     - i. *if $\lambda_k \theta + (1 - \lambda_k) \bar{\theta} < p \leq \lambda_{k+1} \theta + (1 - \lambda_{k+1}) \bar{\theta}$ then $Q(\theta, p)$ keeps the same structure and therefore $q^d(\theta, p) = q^H_{k+1}$ is constant, for $k \leq K - 1$.*
     - ii. *if $p > \lambda_K \theta + (1 - \lambda_K) \bar{\theta}$ then $Q(\theta, p) = \{\emptyset\}$ and $q^d(\theta, p) = 0$.*
2. *Assume that $\theta < \bar{\theta}$. Then, there exist numbers $\{\lambda_k\}_{k=1}^{K}$ with $0 \leq \lambda_K \leq \lambda_{K-1} \leq ... \leq \lambda_1 = \lambda^{\max}(N) \leq \delta$*
   - (a) *If $0 \leq p \leq \lambda_1 \theta + (1 - \lambda_1) \bar{\theta}$ then $Q(\theta, p) = \{N\}$ and $q^d(\theta, p) = q^L_1 = n$.*
   - (b) *If $\lambda_k \theta + (1 - \lambda_k) \bar{\theta} < p \leq \lambda_{k+1} \theta + (1 - \lambda_{k+1}) \bar{\theta}$ then $Q(\theta, p)$ keeps the same structure and $q^d(\theta, p) = q^L_{k+1}$ is constant.*
   - (c) *If $\lambda_K \theta + (1 - \lambda_K) \bar{\theta} < p \leq \bar{\theta}$ then $q^d(\theta, p) = \varsigma(g)$.*
   - (d) *If $p > \bar{\theta}$ then $Q(\theta, p) = \{\emptyset\}$ and $q^d(\theta, p) = 0$.*

Proof of Lemma 4.1. Fix a set of consumers $B \subsetneq N$. Recall that $\lambda^{\max}(B) = \max_{i \in B} \lambda_i(B, g)$ and $\lambda^{\min}(B) = \min_{i \in B} \lambda_i(B, g)$ are the maximal and the minimal information measure inside $B \subseteq N$, respectively, and $\lambda^{\max}(\neg B) = \max_{i \notin B} \lambda_i(B, g)$ and $\lambda^{\min}(\neg B) = \min_{i \notin B} \lambda_i(B, g)$ are the maximal and minimal information, resp., outside $B \subset N$. Consider first the case when the producer chooses a θ such that $\theta > \bar{\theta}$. I first identify the conditions for a set B to be a buyers configuration. These conditions will be needed to prove each of the different cases stated in the first part of Lemma 4.1. Recall that, by definition of a buyers configuration, if $B \in Q(\theta, p)$ then (i) for all $i \in B$: $\lambda_i(B, g) \theta + [1 - \lambda_i(B, g)] \bar{\theta} \geq p$ and (ii) for all $i \notin B$: $\lambda_i(B, g) \theta + [1 - \lambda_i(B, g)] \bar{\theta} < p$. Note that if $\theta > \bar{\theta}$ then $\lambda_i(B, g) \theta + [1 - \lambda_i(B, g)] \bar{\theta} \geq \lambda_j(B, g) \theta + [1 - \lambda_j(B, g)] \bar{\theta}$ for i, $j \in N$ if and only if $\lambda_i(B, g) \geq \lambda_j(B, g)$. Hence, if $\theta > \bar{\theta}$ and $B \in Q(\theta, p)$ it has to be that:

- $\lambda^{min}(B)\,\theta + \left[1-\lambda^{min}(B)\right]\bar{\theta} \geq p$ if $B \neq \emptyset$, and
- $\lambda^{max}(\neg B)\,\theta + \left[1-\lambda^{max}(\neg B)\right]\bar{\theta} < p$ if $B \neq N$.

Note that if B is a union of components (singletons or not) or if $B = \emptyset$, then $\lambda^{max}(\neg B) = 0$. Furthermore, if B includes an agent i not connected to any other agent in B, in particular, if B is a singleton or includes a component that is a singleton, then $\lambda^{min}(B) = 0$. Let $\mathcal{VC}(g)$ denote the set of components of g containing at least two members and unions of them, i.e., $S \in \mathcal{VC}(g)$ if and only if either $S \in \mathcal{C}(g)$ and $|S| > 1$ or there are two sets S_1 and S_2 both in $\mathcal{VC}(g)$ with $S = S_1 \cup S_2$. From all this, if $\theta > \bar{\theta}$ and $B \in Q(\theta, p)$:

1. $p \leq \lambda^{\min}(N)\,\theta + [1-\lambda^{\min}(N)]\bar{\theta}$ if $B = N$,
2. $\bar{\theta} < p \leq \lambda^{\min}(B)\,\theta + [1-\lambda^{\min}(B)]\bar{\theta}$ if $B \in \mathcal{VC}(g)$,
3. $\lambda^{\max}(\neg B)\,\theta + [1-\lambda^{\max}(\neg B)]\bar{\theta} < p \leq \lambda^{\min}(B)\,\theta + [1-\lambda^{\min}(B)]\bar{\theta}$ if $B \notin \mathcal{VC}(g)$, $B \neq N$ and $B \neq \emptyset$,
4. $\bar{\theta} < p$ if $B = \emptyset$.

Note that $\lambda^{\min}(N)\,\theta + [1-\lambda^{\min}(N)]\bar{\theta}$ and $\lambda^{\max}(\neg B)\,\theta + [1-\lambda^{\max}(\neg B)]\bar{\theta}$, for $B \notin \mathcal{C}(g)$, $B \neq N$ and $B \neq \emptyset$, are both greater than $\bar{\theta}$ since $\theta > \bar{\theta}$. This implies that for all p such that $p \leq \bar{\theta}$ the only buyers configuration is N. For prices greater than $\bar{\theta}$ note that $B = \emptyset$ is always a buyers configuration, while other configurations may arise, depending on the family of functions λ, which eventually depends on the network structure. It is easy to see that if B is a buyers configuration for a given price that is neither a union of components of g of at least two members nor the empty set then it has to satisfy that $\lambda^{\max}(\neg B) < \lambda^{\min}(B)$.

From above, and by network-consistent beliefs of the producer, we can partition the set of possible prices into intervals, each interval yielding an expected number of consumers as in the statement of Lemma 4.1. Note that for any quality θ and price p there is always at least one buyers configuration: For $p \leq \bar{\theta}$ everybody buys the product, while for $p > \bar{\theta}$ nobody buying the product is a buyers configuration, while other configurations may arise. In particular, for $\bar{\theta} < p \leq \lambda^{\min}(N)\,\theta + \left[1-\lambda^{\min}(N)\right]\bar{\theta}$, $B = N$ is a buyers configuration.

Assume now that $\theta < \bar{\theta}$. Recall that, given a set of consumers $B \subseteq N$, $\lambda^{\max}(B) = \max_{i\in B}\lambda_i(B,g)$, $\lambda^{\min}(B) = \min_{i\in B}\lambda_i(B,g)$, $\lambda^{\max}(\neg B) = \max_{i\notin B}\lambda_i(B,g)$ and $\lambda^{\min}(\neg B) = \min_{i\notin B}\lambda_i(B,g)$. Again, if $B \in Q(\theta,p)$ then (i) for all $i \in B$: $\lambda_i(B,g)\,\theta + (1-\lambda_i(B,g))\,\bar{\theta} \geq p$ and (ii) for all $i \notin B$: $\lambda_i(B,g)\,\theta + (1-\lambda_i(B,g))\,\bar{\theta} < p$. Note that if $\theta < \bar{\theta}$ then (i) if $\lambda_i(B,g)\,\theta + (1-\lambda_i(B,g))\,\bar{\theta} \geq p$ for some i then $p \leq \bar{\theta}$ and at least one consumer per component will buy the good, and (ii) $\lambda_i(B,g)\,\theta + (1-\lambda_i(B,g))\,\bar{\theta} \geq \lambda_j(B,g)\,\theta + (1-\lambda_j(B,g))\,\bar{\theta}$ for i, $j \in N$ if and only if $\lambda_i(B,g) \leq \lambda_j(B,g)$. Hence, if $\theta < \bar{\theta}$ and $B \in Q(\theta,p)$ it has to be that:

- $\lambda^{max}(B)\,\theta + [1-\lambda^{max}(B)]\,\bar{\theta} \geq p$ if $B \neq \emptyset$, and

- $\lambda^{min}(\neg B)\,\theta + \left[1-\lambda^{min}(\neg B)\right]\bar{\theta} < p$ if $B \neq N$.

Note that $\lambda^{\min}(B)\,\theta + [1-\lambda^{\min}(B)]\bar{\theta}$ and $\lambda^{\max}(\neg B)\,\theta + [1-\lambda^{\max}(\neg B)]\bar{\theta}$, for any $B \subseteq N$, are both smaller than $\bar{\theta}$ since $\theta < \bar{\theta}$. This implies that for all p such that $p > \bar{\theta}$ the only buyers configuration is the empty set. For $p \leq \bar{\theta}$, any $B \neq N$ with $B \in Q(\theta, p)$ necessary satisfies $\lambda^{\min}(\neg B) > \lambda^{\max}(B)$ and includes at least one agent per component, since p has to be smaller than $\bar{\theta}$ as far as B is not a singleton. From all this, if $\theta > \bar{\theta}$ and $B \in Q(\theta, p)$:

1. $p \leq \lambda^{\max}(N)\,\theta + [1-\lambda^{\max}(N)]\bar{\theta}$ if $B = N$,

2. $\lambda^{\min}(\neg B)\,\theta + [1-\lambda^{\min}(\neg B)]\bar{\theta} < p \leq \lambda^{\max}(B)\,\theta + [1-\lambda^{\max}(B)]\bar{\theta}$, if $\lambda^{\max}(B)$ and $\lambda^{\min}(\neg B)$ are both strictly greater than 0. This implies that $|B \cap S| \geq 1$ for any $S \in C(g)$ and there is at least one S in $C(g)$ such that $|B \cap S| > 1$,

3. $\lambda^{\min}(\neg B)\,\theta + [1-\lambda^{\min}(\neg B)]\bar{\theta} < p \leq \bar{\theta}$ if $B \neq \emptyset$, $\lambda^{\min}(\neg B) > 0$ and $\lambda^{\max}(B) = 0$. Note that $\lambda^{min}(\neg B) > 0$ implies that B selects at least one agent in each possible component of g and that $\lambda^{max}(B) = 0$ implies that B cannot select more than one agent in each possible component of g. Therefore, $|B \cap S| = 1$ for all $S \in C(g)$,

4. $\bar{\theta} < p$ if $B = \emptyset$.

I show now that for any p between 0 and $\bar{\theta}$, given $\theta < \bar{\theta}$, there always exists one buyers configuration. In order to do that, I prove first the following two claims.

Claim 1. For $\theta < \bar{\theta}$ there exist at least one family of sets of buyers $\{B_1, B_2, ..., B_k, ..., B_K\}$ such that

1. $\lambda^{\min}(\neg B_k) \geq \lambda^{\max}(B_k)$, for all $k = 2, .., K$,

2. $B_1 = N$, $B_{k+1} \subsetneqq B_k$ for all $k = 1, .., K-1$, and $|B_K \cap S| = 1$ for all $S \in C(g)$,

3. $\lambda^{\max}(B_k) = \lambda^{\min}(\neg B_{k+1})$ for all $k = 1, ..., K-1$.

Proof of Claim 1. The proof is made by induction.

Step 1. Starting by $B_1 = N$, and assume that $\lambda^{\max}(N) > 0$. Let $M_1 = \arg\max_{i \in N} \lambda_i(N, g)$. If $|M_1| > 1$ choose a nonempty subset $I_1 \subseteq M_1$ such that (i) if there are two buyers i and j in I_1 they have to be non connected (directly or indirectly) in g and (ii) there is at least one agent in $N \backslash I_1$ connected to one agent in I_1. In words, I_1 chooses at most one agent for each component that is not a singleton. Define B_2 as $B_1 \backslash I_1$. I show now that $\lambda^{\max}(B_1) = \lambda^{\min}(\neg B_2)$ and that $\lambda^{\min}(\neg B_2) \geq \lambda^{\max}(B_2)$ since by definition $B_2 \subsetneqq B_1$ as far as $\lambda^{max}(N) > 0$[4].

Recall that, by definition of B_1 and B_2,

$$\lambda^{\min}(\neg B_2) = \min_{i \notin B_2} \lambda_i(B_2, g) = \min_{i \in I_1} \lambda_i(N \backslash I_1, g),$$

and

$$\lambda^{\max}(B_2) = \max_{i \in B_2} \lambda_i(B_2, g) = \max_{i \in N \backslash I_1} \lambda_i(N \backslash I_1, g).$$

Note that any agent i in I_1 extracts the same information from $N \backslash I_1$ as from N, since either I_1 is a singleton or any two agents in I_1 are disconnected. Hence, $\lambda^{\min}(\neg B_2) = \min_{i \in I_1} \lambda_i(N \backslash I_1, g) = \min_{i \in I_1} \lambda_i(N, g) = \lambda^{\max}(N)$, since $\lambda_i(N, g) = \lambda^{\max}(N)$ for all $i \in I_1$. This implies that $\lambda^{\min}(\neg B_2) = \lambda^{\max}(B_1)$. Finally, agents in $N \backslash I_1$ could be connected or not to an agent in I_1. For any agent j in $N \backslash I_1$ we know that $\lambda_j(N \backslash I_1) \leq \lambda_j(N) \leq \lambda^{\max}(N)$ by monotonicity of the λ functions. This implies that $\lambda^{\max}(B_2) = \max_{i \in N \backslash I_1} \lambda_i(N \backslash I_1, g) \leq \lambda^{\max}(N)$. As we have seen just before, $\lambda^{\max}(N) = \lambda^{\min}(\neg B_2)$ and therefore $\lambda^{\max}(B_2) \leq \lambda^{\min}(\neg B_2)$.

Recall that we have assumed from the beginning of Step 1 that $\lambda^{\max}(N) > 0$. Note that if $\lambda^{\max}(N) = 0$ then it has to be that g is an empty network, as nobody can extract information even when everybody is holding the good. But if g is an empty network, $\mathcal{C}(g) = \{\{1\}, \{2\}, ..., \{N\}\}$ or, in other words, all components are singletons, with $|N \cap S|$ for any component S being trivially equal to 1. Then, $B_1 = B_K$ and the family of sets of consumers is just a singleton.

Step 2. Assume now $k > 2$ and that there are sets of consumers $B_1, B_2, ..., B_{k-1}$ such that (i) $\lambda^{\min}(\neg B_{k'}) \geq \lambda^{\max}(B_{k'})$, for all $k' < k$, (ii) $B_{k'+1} \subsetneqq B_{k'}$ for all $k' < k-2$, and (iii) $\lambda^{\max}(B_{k'}) = \lambda^{\min}(\neg B_{k'+1}) > 0$ for all $k' < k-2$. I prove now that there is a $B_k \subsetneqq B_{k-1}$ such that (i) $\lambda^{\min}(\neg B_k) \geq \lambda^{\max}(B_k)$, and (ii) $\lambda^{\max}(B_{k-1}) = \lambda^{\min}(\neg B_k)$. Using a similar argument as in Step 1, let $M_{k-1} = \arg\max_{i \in B_{k-1}} \lambda_i(B_{k-1}, g)$. If $|M_{k-1}| > 1$ choose a nonempty subset $I_{k-1} \subseteq M_{k-1}$ such that if there are two agents i and j in I_{k-1} they have to be non connected (directly or indirectly) in g. Note that if $\lambda^{\max}(B_{k-1}) > 0$ there has to be at least one agent in $B_{k-1} \backslash I_{k-1}$ connected to one agent in I_{k-1}. Assume then that $\lambda^{max}(B_{k-1}) > 0$. (For $\lambda^{max}(B_{k-1}) = 0$ see Step 3.) Define B_k as $B_{k-1} \backslash I_{k-1}$. I show now that $\lambda^{\max}(B_{k-1}) = \lambda^{\min}(\neg B_k)$ and that $\lambda^{\min}(\neg B_k) \geq \lambda^{\max}(B_k)$ since by definition $B_k \subsetneqq B_{k-1}$.

Recall that,

$$\lambda^{\min}(\neg B_k) = \min_{i \notin B_k} \lambda_i(B_k, g) = \min_{i \notin B_{k-1} \backslash I_{k-1}} \lambda_i(B_{k-1} \backslash I_{k-1}, g),$$

and

$$\lambda^{\max}(B_k) = \max_{i \in B_k} \lambda_i(B_k, g) = \max_{i \in B_{k-1} \backslash I_{k-1}} \lambda_i(B_{k-1} \backslash I_{k-1}, g).$$

[4]By definition of the family of functions λ, if $\lambda^{max}(N) > 0$ then any agent in M_1 belongs to a component containing at least two agents.

Note that any agent i not in B_k is either an agent not in B_{k-1} or an agent in I_{k-1}, therefore $\lambda^{\min}(\neg B_k) = \min\{\lambda^{\min}(\neg B_{k-1}), \min_{i \in I_{k-1}} \lambda_i(B_{k-1} \backslash I_{k-1}, g)\}$. Any agent i in I_{k-1} extracts the same information from $B_{k-1} \backslash I_{k-1}$ as from B_{k-1}. This is so given that either I_{k-1} is a singleton or any two agents in I_{k-1} are disconnected. Hence, $\min_{i \in I_{k-1}} \lambda_i(B_{k-1} \backslash I_{k-1}, g) = \min_{i \in I_{k-1}} \lambda_i(B_{k-1}, g) = \lambda^{\max}(B_{k-1})$, since $\lambda_i(B_{k-1} \backslash \{i\}, g) = \lambda^{\max}(B_{k-1})$ for all $i \in I_{k-1}$. This implies that $\lambda^{\min}(\neg B_k) = \min\{\lambda^{\min}(\neg B_{k-1}), \lambda^{\max}(B_{k-1})\}$. By the induction hypothesis, $\lambda^{\min}(\neg B_{k-1}) \geq \lambda^{\max}(B_{k-1})$ and therefore $\lambda^{\min}(\neg B_k) = \lambda^{\max}(B_{k-1})$. Finally, agents in $B_{k-1} \backslash I_{k-1}$ could be connected or not to an agent in I_{k-1}. For any agent j in $B_{k-1} \backslash I_{k-1}$ we know that $\lambda_j(B_{k-1} \backslash I_{k-1}) \leq \lambda_j(B_{k-1})$, and that $\lambda_j(B_{k-1}) \leq \lambda^{\max}(B_{k-1})$ by monotonicity of the λ functions. This implies that $\lambda^{\max}(B_k) = \max_{j \in B_{k-1} \backslash I_{k-1}} \lambda_i(B_{k-1} \backslash I_{k-1}, g) \leq \lambda^{\max}(B_{k-1})$. As we have just seen, $\lambda^{\max}(B_{k-1}) = \lambda^{\min}(\neg B_k)$ and therefore $\lambda^{\max}(B_k) \leq \lambda^{\min}(\neg B_k)$.

Step 3. Assume $\{B_1, ..., B_K\}$ is generated starting with $B_1 = N$ and at each $k > 1$ removing I_{k-1} agents from B_{k-1}, where I_{k-1} is defined as above, until $\lambda^{\max}(B_{k-1}) = 0$. B_K is then the first k such that $\lambda^{\max}(B_k) = 0$ when applying this logarithm. Note that by steps 1 and 2 we know that

1. $\lambda^{\min}(\neg B_k) \geq \lambda^{\max}(B_k)$, for all $k = 2, .., K$,
2. $B_1 = N$, $B_{k+1} \subsetneqq B_k$ for all $k = 1, .., K-1$, and
3. $\lambda^{\max}(B_k) = \lambda^{\min}(\neg B_{k+1})$ for all $k = 1, ..., K-1$.

It remains to show that $|B_K \cap S| = 1$ for all $S \in C(g)$. By construction, B_K is defined as $\lambda^{\max}(B_K) = 0$, with $\lambda^{\max}(B_{K-1}) > 0$. First note that if $\lambda^{\max}(B_K) = 0$ this means that we can have at most one agent per component. As $\lambda^{\max}(B_k) > 0$ for all $k \leq K-1$ we know that B_k for all $k < K$ has at least two agents i and j that are connected in g. In particular, the set $\arg\max_{i \in B_k} \lambda_i(B_k, g) = M_k$ for each $k \leq K-1$ includes only players that are connected to at least another agent in B_k. Otherwise, their $\lambda's$ would be zero. By definition of I_k we can only select players in M_k not connected among them. This means, first, that any agent i disconnected from $B_k \backslash \{i\}$ in g cannot belong to M_k and therefore neither do they belong to I_k. This in turn means that any agent i disconnected from $B_k \backslash \{i\}$ belongs to B_{k+1} for all $k \leq K-1$. Second, at least one agent in each component remains in B_{k+1} since, as agents in I_k are disconnected, we remove from B_k at most one agent per component when computing B_{k+1}. Therefore, when there is a B_k such that $\lambda^{\max}(B_k) = 0$ when $\lambda^{\max}(B_{k-1}) > 0$ it can only mean that B_k has exactly one agent per component. Finally note that we arrive to such a B_k after a finite number of steps given that N is a finite set. This completes the proof of of Claim 1. ■

Claim 2. Assume $\{B_1, ..., B_K\}$ is a family of sets of buyers such that

1. $\lambda^{\min}(\neg B_k) \geq \lambda^{\max}(B_k)$, for all $k = 2, .., K$,

2. $B_1 = N$, $B_{k+1} \subsetneqq B_k$ for all $k = 1, .., K-1$, and $|B_K \cap S| = 1$ for all $S \in C(g)$,

3. $\lambda^{\max}(B_k) = \lambda^{\min}(\neg B_{k+1})$ for all $k = 1, ..., K-1$.

Assume that there exists one $B_p \in \{B_1, ..., B_K\}$ such that $\lambda^{\min}(\neg B_p) = \lambda^{\max}(B_p)$. Define a new family of sets of buyers $\{B'_1, ..., B'_{K-1}\}$ where $B'_k = B_k$, if $k < p$, and $B'_k = B_{k+1}$, otherwise. In words, $\{B'_1, ..., B'_{K-1}\}$ removes such a B_p from $\{B_1, ..., B_K\}$. This new family $\{B'_1, ..., B'_{K-1}\}$ holds the properties that

1. $\lambda^{\min}(\neg B'_k) \geq \lambda^{\max}(B'_k)$, for all $k = 2, .., K-1$,

2. $B_1 = N$, $B'_{k+1} \subsetneqq B'_k$ for all $k = 1, .., K-2$, and $|B'_{K-1} \cap S| = 1$ for all $S \in C(g)$,

3. $\lambda^{\max}(B'_k) = \lambda^{\min}\left(\neg B'_{k+1}\right)$ for all $k = 1, ..., K-2$.

Proof of Claim 2. Note that B_p cannot be equal to B_K as far as g is not the empty network, for $\lambda^{\max}(B_K) = 0$. The only problem is in the comparisons of properties across $B'_{p-1} = B_{p-1}$ and $B'_p = B_{p+1}$. By definition of the family $\{B_1, ..., B_K\}$, to which B_p belongs, $\lambda^{\max}(B_{p-1}) = \lambda^{\min}(\neg B_p)$ and $\lambda^{\max}(B_p) = \lambda^{\min}(\neg B_{p+1})$. This together with the fact that $\lambda^{\min}(\neg B_p) = \lambda^{\max}(B_p)$ implies that $\lambda^{\max}(B_{p-1}) = \lambda^{\min}(\neg B_{p+1})$. In other words, $\lambda^{\max}\left(B'_{p-1}\right) = \lambda^{\min}\left(\neg B'_p\right)$. Furthermore, since $B_{p+1} \subsetneqq B_p \subsetneqq B_{p-1}$ it is trivial that $B'_p \subsetneqq B'_{p-1}$.This completes the proof of Claim 2. ■

Consider the following algorithm. Choose a family of functions with the properties defined in the statement of Claim 1. We know by Claim 1 that at least one such family exists. Remove sequentially one set B of such a family with the property that $\lambda^{\min}(\neg B) = \lambda^{\max}(B)$ until for all remaining sets B in the original family it is true that $\lambda^{\min}(\neg B) > \lambda^{\max}(B)$. Note that this iterative process has a finite number of steps, as the family of sets of buyers is finite. By Claim 2, the surviving subfamily, denoted by $\{B_1, ..., B_K\}$ has the following three properties:

1. $\lambda^{\min}(\neg B_k) > \lambda^{\max}(B_k)$, for all $k = 2, .., K$,

2. $B_1 = N$, $B_{k+1} \subsetneqq B_k$ for all $k = 1, .., K-1$, and $|B_K \cap S| = 1$ for all $S \in C(g)$,

3. $\lambda^{\max}(B_k) = \lambda^{\min}(\neg B_{k+1})$ for all $k = 1, ..., K-1$.

By definition of a buyers configuration and property 1, any set B_k of the surviving family $\{B_1, ..., B_K\}$ with $k \geq 2$ is a buyers configuration if $\lambda^{\min}(\neg B_k)\,\theta + [1 - \lambda^{\min}(\neg B_k)]\bar{\theta} < p \leq \lambda^{\max}(B_k)\,\theta + [1 - \lambda^{\max}(B_k)]\bar{\theta}$. Recall that for $p \leq \lambda^{\max}(N)\,\theta + [1 - \lambda^{\max}(N)]\bar{\theta}$ the full set $N = B_1$ is the only buyers configuration. By property 3 of the surviving family $\{B_1, ..., B_K\}$, we can guarantee that for any price p in between 0 and $\bar{\theta}$, given $\theta < \bar{\theta}$, there is always one set from this family included in $Q(\theta, p)$. Given that we know that at least one family of sets with these properties exists, we know that a buyers configuration always exists. In particular, each family of sets with the properties 1 to 3 stated above **partitions the interval** 0 **to** $\bar{\theta}$ **of prices** into a finite number of intervals.

Finally, note that in general there might be different families of sets with the properties stated above, yielding sequences of buyers configurations. Take the partition defined by the intersections of partitions defined by all possible families of sets with the properties above. In each interval of this finest partition the expected number of consumers is constant by network-consistent beliefs of the producer. In particular, the lowest segment of prices yields N as the only possible buyers configuration, while the last segment yields buyers configurations include only one agent per component, therefore being the number of expected buyers equal to the number of components in g. This completes the proof of Lemma 4.1.■

Proof of Theorem 3.1.

In order to present the proof of this result, I solve the producer's decision backwards, as he decides first the level of quality θ^*, and afterwards, he chooses price. Let $\pi^*(\theta)$ denote the maximum value of profit when the producer chooses a quality level θ. The function $\pi^*(\theta)$ is thus built by letting the producer choose the price maximizing profit, given this quality level θ and consistent beliefs. In other words,

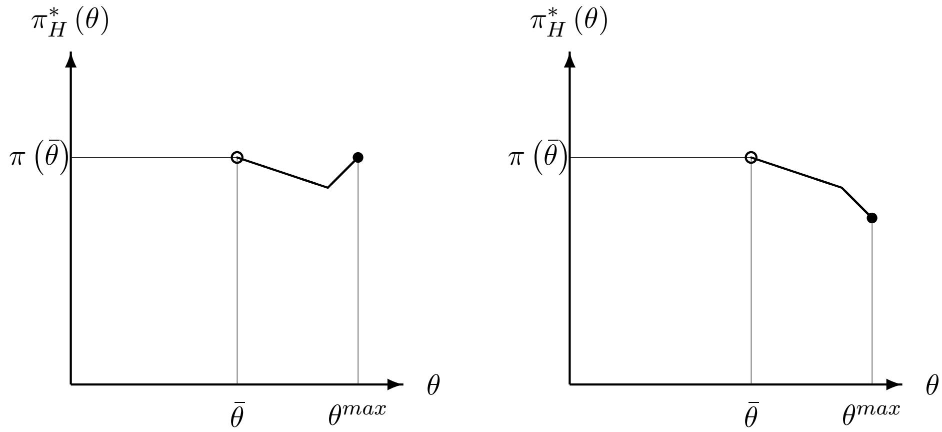
$$\pi^*(\theta) = \max_p q^d(\theta, p)(p - c\theta). \tag{4}$$

Note first that I can write

$$\pi^*(\theta) = \begin{cases} \pi_H^*(\theta), & \text{if } \theta > \bar{\theta}, \\ \pi(\bar{\theta}), & \text{if } \theta = \bar{\theta}, \\ \pi_L^*(\theta), & \text{otherwise,} \end{cases}$$

where $\pi_H^*(\theta) = \max_p q^d(\theta, p)(p - c\theta)$, $\pi_L^*(\theta) = \max_p q^d(\theta, p)(p - c\theta)$ for $\theta > \bar{\theta}$ and $\pi(\bar{\theta}) = n(1-c)\bar{\theta}$. Note that $\pi(\bar{\theta})$ is the profit when the producer chooses a quality level equal to $\bar{\theta}$. This will make all consumers indifferent between acquiring the good or not, independently of the set of consumers who are buying. In case of indifference, we have assumed that consumers acquire the good. I analyze the shape of $\pi^*(\theta)$ by analyzing each different part of it. This is done in the following two claims.

Claim 1. $\pi_H^*(\theta)$ tends to $\pi(\bar{\theta})$ as θ tends to $\bar{\theta}$ and is continouos on $(\bar{\theta}, \theta^{\max}]$. Furthermore, there are thresholds c_H and θ_H such that (i) if $c_H > c$ and $\theta^{\max} > \theta_H$ then the function $\pi_H^*(\theta)$ decreases to the right of $\bar{\theta}$ until it reaches a minimum in between $\bar{\theta}$ and $\theta^{\max}$ and increases all the way to the right until it reaches $\theta^{\max}$, and (ii) the function $\pi_H^*(\theta)$ is decreasing in $(\bar{\theta}, \theta^{\max}]$ otherwise. Figure 2 shows this statement graphically.



(i) $c_H > c$ and $\theta^{max} > \theta$ (ii) $c_H > c$ and $\theta^{max} < \theta$, or $c_H < c$

FIGURE 2

Proof of Claim 1. Assume then that $\theta > \bar{\theta}$. Recall that, by Lemma 4.1,

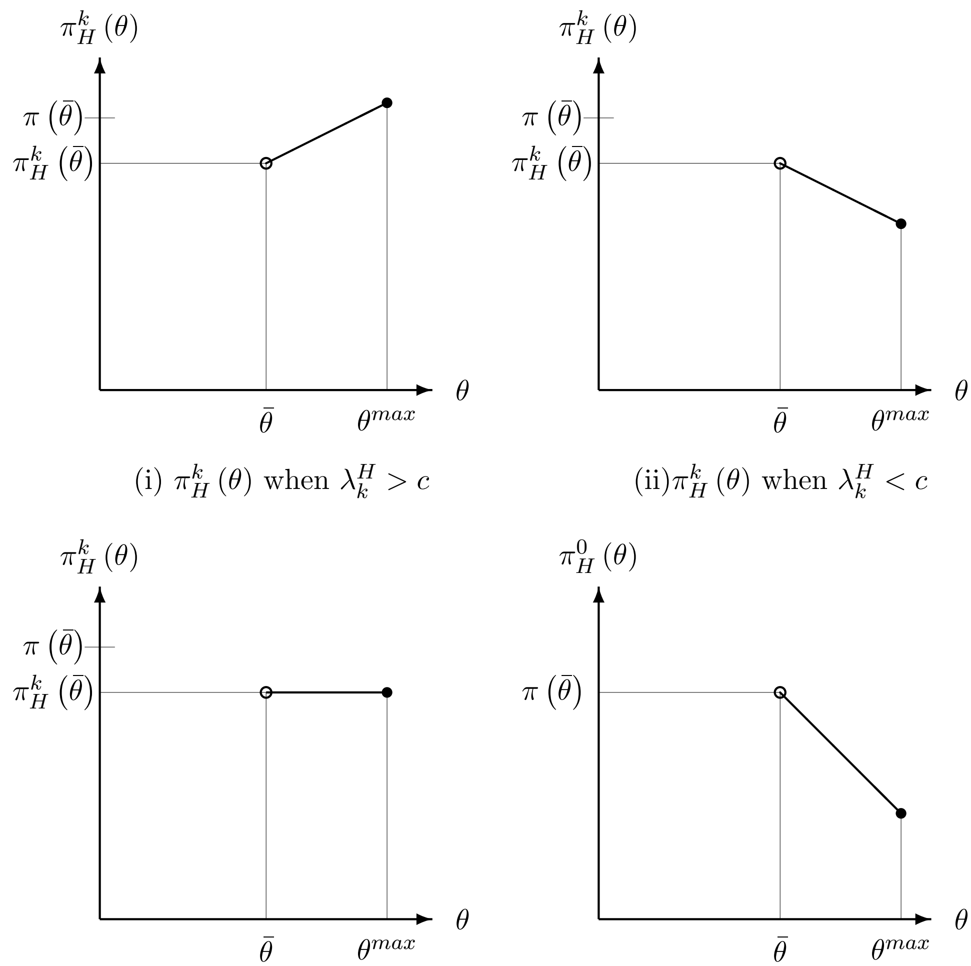
1. If $0 \leq p \leq \bar{\theta}$ then $Q(\theta, p) = \{N\}$ and $q^d(\theta, p) = n$.
2. If $p > \bar{\theta}$ then, there exist numbers $\{\lambda_k\}_{k=1}^K$ with $0 = \lambda_1 \leq \lambda_2 \leq ... \leq \lambda_k \leq ... \leq \lambda_K \leq \delta$ such that
   (a) if $\lambda_k\theta + (1 - \lambda_k)\bar{\theta} < p \leq \lambda_{k+1}\theta + (1 - \lambda_{k+1})\bar{\theta}$ then $Q(\theta, p)$ keeps the same structure and therefore $q^d(\theta, p) = q_{k+1}$ is constant, for $k \leq K - 1$.
   (b) if $p > \lambda_K\theta + (1 - \lambda_K)\bar{\theta}$ then $Q(\theta, p) = \{\emptyset\}$ and $q^d(\theta, p) = 0$.

Let $\{\lambda_k^H\}_{k=1}^K$ and $q^H(\lambda_k^H)$ for each k in $\{2, .., K\}$ be numbers satisfing that $\lambda_k^H\theta + (1 - \lambda_k^H)\bar{\theta} < p \leq \lambda_{k+1}^H\theta + (1 - \lambda_{k+1}^H)\bar{\theta}$ if and only if $q^d(\theta, p) = q^H(\lambda_{k+1}^H)$ for $k \geq 1$. It is easy to see that for each interval of prices where the expected demand is constant the producer will choose the highest price. Therefore, (i) the price chosen is a convex combination of θ^{max} and $\bar{\theta}$, and (ii) $\pi_H^*(\theta) = \max\{n(\bar{\theta} - c\theta), \max_{k\in\{1,..,K-1\}} \pi_H^k(\theta)\}$, where $\pi_H^k(\theta) = q^H(\lambda_k^H)\left[(\lambda_k^H - c)\theta + (1 - \lambda_k^H)\bar{\theta}\right]$, for each $k \in \{2, ..., K\}$. Note that each of these $\pi_H^k(\theta)$ have the following properties:

1. $\pi_H^k(\theta)$ is an affine function that could be monotonically increasing or decreasing on θ, depending whether $\lambda_k^H > c$ or $\lambda_k^H < c$, or constant, if $\lambda_k^H = c$.
2. $\pi_H^k(\theta)$ tends to $q^H(\lambda_k^H)(1 - c)\bar{\theta}$ as θ tends to $\bar{\theta}$. Note that $n(\bar{\theta} - c\theta)$ tends to $n(1 - c)\bar{\theta}$ as θ tends to $\bar{\theta}$ and that $n(1 - c)\bar{\theta} \geq q^H(\lambda_k^H)(1 - c)\bar{\theta}$.

3. $\pi_H^k(\theta^{\max})$ is equal to $q^H(\lambda_k^H)\left[(\lambda_k^H - c)\theta^{\max} + (1-\lambda_k^H)\bar{\theta}\right] > n(\bar{\theta} - c\theta^{\max})$ only if $\theta^{\max} > \frac{q^H(\lambda_k^H)(1-\lambda_k^H)}{q^H(\lambda_k^H)(\lambda_k^H - c)+cn}\bar{\theta}$ (note that $q^H(\lambda_k^H)(\lambda_k^H - c) + cn = c\left(n - q^H(\lambda_k^H)\right) + q^H(\lambda_k^H)\lambda_k^H$ is always positive, as $n \geq q^H(\lambda_k^H)$.)

Figure 3(i), (ii) and (iii) show each of the cases for $\pi_H^k(\theta)$ enumerated above.



(i) $\pi_H^k(\theta)$ when $\lambda_k^H > c$

(ii)$\pi_H^k(\theta)$ when $\lambda_k^H < c$

(iii) $\pi_H^k(\theta)$ when $\lambda_k^H = c$

(iv) $\pi_H^0(\theta) = n(\bar{\theta} - c\theta)$

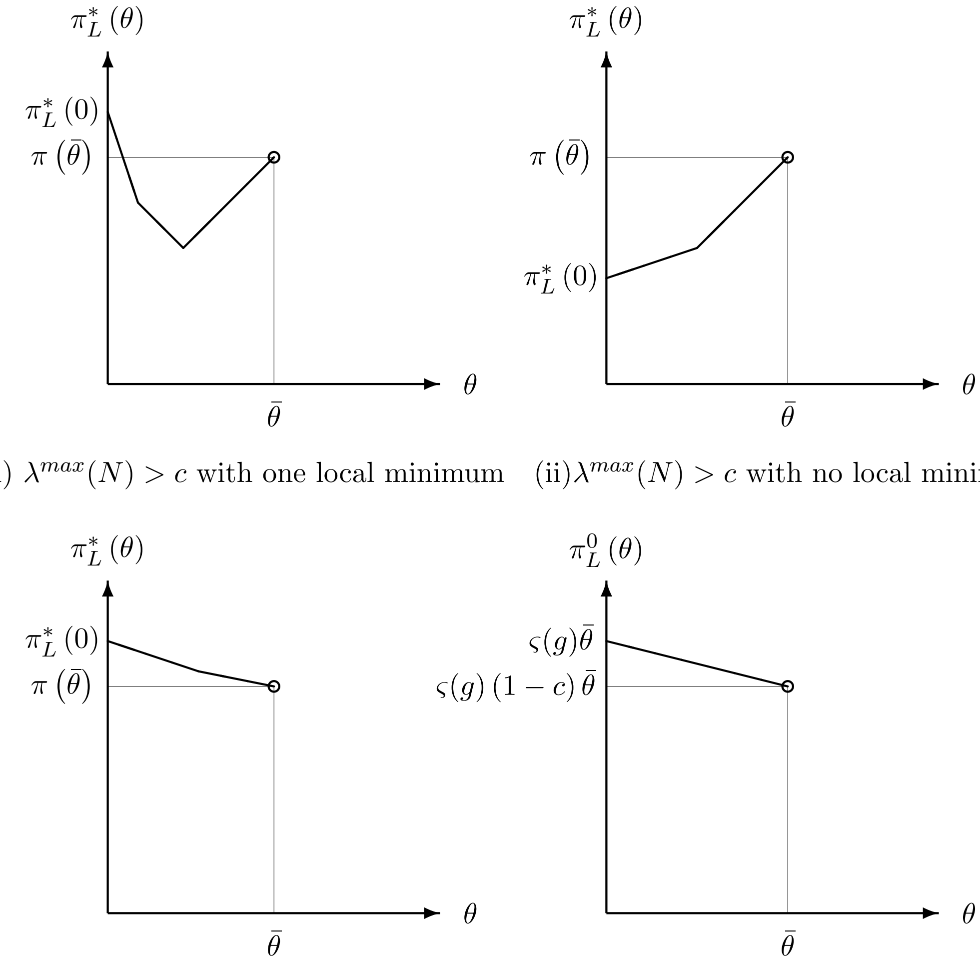
FIGURE 3

Figure 3 (iv) shows $\pi_H^0 = n(\bar{\theta} - c\theta)$ graphically. By definition, $\pi_H^*(\theta)$ is the upper envelope of π_H^0 and $\pi_H^k(\theta)$ for each $k \in \{1, ..., K\}$. Therefore, the function $\pi_H^*(\theta)$ is piecewise affine and continuous (recall that it is the upper envelope of affine functions). We know (by point 2 above) that in the neighborhood to the right of $\bar{\theta}$ (i) $\pi_H^*(\theta) = n\left(\bar{\theta} - c\theta\right)$ is decreasing, and (ii) it tends to $n(1-c)\bar{\theta}$ as θ tends to $\bar{\theta}$. If $\theta^{\max} \leq \frac{q^H\left(\lambda_k^H\right)\left(1-\lambda_k^H\right)}{q^H\left(\lambda_k^H\right)\left(\lambda_k^H - c\right)+cn}\bar{\theta}$ for all $k \in \{2, ..., K\}$ and given that $\pi_H^k(\theta) < \pi_H^0(\theta)$ as θ tends to $\bar{\theta}$, it can only be that $\pi_H^*(\theta) = n\left(\bar{\theta} - c\theta\right)$ and $\pi_H^*(\theta)$ is decreasing in θ in a neighborhood to the right of $\bar{\theta}$. Otherwise, if there is at least one k such that both $\theta^{\max} > \frac{q^H\left(\lambda_k^H\right)\left(1-\lambda_k^H\right)}{q^H\left(\lambda_k^H\right)\left(\lambda_k^H - c\right)+cn}\bar{\theta}$ and $\lambda_k^H > c$ then we know that $\pi_H^*(\theta)$ might start increasing at some point, and in fact it will for $\theta^{\max}$ big enough. Notice that, since it is true that for each k $\pi_H^k(\theta)$ is an affine function such that $\pi_H^k\left(\bar{\theta}\right) < \pi_H^0\left(\bar{\theta}\right)$, we know that $\pi_H^*(\theta)$ cannot be decreasing again once it has started increasing. Therefore, if there is at least one k such that $\lambda_k^H > c$ and $\theta^{\max}$ is big enough the function $\pi_H^*(\theta)$ reaches a minimum in between $\bar{\theta}$ and $\theta^{\max}$, and it is decreasing otherwise. In particular the threashold θ_H for $\theta^{\max}$ is the one for which

$$\max_{k:\lambda_k^H > c} q^H\left(\lambda_k^H\right)\left[\left(\lambda_k^H - c\right)\theta_H + \left(1-\lambda_k^H\right)\bar{\theta}\right] = \max_{k:\lambda_k^H \leq c} q^H\left(\lambda_k^H\right)\left[\left(\lambda_k^H - c\right)\theta_H + \left(1-\lambda_k^H\right)\bar{\theta}\right]$$

This completes the proof of Claim 1. ■

Claim 2. $\pi_L^*(\theta)$ tends to $\pi\left(\bar{\theta}\right)$ as θ tends to $\bar{\theta}$ and it is continuous in $[0, \bar{\theta})$. Furthermore, (i) if $c < \lambda^{\max}(N)$ then $\pi_L^*(\theta)$ is increasing in a neighborhood to the left of $\bar{\theta}$, with at most one local minimum in $\left(0, \bar{\theta}\right)$, and (ii) the function $\pi_L^*(\theta)$ is decreasing in $[0, \bar{\theta})$. Figure 4 (i), (ii) and (iii) show graphically each of the cases of the statement.



(i) $\lambda^{max}(N) > c$ with one local minimum (ii)$\lambda^{max}(N) > c$ with no local minimum

(iii) $\lambda^{max}(N) < c$ (iv) $\pi_L^0(\theta) = \varsigma(g)\left(\bar{\theta} - c\theta\right)$

FIGURE 4

Proof of Claim 2. Assume then that $\theta > \bar{\theta}$. Recall that, by Lemma 4.1, there exist numbers $\{\lambda_k\}_{k=0}^{K}$ with $0 \leq \lambda_K \leq \lambda_{K-1} \leq ... \leq \lambda_1 = \lambda^{\max}(N) \leq \delta$

1. If $0 \leq p \leq \lambda_1\theta + (1-\lambda_1)\bar{\theta}$ then $Q(\theta, p) = \{N\}$ and $q^d(\theta, p) = q_1 = n$.

2. If $\lambda_k\theta + (1-\lambda_k)\bar{\theta} < p \leq \lambda_{k+1}\theta + (1-\lambda_{k+1})\bar{\theta}$ then $Q(\theta, p)$ keeps the same structure and $q^d(\theta, p) = q_{k+1}$ is constant.

3. If $\lambda_K\theta + (1-\lambda_K)\bar{\theta} < p \leq \bar{\theta}$ then $q^d(\theta, p) = \varsigma(g)$.

4. If $p > \bar{\theta}$ then $Q(\theta, p) = \{\emptyset\}$ and $q^d(\theta, p) = 0$.

Let $\{\lambda_k^L\}_{k=1}^K$ and $q^L(\lambda_k^L)$ for each k in $\{2, ..., K\}$ be numbers satisfying that $\lambda_k^L \theta + (1 - \lambda_k^L)\bar{\theta} < p \leq \lambda_{k+1}^L \theta + (1 - \lambda_{k+1}^L)\bar{\theta}$ if and only if $q^d(\theta, p) = q^L(\lambda_{k+1}^L)$. Again, it is easy to see that for each interval of prices where the expected demand is constant the producer will choose the highest price. Therefore, (i) the price chosen is a convex combination of θ^{max} and $\bar{\theta}$, and (ii) $\pi_L^*(\theta) = \max\{\varsigma(g)(\bar{\theta} - c\theta), \max_{k \in \{1,..,K\}} \pi_L^k(\theta)\}$, where $\pi_L^k(\theta) = q^L(\lambda_k^L)\left[(\lambda_k^L - c)\theta + (1 - \lambda_k^L)\bar{\theta}\right]$, for each $k \in \{1, ..., K\}$, where $q^L(\lambda_1^k) = n$. Note that each of these $\pi_L^k(\theta)$ have the following properties:

1. $\pi_L^k(\theta)$ is an affine function that could be monotonically increasing or decreasing on θ, depending whether $\lambda_k^L > c$ or $\lambda_k^L < c$, or constant, if $\lambda_k^L = c$.
2. $\pi_L^k(\theta)$ tends to $q^L(\lambda_k^L)(1 - c)\bar{\theta}$ as θ tends to $\bar{\theta}$. Note that $n\left[(\lambda^{\max}(N) - c)\theta + (1 - \lambda^{\max}(N))\bar{\theta}\right]$ tends to $n(1 - c)\bar{\theta}$ as θ tends to $\bar{\theta}$ and that $n(1 - c)\bar{\theta} \geq q^L(\lambda_k^L)(1 - c)\bar{\theta}$ for any q_k^L.
3. $\pi_L^k(0)$ is equal to $q^L(\lambda_k^L)(1 - \lambda_k^L)\bar{\theta} > n\lambda^{\max}(N)\bar{\theta}$ only if $\lambda_k^L > 1 - \frac{n}{q^L(\lambda_k^L)}\lambda^{\max}(N)$.

By definition, $\pi_L^*(\theta)$ is the upper envelope of $\pi_L^0 = \varsigma(g)(\bar{\theta} - c\theta)$ and $\pi_L^k(\theta)$ for each $k \in \{1, ..., K\}$. Therefore, the function $\pi_L^*(\theta)$ is piecewise affine and continuous (recall that it is the upper envelope of affine functions). Furthermore, from Property 2 above it can only be that $\pi_L^*(\theta) = n\left[(\lambda^{\max}(N) - c)\theta + (1 - \lambda^{\max}(N))\bar{\theta}\right]$ in a neighborhood to the left of $\bar{\theta}$, which tends to $n(1 - c)\bar{\theta}$ as θ tends to $\bar{\theta}$. Note that $\pi_L^*(\theta)$ is strictly increasing in a neighborhood to the left of $\bar{\theta}$ if and only if $\lambda^{\max}(N) > c$, and decreasing otherwise.

If $c \geq \lambda^{\max}(N)$ we know that $c \geq \lambda_k^L$ for all $k \in \{1, ..., K\}$. Therefore, if $c \geq \lambda^{\max}(N)$ all functions $\pi_L^k(\theta)$ are decreasing in θ, which implies that $\pi_L^*(\theta)$ is decreasing in $[0, \bar{\theta})$. If there is at least one k such that both $\lambda_k^L > 1 - \frac{n}{q^L(\lambda_k^L)}\lambda^{\max}(N)$ and $\lambda_k^L > c$ then we know that $\pi_L^*(\theta)$ might start increasing as we approach $\theta = 0$ at some point. Notice that for each k, $\pi_L^k(\theta)$ is an affine function such that $\pi_L^k(\bar{\theta}) < \pi_L^1(\bar{\theta})$. We know then that $\pi_L^*(\theta)$ cannot be decreasing again as we are approaching $\theta = 0$, and therefore they will be at most one local minimum. This completes the proof of Claim 2. ■

From Claims 1 and 2 we can therefore conclude that $\pi^*(\theta)$ is a continuous function. We need to distinguish several cases. First note that from the proof of Claim 1 the threshold c_H is in fact a λ_k^H. Recall that λ_k^H is either a $\lambda^{\min}(B)$ or a $\lambda^{\max}(\neg B)$ for a $B \subset N$ that is a buyers configuration. By monotonicity of the λ functions, $\lambda^{\min}(B) \leq \lambda^{\min}(N) \leq \lambda^{\max}(N)$ and $\lambda^{\max}(\neg B) = \max_{i \notin B} \lambda_i(B, g) \leq \max_{i \notin B} \lambda_i(N, g) \leq \lambda^{\max}(N)$. This means that if $c > \lambda^{\max}(N)$ then c has to be greater than the threshold c_H. We only then need to consider 2 cases.

1. If $c > \lambda^{\max}(N)$ then the function $\pi^*(\theta)$ is decreasing in $[0, \theta^{\max}]$.

2. If $\lambda^{\max}(N) \geq c$ then the function $\pi^*(\theta)$ has a local maximum at $\theta = \bar{\theta}$.

In order to picture those cases graphically, the reader can combine figures 2 and 4 (parts (i), (ii) and (iii)), which show how the function $\pi^*(\theta)$ looks like to the right and to the left of $\bar{\theta}$, respectively.

Let θ^* be the global maximum of $\pi^*(\theta)$, where $\pi^*(\theta)$ can only be defined in the interval $[0, \theta^{\max}]$. Consider case 1. Since the function $\pi^*(\theta)$ is decreasing in the domain, we know that the only global maximum is $\theta^* = 0$.

Consider now case 2. We have a local maximum at $\pi^*\left(\bar{\theta}\right) = n(1-c)\bar{\theta}$. In order to determine whether $\bar{\theta}$ is a global maximum or not, we will have to compare this value with the values at the corners: $\pi^*(0)$ and $\pi^*(\theta^{\max})$. Recall that $\pi^*(0) = \pi_L^*(0) = \max_k q^L\left(\lambda_k^L\right)\left(1-\lambda_k^L\right)\bar{\theta}$. Then, $\pi^*(0) \leq \pi^*\left(\bar{\theta}\right)$ if and only if $c \leq 1 - \frac{1}{n}\max_k q^L\left(\lambda_k^L\right)\left(1-\lambda_k^L\right)$.

The case of $\pi^*(\theta^{\max})$ is more complex as it is not guaranteed that the function $\pi^*(\theta)$ is increasing to the right of $\bar{\theta}$. For the case when $\pi_H^*(\theta)$ is decreasing, we know that the global maximum will be either $\bar{\theta}$ or 0, but never $\theta^{\max}$. Recall that this case holds when either all λ_k^H are smaller or equal to c or when $\theta^{\max} < \theta^H$, where θ^H was defined as

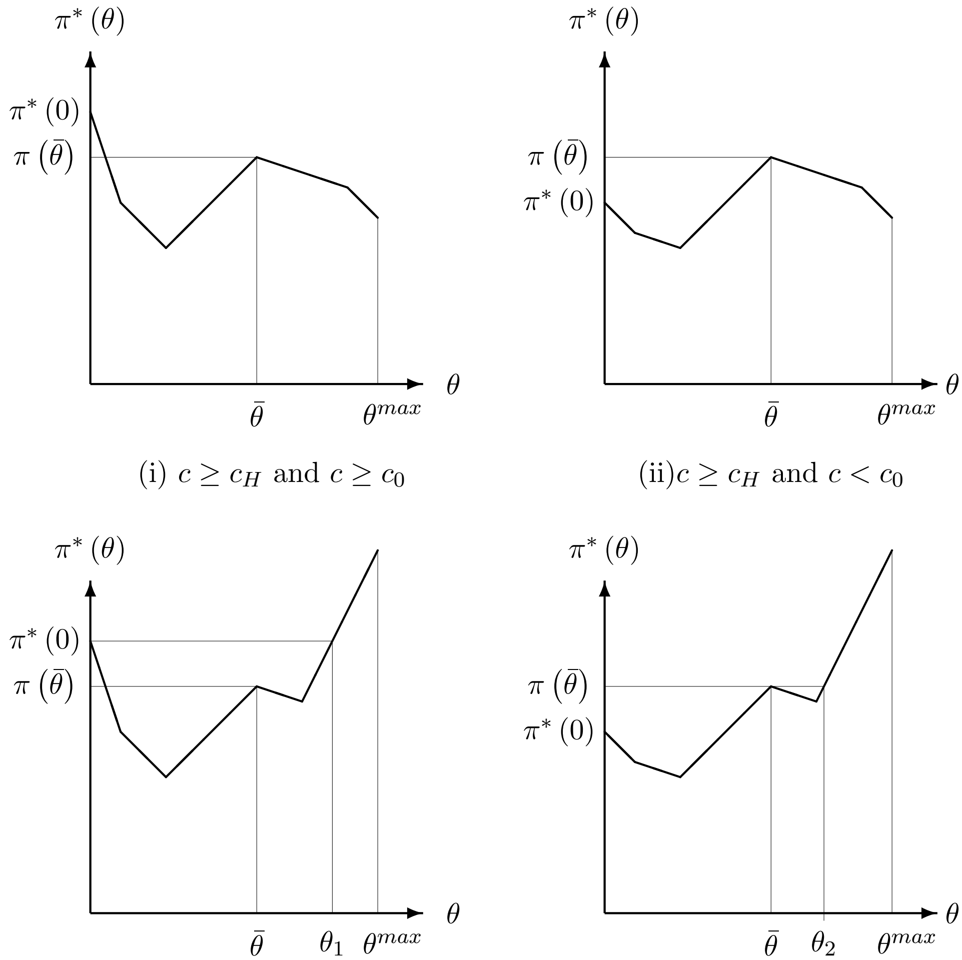
$$\max_{k:\lambda_k^H > c} q^H\left(\lambda_k^H\right)\left[\left(\lambda_k^H - c\right)\theta_H + \left(1-\lambda_k^H\right)\bar{\theta}\right] = \max_{k:\lambda_k^H \leq c} q^H\left(\lambda_k^H\right)\left[\left(\lambda_k^H - c\right)\theta_H + \left(1-\lambda_k^H\right)\bar{\theta}\right].$$

Note that each of the λ_k^H are equal to $\lambda^{\min}(B)$ or to $\lambda^{\max}(\neg B)$ for some B such that $\lambda^{\min}(B) > \lambda^{\max}(\neg B)$. If $c \geq \lambda^{\min}(B)$ for all B such that $\lambda^{\min}(B) > \lambda^{\max}(\neg B)$ we know that there is no $\lambda_k^H > c$.

Assume now that there is at least one λ_k^H such that $\lambda_k^H > c$ and $\theta^{\max} > \theta_H$. Define θ_1 and θ_2, both greater than θ_H, as the quality levels such that $\pi^*(\theta_1) = \pi^*(0)$ and $\pi^*(\theta_2) = \pi^*\left(\bar{\theta}\right)$.

Since both θ_1 and θ_2 are greater than θ_H it can only be that $\pi^*(\theta)$ is increasing to the right of both θ_1 and θ_2. This in turn implies that for any $\theta^{\max} \geq \theta_1$ it is true that $\pi^*(\theta^{\max}) \geq \pi^*(0)$ and that for any $\theta^{\max} \geq \theta_2$ it is true that $\pi^*(\theta^{\max}) \geq \pi^*\left(\bar{\theta}\right)$.

Combining figures 2 and 4 (i) and (ii) one can see that function $\pi^*(\theta)$ for $\lambda^{max}(N) > c$ looks as in Figure 5, where $c_H = \max\limits_{B \subset N:\lambda^{\min}(B) > \lambda^{\max}(B)} \lambda^{\min}(B) \geq c > 0$. Note that cases (ii) and (iv) are shown for the case of one local minimum to the left of $\bar{\theta}$, although there could be no local minimum at all, and that (iii) and (iv) are drawn assuming that θ^{max} is greater than the corresponding thresholds θ_1 and θ_2.



(i) $c \geq c_H$ and $c \geq c_0$

(ii)$c \geq c_H$ and $c < c_0$

(iii) $c < c_H$ and $c \geq c_0$

(iv) $c < c_H$ and $c < c_0$

FIGURE 5

From above, the optimal choice of quality level by a network-consistent producer is as follows:

1. Assume $c \geq \lambda^{\max}(N)$. Then $\theta^* = 0$

2. Assume $\lambda^{\max}(N) \geq c > \max_{B \subset N : \lambda^{\min}(B) > \lambda^{\max}(B)} \lambda^{\min}(B)$. Then

$$\theta^* = \begin{cases} 0, & \text{if } c \geq 1 - \frac{1}{n} \max_k q^L\left(\lambda_k^L\right)\left[1 - \lambda_k^L\right], \\ \bar{\theta}, & \text{if } c \leq 1 - \frac{1}{n} \max_k q^L\left(\lambda_k^L\right)\left[1 - \lambda_k^L\right]. \end{cases}$$

3. Assume now that $\max\limits_{B \subset N : \lambda^{\min}(B) > \lambda^{\max}(B)} \lambda^{\min}(B) \geq c > 0$. Then

$$\theta^* = \begin{cases} 0, & \text{if } c \geq 1 - \frac{1}{n} \max_k q^L\left(\lambda_k^L\right)\left[1 - \lambda_k^L\right] \text{ and } \theta^{\max} \leq \theta_1, \\ \bar{\theta}, & \text{if } c \leq 1 - \frac{1}{n} \max_k q^L\left(\lambda_k^L\right)\left[1 - \lambda_k^L\right] \text{ and } \theta^{\max} \leq \theta_2, \\ \theta^{\max}, & \begin{array}{c} \text{either if } c \geq 1 - \frac{1}{n} \max_k q^L\left(\lambda_k^L\right)\left[1 - \lambda_k^L\right] \text{ and } \theta^{\max} \geq \theta_1, \\ \text{or, if } c \leq 1 - \frac{1}{n} \max_k q^L\left(\lambda_k^L\right)\left[1 - \lambda_k^L\right] \text{ and } \theta^{\max} \geq \theta_2. \end{array} \end{cases}$$

Since $c_0 = 1 - \frac{1}{n} \max_k q^L\left(\lambda_k^L\right)\left[1 - \lambda_k^L\right]$, this completes the proof of Theorem 3.1. $\square$